Dynamic Mechanism Design Tutorial

Susan Athey

July 7, 2009

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions

Preview

- Static “expected externality” (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment
  - Athey-Segal: IR constraints can be satisfied in an ergodic Markov model with patient agents

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment
  - Athey-Segal: IR constraints can be satisfied in an ergodic Markov model with patient agents
    - Model: incorporate an "exit option" that can be taken in each period

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment
  - Athey-Segal: IR constraints can be satisfied in an ergodic Markov model with patient agents
    - Model: incorporate an "exit option" that can be taken in each period
    - In this case the mechanism can be self-enforcing

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
  - Implements efficient decisions
  - Has a balanced budget
  - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment
  - Athey-Segal: IR constraints can be satisfied in an ergodic Markov model with patient agents
    - Model: incorporate an "exit option" that can be taken in each period
    - In this case the mechanism can be self-enforcing
  - Recursive Mechanisms with Transfers

Preview

- Static "expected externality" (AGV) mechanism is not IC - does not prevent contingent deviations
- Athey-Segal constructs a dynamic mechanism that
    - Implements efficient decisions
    - Has a balanced budget
    - IC – prevents contingent deviations
- Dynamic Games without Enforcement/Commitment
    - Athey-Segal: IR constraints can be satisfied in an ergodic Markov model with patient agents
        - Model: incorporate an "exit option" that can be taken in each period
        - In this case the mechanism can be self-enforcing
    - Recursive Mechanisms with Transfers
- Dynamic Games without Enforcement and without Transfers

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2}x_t^2/\theta_S$ in each period $t = 1, 2$.

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2} x_t^2 / \theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \quad \chi_2(\theta_S, \theta_B) = \theta_S \theta_B$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2} x_t^2 / \theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \quad \chi_2(\theta_S, \theta_B) = \theta_S \theta_B$
  - Note: B infers θ_S from $\chi_1(\theta_S)$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2}x_t^2/\theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \quad \chi_2(\theta_S, \theta_B) = \theta_S\theta_B$
  - Note: B infers θ_S from $\chi_1(\theta_S)$
- Team Transfers (not BB):

$$\begin{aligned}\gamma_S(\hat{\theta}_B, \hat{\theta}_S) &= \chi_1(\hat{\theta}_S) + \hat{\theta}_B \cdot \chi_2\left(\hat{\theta}_S, \hat{\theta}_B\right), \\ \gamma_B\left(\hat{\theta}_B, \hat{\theta}_S\right) &= -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right).\end{aligned}$$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2}x_t^2/\theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \chi_2(\theta_S, \theta_B) = \theta_S\theta_B$
  - Note: B infers θ_S from $\chi_1(\theta_S)$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2} x_t^2 / \theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \chi_2(\theta_S, \theta_B) = \theta_S \theta_B$
  - Note: B infers θ_S from $\chi_1(\theta_S)$
- Static AGV ("Expected Externality")–note beliefs are CK:

$$\begin{aligned}
\gamma_S(\hat{\theta}_S) &= \chi_1(\hat{\theta}_S) + \mathbb{E}_{\tilde{\theta}_B}\left[\tilde{\theta}_B \cdot \chi_2\left(\hat{\theta}_S, \tilde{\theta}_B\right)\right], \\
\gamma_B\left(\hat{\theta}_B\right) &= -\mathbb{E}_{\tilde{\theta}_S}\left[c\left(\chi_2(\tilde{\theta}_S, \hat{\theta}_B), \tilde{\theta}_S\right)\right].
\end{aligned}$$

A Simple Example

1a. Seller learns θ_S
1b. Buyer buys x_1 from Seller
2a. Buyer learns θ_B
2b. Buyer buys x_2 from Seller

- Buyer's total value: $x_1 + \theta_B x_2$
- Seller's cost $c(x_t, \theta_S) = \frac{1}{2} x_t^2 / \theta_S$ in each period $t = 1, 2$.
- Efficient plan: $\chi_1(\theta_S) = \theta_S, \chi_2(\theta_S, \theta_B) = \theta_S \theta_B$
  - Note: B infers θ_S from $\chi_1(\theta_S)$
- Static AGV ("Expected Externality")–note beliefs are CK:

$$\begin{aligned} \gamma_S(\hat{\theta}_S) &= \chi_1(\hat{\theta}_S) + \mathbb{E}_{\tilde{\theta}_B}\left[\tilde{\theta}_B \cdot \chi_2\left(\hat{\theta}_S, \tilde{\theta}_B\right)\right], \\ \gamma_B\left(\hat{\theta}_B\right) &= -\mathbb{E}_{\tilde{\theta}_S}\left[c\left(\chi_2(\tilde{\theta}_S, \hat{\theta}_B), \tilde{\theta}_S\right)\right]. \end{aligned}$$

- Total payment from B to S: $\psi_S\left(\theta_B, \theta_S\right) = -\psi_B(\theta_B, \theta_S) = \gamma_S(\theta_S) - \gamma_B(\theta_B)$

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B \left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right)?$$

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right)?$$

- But then S, who pays γ_B, would lie to manipulate it!

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right) ?$$

- But then S, who pays γ_B, would lie to manipulate it!
- Let B's $\gamma_B =$ *change* in S's expected [CP] cost induced by B's report:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right) + \mathbb{E}_{\tilde{\theta}_B}\left[c\left(\chi_2(\hat{\theta}_S, \tilde{\theta}_B), \hat{\theta}_S\right)\right].$$

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right)?$$

- But then S, who pays γ_B, would lie to manipulate it!
- Let B's $\gamma_B =$ *change* in S's expected [CP] cost induced by B's report:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right) + \mathbb{E}_{\tilde{\theta}_B}\left[c\left(\chi_2(\hat{\theta}_S, \tilde{\theta}_B), \hat{\theta}_S\right)\right].$$

- γ_B lets B internalize S's cost $\Rightarrow$ B will not lie regardless of what θ_S he infers

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right)?$$

- But then S, who pays γ_B, would lie to manipulate it!
- Let B's $\gamma_B =$ *change* in S's expected [CP] cost induced by B's report:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right) + \mathbb{E}_{\tilde{\theta}_B}\left[c\left(\chi_2(\hat{\theta}_S, \tilde{\theta}_B), \hat{\theta}_S\right)\right].$$

- γ_B lets B internalize S's cost $\Rightarrow$ B will not lie regardless of what θ_S he infers
- $\mathbb{E}_{\tilde{\theta}_B}\ \gamma_B(\tilde{\theta}_B, \theta_S) \equiv 0 \Rightarrow$ having S pay γ_B does not alter S's incentives if B is truthful

Building an IC Dynamic Mechanism

- Instead of $\mathbb{E}_{\theta_S}$, calculate γ_B using S's reported $\hat{\theta}_S$:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right)?$$

- But then S, who pays γ_B, would lie to manipulate it!
- Let B's $\gamma_B =$ *change* in S's expected [CP] cost induced by B's report:

$$\gamma_B\left(\hat{\theta}_S, \hat{\theta}_B\right) = -c\left(\chi_2(\hat{\theta}_S, \hat{\theta}_B), \hat{\theta}_S\right) + \mathbb{E}_{\tilde{\theta}_B}\left[c\left(\chi_2(\hat{\theta}_S, \tilde{\theta}_B), \hat{\theta}_S\right)\right].$$

- γ_B lets B internalize S's cost $\Rightarrow$ B will not lie regardless of what θ_S he infers
- $\mathbb{E}_{\tilde{\theta}_B}\ \gamma_B(\tilde{\theta}_B, \theta_S) \equiv 0 \Rightarrow$ having S pay γ_B does not alter S's incentives if B is truthful
- Thus letting $\psi_S\left(\theta_B, \theta_S\right) = -\psi_B(\theta_B, \theta_S) = \gamma_S(\theta_S) - \gamma_B(\theta_B, \theta_S)$ yields a BIC balanced-budget mechanism

Generalizing Example: Add Another Period of Trade

- Seller type constant across repetitions, buyer type serially correlated

  1a. Seller learns θ_S
  1b. Buyer buys x_1 from Seller
  2a. Buyer learns $\theta_{B,2}$
  2b. Buyer buys x_2 from Seller
  3a. Buyer learns $\theta_{B,3}$
  3b. Buyer buys x_3 from Seller

Generalizing Example: Add Another Period of Trade

- Seller type constant across repetitions, buyer type serially correlated

  1a. Seller learns θ_S
  1b. Buyer buys x_1 from Seller
  2a. Buyer learns $\theta_{B,2}$
  2b. Buyer buys x_2 from Seller
  3a. Buyer learns $\theta_{B,3}$
  3b. Buyer buys x_3 from Seller

- Pay buyer $\gamma_B =$ *change* in S's expected cost induced by B's report in each repetition. Implies $t = 3$ incentive payment to buyer is:

$$\begin{aligned}\gamma_{B,3}\left(\hat{\theta}_S, \hat{\theta}_{B,3}, \hat{\theta}_{B,2}\right) &= -c\left(\chi_3(\hat{\theta}_S, \hat{\theta}_{B,3}), \hat{\theta}_S\right) \\ &\quad + \mathbb{E}_{\tilde{\theta}_{B,3}}\left[\, c\left(\chi_1(\hat{\theta}_S, \tilde{\theta}_{B,3}), \hat{\theta}_S\right) \middle| \, \hat{\theta}_{B,2}\right].\end{aligned}$$

Generalizing Example: Add Another Period of Trade

- Seller type constant across repetitions, buyer type serially correlated
  - 1a. Seller learns θ_S
  - 1b. Buyer buys x_1 from Seller
  - 2a. Buyer learns $\theta_{B,2}$
  - 2b. Buyer buys x_2 from Seller
  - 3a. Buyer learns $\theta_{B,3}$
  - 3b. Buyer buys x_3 from Seller
- Pay buyer $\gamma_B =$ *change* in S's expected cost induced by B's report in each repetition. Implies $t = 3$ incentive payment to buyer is:

$$\begin{aligned}\gamma_{B,3}\left(\hat{\theta}_S, \hat{\theta}_{B,3}, \hat{\theta}_{B,2}\right) &= -c\left(\chi_3(\hat{\theta}_S, \hat{\theta}_{B,3}), \hat{\theta}_S\right) \\ &\quad + \mathbb{E}_{\tilde{\theta}_{B,3}}\left[c\left(\chi_1(\hat{\theta}_S, \tilde{\theta}_{B,3}), \hat{\theta}_S\right) \middle| \hat{\theta}_{B,2}\right].\end{aligned}$$

  - In $t = 2$, buyer sees add'l effect of reporting $\hat{\theta}_{B,2}$: affects beliefs

Generalizing Example: Add Another Period of Trade

- Seller type constant across repetitions, buyer type serially correlated
  - 1a. Seller learns θ_S
  - 1b. Buyer buys x_1 from Seller
  - 2a. Buyer learns $\theta_{B,2}$
  - 2b. Buyer buys x_2 from Seller
  - 3a. Buyer learns $\theta_{B,3}$
  - 3b. Buyer buys x_3 from Seller
- Pay buyer γ_B = *change* in S's expected cost induced by B's report in each repetition. Implies $t = 3$ incentive payment to buyer is:

$$\begin{aligned}\gamma_{B,3}\left(\hat{\theta}_S, \hat{\theta}_{B,3}, \hat{\theta}_{B,2}\right) &= -c\left(\chi_3(\hat{\theta}_S, \hat{\theta}_{B,3}), \hat{\theta}_S\right) \\ &\quad + \mathbb{E}_{\tilde{\theta}_{B,3}}\left[c\left(\chi_1(\hat{\theta}_S, \tilde{\theta}_{B,3}), \hat{\theta}_S\right) \middle| \hat{\theta}_{B,2}\right].\end{aligned}$$

  - In $t = 2$, buyer sees add'l effect of reporting $\hat{\theta}_{B,2}$: affects beliefs
  - "Correction term" was there to neutralize seller's incentive to manipulate $\gamma_{B,3}$ through report of $\hat{\theta}_S$

Generalizing Example: Add Another Period of Trade

- Seller type constant across repetitions, buyer type serially correlated
  - 1a. Seller learns θ_S
  - 1b. Buyer buys x_1 from Seller
  - 2a. Buyer learns $\theta_{B,2}$
  - 2b. Buyer buys x_2 from Seller
  - 3a. Buyer learns $\theta_{B,3}$
  - 3b. Buyer buys x_3 from Seller
- Pay buyer $\gamma_B =$ *change* in S's expected cost induced by B's report in each repetition. Implies $t = 3$ incentive payment to buyer is:

$$\begin{aligned}\gamma_{B,3}\left(\hat{\theta}_S, \hat{\theta}_{B,3}, \hat{\theta}_{B,2}\right) &= -c\left(\chi_3(\hat{\theta}_S, \hat{\theta}_{B,3}), \hat{\theta}_S\right) \\ &\quad + \mathbb{E}_{\tilde{\theta}_{B,3}}\left[c\left(\chi_1(\hat{\theta}_S, \tilde{\theta}_{B,3}), \hat{\theta}_S\right) \middle| \hat{\theta}_{B,2}\right].\end{aligned}$$

  - In $t = 2$, buyer sees add'l effect of reporting $\hat{\theta}_{B,2}$: affects beliefs
  - "Correction term" was there to neutralize seller's incentive to manipulate $\gamma_{B,3}$ through report of $\hat{\theta}_S$
  - But in period 2, this correction distorts buyer's incentives

The Model

- In each period $t = 1, 2, \ldots$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i
- Histories: $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t = \prod_{\tau=1}^{t} \prod_i \Theta_{i,t}$; similarly $x^t \in X^t$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i
- Histories: $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t = \prod_{\tau=1}^{t} \prod_i \Theta_{i,t}$; similarly $x^t \in X^t$
- Technology: $\theta_t \sim \nu_t \left(\cdot | x^{t-1}, \theta^{t-1} \right)$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i
- Histories: $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t = \prod_{\tau=1}^{t} \prod_i \Theta_{i,t}$; similarly $x^t \in X^t$
- Technology: $\theta_t \sim \nu_t \left(\cdot | x^{t-1}, \theta^{t-1} \right)$
- Preferences: Agent i's utility

$$\sum_{t=1}^{\infty} \delta^t \left[u_{i,t}(x^t, \theta^t) + y_{i,t} \right]$$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i
- Histories: $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t = \prod_{\tau=1}^{t} \prod_i \Theta_{i,t}$; similarly $x^t \in X^t$
- Technology: $\theta_t \sim \nu_t\left(\cdot | x^{t-1}, \theta^{t-1}\right)$
- Preferences: Agent i's utility

$$\sum_{t=1}^{\infty} \delta^t \left[u_{i,t}(x^t, \theta^t) + y_{i,t}\right]$$

  - $0 < \delta < 1$

The Model

- In each period $t = 1, 2, \ldots$
  1. Each agent $i = 1, \ldots, N$ privately observes signal $\theta_{i,t} \in \Theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i makes private decision $x_{i,t} \in X_{i,t}$
  4. Mechanism makes public decision $x_{0,t} \in X_{0,t}$, transfers $y_{i,t} \in \mathbb{R}$ to each i
- Histories: $\theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t = \prod_{\tau=1}^{t} \prod_i \Theta_{i,t}$; similarly $x^t \in X^t$
- Technology: $\theta_t \sim \nu_t \left(\cdot | x^{t-1}, \theta^{t-1}\right)$
- Preferences: Agent i's utility

$$\sum_{t=1}^{\infty} \delta^t \left[u_{i,t}(x^t, \theta^t) + y_{i,t}\right]$$

  - $0 < \delta < 1$
  - $u_{i,t}$ uniformly bounded

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \longrightarrow X_t$

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \rightarrow X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ
- Transfers: $\psi_{i,t} : \Theta^t \to \mathbb{R}$; PDV $\Psi_i(\theta) = \sum_{t=0}^{\infty} \delta^t \psi_{i,t}(\theta^t)$

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ
- Transfers: $\psi_{i,t} : \Theta^t \to \mathbb{R}$; PDV $\Psi_i(\theta) = \sum_{t=0}^{\infty} \delta^t \psi_{i,t}(\theta^t)$
  - Measurable, uniformly bounded

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \rightarrow X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ
- Transfers: $\psi_{i,t} : \Theta^t \rightarrow \mathbb{R}$; PDV $\Psi_i(\theta) = \sum_{t=0}^{\infty} \delta^t \psi_{i,t}(\theta^t)$
  - Measurable, uniformly bounded
  - Budget balance: $\sum_i \psi_{i,t}(\theta) \equiv 0$

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ
- Transfers: $\psi_{i,t} : \Theta^t \to \mathbb{R}$; PDV $\Psi_i(\theta) = \sum_{t=0}^{\infty} \delta^t \psi_{i,t}(\theta^t)$
  - Measurable, uniformly bounded
  - Budget balance: $\sum_i \psi_{i,t}(\theta) \equiv 0$
- Information Disclosure: All announcements are public

Direct Mechanisms

- Measurable decision plan: $\chi_t : \Theta^t \to X_t$
  - $\chi_{0,t}$ are prescribed public decisions
  - $\chi_{i,t}$ are recommended private decisions for agent $i \geq 1$
- Decision plan induces stochastic process $\mu[\chi]$ on Θ
- Transfers: $\psi_{i,t} : \Theta^t \to \mathbb{R}$; PDV $\Psi_i(\theta) = \sum_{t=0}^{\infty} \delta^t \psi_{i,t}(\theta^t)$
  - Measurable, uniformly bounded
  - Budget balance: $\sum_i \psi_{i,t}(\theta) \equiv 0$
- Information Disclosure: All announcements are public
  - Disclosing less less would preserve equilibrium as long as agents can still infer recommended private decisions

Strategies

- Agent i's strategy defines

Strategies

- Agent i's strategy defines
  - Reporting plan $\beta_{i,t} : \Theta_i^t \times \Theta_{-i}^{t-1} \longrightarrow \Theta_{i,t}$

Strategies

- Agent i's strategy defines
  - Reporting plan $\beta_{i,t} : \Theta_i^t \times \Theta_{-i}^{t-1} \longrightarrow \Theta_{i,t}$
  - Private action plan $\alpha_{i,t} : \Theta_i^t \times \Theta_{-i}^t \longrightarrow X_{i,t}$

Strategies

- Agent i's strategy defines
  - Reporting plan $\beta_{i,t} : \Theta_i^t \times \Theta_{-i}^{t-1} \to \Theta_{i,t}$
  - Private action plan $\alpha_{i,t} : \Theta_i^t \times \Theta_{-i}^t \to X_{i,t}$
- Strategy also defines behavior following agent's own deviations, but this is irrelevant for the normal form

Strategies

- Agent i's strategy defines
  - Reporting plan $\beta_{i,t} : \Theta_i^t \times \Theta_{-i}^{t-1} \to \Theta_{i,t}$
  - Private action plan $\alpha_{i,t} : \Theta_i^t \times \Theta_{-i}^t \to X_{i,t}$
- Strategy also defines behavior following agent's own deviations, but this is irrelevant for the normal form
- Strategy is *truthful-obedient* if for all θ^t,

$$\begin{aligned} \beta_{i,t}(\theta_i^t, \theta_{-i}^{t-1}) &= \theta_{i,t}, \\ \alpha_{i,t}(\theta^t) &= \chi_{i,t}\left(\theta^t\right) \end{aligned}$$

Balanced Team Mechanism

- $U_i\left(\chi^*(\theta),\theta\right)=\sum_{t=1}^{\infty}\delta^t u_{i,t}\left(\chi_t\left(\tilde{\theta}^t\right),\tilde{\theta}\right)$

Balanced Team Mechanism

- $U_i\left(\chi^*(\theta),\theta\right) = \sum_{t=1}^{\infty} \delta^t u_{i,t}\left(\chi_t\left(\tilde{\theta}^t\right),\tilde{\theta}\right)$
- Efficient decision χ^*: $\max_\chi \mathbb{E}_{\tilde{\theta}}^{\mu[\chi]}\left[\sum_i U_i\left(\chi^*(\theta),\theta\right)\right]$

Balanced Team Mechanism

- $U_i\left(\chi^*(\theta),\theta\right)=\sum_{t=1}^{\infty}\delta^t u_{i,t}\left(\chi_t\left(\tilde{\theta}^t\right),\tilde{\theta}\right)$
- Efficient decision χ^*: $\max_\chi \mathbb{E}_{\tilde{\theta}}^{\mu[\chi]}\left[\sum_i U_i\left(\chi^*(\theta),\theta\right)\right]$
- Balanced Team Transfers:

$$\psi_{i,t}^B(\theta^t) = \gamma_{i,t}\left(\theta_{i,t},\theta^{t-1}\right)-\frac{1}{I-1}\sum_{j\neq i}\gamma_{j,t}(\theta_{j,t},\theta^{t-1}),\ \text{where}$$

$$\gamma_{j,t}(\theta_{j,t},\theta^{t-1}) = \delta^{-t}\left(\begin{array}{c}\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\theta_{j,t},\theta^{t-1}}\left[\sum_{i\neq j}U_i\left(\chi^*(\theta),\theta\right)\right]\\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\theta^{t-1}}\left[\sum_{i\neq j}U_i\left(\chi^*(\theta),\theta\right)\right]\end{array}\right)$$

Balanced Team Mechanism

- $U_i\left(\chi^*(\theta),\theta\right) = \sum_{t=1}^{\infty} \delta^t u_{i,t}\left(\chi_t\left(\tilde{\theta}^t\right),\tilde{\theta}\right)$
- Efficient decision χ^*: $\max_\chi \mathbb{E}_{\tilde{\theta}}^{\mu[\chi]}\left[\sum_i U_i\left(\chi^*(\theta),\theta\right)\right]$
- Balanced Team Transfers:

$$\psi_{i,t}^{B}(\theta^t) = \gamma_{i,t}\left(\theta_{i,t},\theta^{t-1}\right) - \frac{1}{I-1}\sum_{j\neq i}\gamma_{j,t}(\theta_{j,t},\theta^{t-1}), \text{ where}$$

$$\gamma_{j,t}(\theta_{j,t},\theta^{t-1}) = \delta^{-t}\left(\begin{array}{c} \mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\theta_{j,t},\theta^{t-1}}\left[\sum_{i\neq j} U_i\left(\chi^*(\theta),\theta\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\theta^{t-1}}\left[\sum_{i\neq j} U_i\left(\chi^*(\theta),\theta\right)\right] \end{array}\right)$$

Theorem

Assume independent types: conditional on x_0^t, agent i's private information θ_i^t, x_i^t does not affect the distribution of $\theta_{j,t}$, for $j \neq i$. Also assume private values: $u_{j,t}\left(x^t,\theta^t\right)$ does not depend on θ_i^t, x_i^t for all t, $i \neq j$. Then balanced team mechanism is BIC.

Balancing: Example

- In initial example:

$$-U_S\left(\chi(\hat{\theta}),\hat{\theta}_S\right) = c\left(\chi_1(\hat{\theta}_S),\hat{\theta}_S\right) + \delta c\left(\chi_2(\hat{\theta}_S,\hat{\theta}_{B,2}),\hat{\theta}_S\right) + \delta^2 c\left(\chi_3(\hat{\theta}_S,\right.$$

$$\begin{aligned}\gamma_{B,3}(\hat{\theta}_{B,2},\hat{\theta}_{B,3},\hat{\theta}_S) &= -c\left(\chi_3(\hat{\theta}_S,\hat{\theta}_{B,3}),\hat{\theta}_S\right)\\ &\quad+\mathbb{E}_{\tilde{\theta}_{B,3}}\left[c\left(\chi_3(\hat{\theta}_S,\tilde{\theta}_{B,3}),\hat{\theta}_S\right)\middle|\,\hat{\theta}_{B,2}\right]\end{aligned}$$

cancel

$$\gamma_{B,2}(\hat{\theta}_{B,2},\hat{\theta}_S) = -c\left(\chi_2(\hat{\theta}_S,\hat{\theta}_{B,2}),\hat{\theta}_S\right) - \delta\mathbb{E}_{\tilde{\theta}_{B,3}}\left[c\left(\chi_3(\hat{\theta}_S,\tilde{\theta}_{B,3}),\hat{\theta}_S\right)\middle|\,\hat{\theta}\right.$$

B2

$$+\mathbb{E}_{\tilde{\theta}_{B,2},\tilde{\theta}_{B,3}}\left[c\left(\chi_2(\hat{\theta}_S,\tilde{\theta}_{B,2}),\hat{\theta}_S\right)+\delta c\left(\chi_3(\hat{\theta}_S,\tilde{\theta}_{B,3}),\hat{\theta}_S\right)\right]$$

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t},\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{+}(\hat{\theta}_{j,t},\hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{-}(\hat{\theta}^{t-1})}$$

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t},\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^+(\hat{\theta}_{j,t},\hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^-(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t}, \hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{+}(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{-}(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$
- $\gamma_{j,t}^{-}(\hat{\theta}^{t-1})$ uses only period $t-1$ information

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t},\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{+}(\hat{\theta}_{j,t},\hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{-}(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$
- $\gamma_{j,t}^{-}(\hat{\theta}^{t-1})$ uses only period $t-1$ information
- $\gamma_{j,t}^{+}(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})$ uses, in addition, agent j's period-t report

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t},\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^+(\hat{\theta}_{j,t},\hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^-(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$
- $\gamma_{j,t}^-(\hat{\theta}^{t-1})$ uses only period $t-1$ information
- $\gamma_{j,t}^+(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})$ uses, in addition, agent j's period-t report
- For any deviation by agent i, if the others are truthful-obedient:

Balancing: Proof Sketch

- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t}, \hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{+}(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^{-}(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$
- $\gamma_{j,t}^{-}(\hat{\theta}^{t-1})$ uses only period $t-1$ information
- $\gamma_{j,t}^{+}(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})$ uses, in addition, agent j's period-t report
- For any deviation by agent i, if the others are truthful-obedient:
    - *Claim 1:* Expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$

Balancing: Proof Sketch

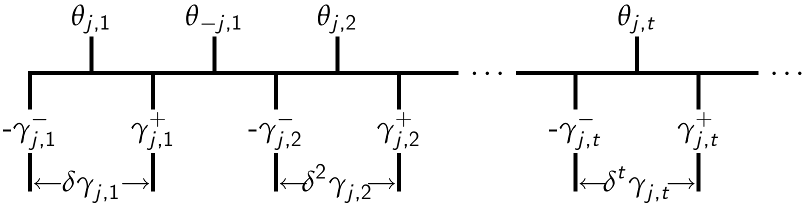
- Let $\Psi_j\left(\tilde{\theta}\right) = \sum_{i \neq j} U_i\left(\chi^*(\theta), \theta\right)$, pv of j's payments:

$$\delta^t \gamma_{j,t}(\hat{\theta}_j^t, \hat{\theta}_{-j}^{t-1}) = \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi]|\hat{\theta}_{j,t}, \hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^+(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})} - \underbrace{\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi]|\hat{\theta}^{t-1}}\left[\Psi_j\left(\tilde{\theta}\right)\right]}_{\gamma_{j,t}^-(\hat{\theta}^{t-1})}$$

- Two terms are expectations of the same function $\Psi_j\left(\tilde{\theta}\right)$
- $\gamma_{j,t}^-(\hat{\theta}^{t-1})$ uses only period $t-1$ information
- $\gamma_{j,t}^+(\hat{\theta}_{j,t}, \hat{\theta}^{t-1})$ uses, in addition, agent j's period-t report
- For any deviation by agent i, if the others are truthful-obedient:
    - *Claim 1:* Expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$
    - *Claim 2:* Expected present value of $\gamma_{j,t}$ is zero for each $j \neq i$

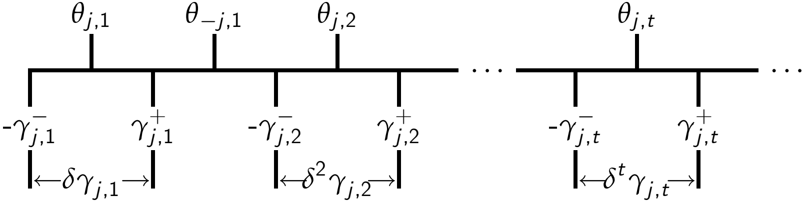
Proof of Claim 2

- For any possible deviation of agent i, expected present value of $\gamma_{j,t}$ is zero for each $j \neq i$:



Proof of Claim 2

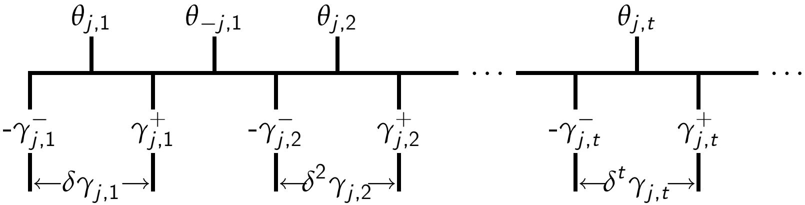
- For any possible deviation of agent i, expected present value of $\gamma_{j,t}$ is zero for each $j \neq i$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{j,t}$

Proof of Claim 2

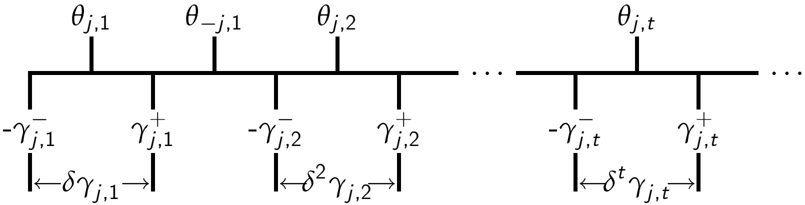
- For any possible deviation of agent i, expected present value of $\gamma_{j,t}$ is zero for each $j \neq i$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{j,t}$
- If agent j is truthful, the expectation of $\gamma^{+}_{j,t}(\tilde{\theta}_{j,t}, \hat{\theta}^{t-1})$ before time t equals $\gamma^{-}_{j,t}(\hat{\theta}^{t-1})$, for any report history $\hat{\theta}^{t-1}$

Proof of Claim 2

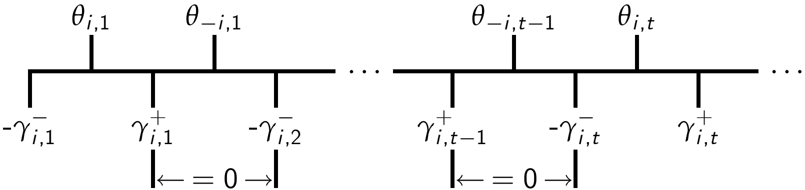
- For any possible deviation of agent i, expected present value of $\gamma_{j,t}$ is zero for each $j \neq i$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{j,t}$
- If agent j is truthful, the expectation of $\gamma^+_{j,t}(\tilde{\theta}_{j,t}, \hat{\theta}^{t-1})$ before time t equals $\gamma^-_{j,t}(\hat{\theta}^{t-1})$, for any report history $\hat{\theta}^{t-1}$
- LIE: ex ante expectation of $\gamma_{j,t}$ equals zero

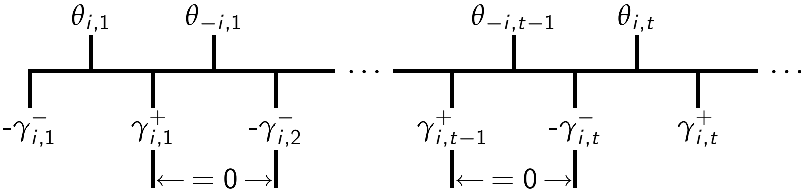
Proof of Claim 1

- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



Proof of Claim 1

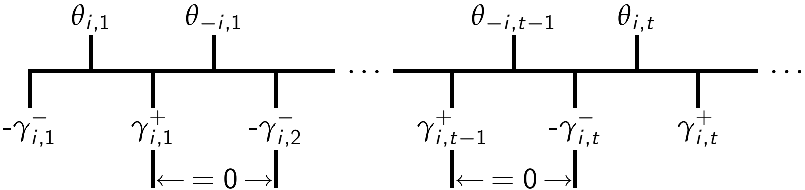
- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{-i,t}$

Proof of Claim 1

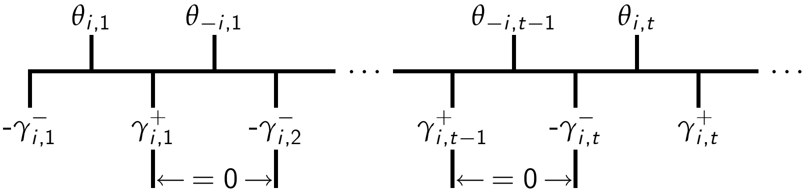
- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{-i,t}$
- If the others are truthful, agent i's time-t expectation of $\gamma^{-}_{i,t+1}(\tilde{\theta}_{-i,t}, \hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ equals $\gamma^{+}_{i,t}(\hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ for any $\hat{\theta}_{i,t}$, $\hat{\theta}^{t-1}$

Proof of Claim 1

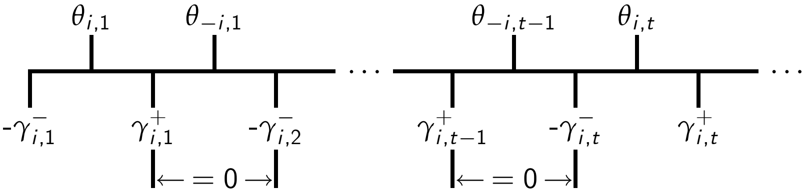
- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{-i,t}$
- If the others are truthful, agent i's time-t expectation of $\gamma^{-}_{i,t+1}(\tilde{\theta}_{-i,t}, \hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ equals $\gamma^{+}_{i,t}(\hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ for any $\hat{\theta}_{i,t}$, $\hat{\theta}^{t-1}$
- LIE: the two terms have the same ex ante expectations as well

Proof of Claim 1

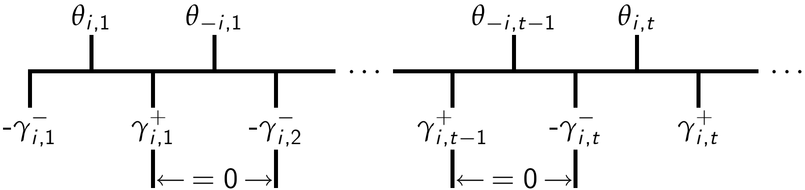
- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{-i,t}$
- If the others are truthful, agent i's time-t expectation of $\gamma^{-}_{i,t+1}(\tilde{\theta}_{-i,t}, \hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ equals $\gamma^{+}_{i,t}(\hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ for any $\hat{\theta}_{i,t}$, $\hat{\theta}^{t-1}$
- LIE: the two terms have the same ex ante expectations as well
- Thus, expectation of $\sum_{\tau=1}^{t} \delta^{\tau} \tilde{\gamma}_{i,\tau}$ equals to that of $\tilde{\gamma}^{+}_{i,t} - \tilde{\gamma}^{-}_{i,1}$

Proof of Claim 1

- For any possible deviation of agent i, expected present value of $\gamma_{i,t}$ equals, up to a constant, that of $\psi_{i,t}$:



- Independent types $\Rightarrow$ agent i's private history $\left(\theta_i^t, x_i^{t-1}\right)$ does not affect beliefs over $\tilde{\theta}_{-i,t}$
- If the others are truthful, agent i's time-t expectation of $\gamma^-_{i,t+1}(\tilde{\theta}_{-i,t}, \hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ equals $\gamma^+_{i,t}(\hat{\theta}_{i,t}, \hat{\theta}^{t-1})$ for any $\hat{\theta}_{i,t}$, $\hat{\theta}^{t-1}$
- LIE: the two terms have the same ex ante expectations as well
- Thus, expectation of $\sum_{\tau=1}^{t} \delta^\tau \tilde{\gamma}_{i,\tau}$ equals to that of $\tilde{\gamma}^+_{i,t} - \tilde{\gamma}^-_{i,1}$
- $\gamma^-_{i,1}$ is unaffected by reports; $\tilde{\gamma}^+_{i,t} \rightarrow \Psi_i\left(\tilde{\theta}\right)$ as $t \rightarrow \infty$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \dots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:
  - Finite action, type spaces, the same in each period

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:
  - Finite action, type spaces, the same in each period
  - Markovian type transitions: $\nu_t\left(\theta_t | \theta^{t-1}, x^{t-1}\right) = \bar{\nu}\left(\theta_t | \theta_{t-1}, x_{t-1}\right)$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:
  - Finite action, type spaces, the same in each period
  - Markovian type transitions: $\nu_t\left(\theta_t|\theta^{t-1}, x^{t-1}\right) = \bar{\nu}\left(\theta_t|\theta_{t-1}, x_{t-1}\right)$
  - Stationary separable payoffs $u_{i,t}\left(x^t, \theta^t\right) = \bar{u}_i\left(x_t, \theta_t\right)$

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:
  - Finite action, type spaces, the same in each period
  - Markovian type transitions: $\nu_t\left(\theta_t | \theta^{t-1}, x^{t-1}\right) = \bar{\nu}\left(\theta_t | \theta_{t-1}, x_{t-1}\right)$
  - Stationary separable payoffs $u_{i,t}\left(x^t, \theta^t\right) = \bar{u}_i\left(x_t, \theta_t\right)$
- $\Rightarrow \exists$ a "Blackwell policy" χ^* - a Markovian decision rule that is efficient for all δ close enough to 1, for any starting state

Decentralized Games (No External Enforcer)

- In each period $t = 1, 2, \ldots$
  1. Each agent i privately observes signal $\theta_{i,t}$
  2. Agents send simultaneous reports
  3. Each agent i chooses private action $x_{i,t}$
  4. Each agent i chooses public action $x_{0,i,t}$, makes public payment $z_{i,j,t} \geq 0$ to each agent j
- $\Rightarrow$ Public action $x_{0,t} = (x_{0,i,t})_{i=1}^{N}$, total transfer $y_{i,t} = \sum_j (z_{j,i,t} - z_{i,j,t})$ to agent i (budget-balanced)
- Markovian Assumptions:
  - Finite action, type spaces, the same in each period
  - Markovian type transitions: $\nu_t\left(\theta_t | \theta^{t-1}, x^{t-1}\right) = \bar{\nu}\left(\theta_t | \theta_{t-1}, x_{t-1}\right)$
  - Stationary separable payoffs $u_{i,t}\left(x^t, \theta^t\right) = \bar{u}_i\left(x_t, \theta_t\right)$
- $\Rightarrow \exists$ a "Blackwell policy" χ^* - a Markovian decision rule that is efficient for all δ close enough to 1, for any starting state
- Can we sustain χ^* in PBE?

Implement the Balanced Team Mechanism

- When no publicly observed deviation, make payments

$$
\begin{aligned}
z_{i,j,t} &= \frac{1}{I-1}\gamma_{j,t}(\theta_j^t, \theta_{-j}^{t-1}) + K_i \\
&= \frac{1}{I-1}\sum_{k\neq j}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{l} \mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi^*]|\theta_j^t,\theta_{-j}^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi^*]|\theta^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \end{array}\right) + K_i
\end{aligned}
$$

Implement the Balanced Team Mechanism

- When no publicly observed deviation, make payments

$$
\begin{aligned}
z_{i,j,t} &= \frac{1}{I-1}\gamma_{j,t}(\theta_j^t, \theta_{-j}^{t-1}) + K_i \\
&= \frac{1}{I-1}\sum_{k\neq j}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{l} \mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi^*]|\theta_j^t,\theta_{-j}^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right), \tilde{\theta}_\tau\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi^*]|\theta^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right), \tilde{\theta}_\tau\right)\right] \end{array}\right) + K_i
\end{aligned}
$$

- Can we prevent public deviations (="quitting") for any history?

Implement the Balanced Team Mechanism

- When no publicly observed deviation, make payments

$$
\begin{aligned}
z_{i,j,t} &= \frac{1}{I-1}\gamma_{j,t}(\theta_j^t, \theta_{-j}^{t-1}) + K_i \\
&= \frac{1}{I-1}\sum_{k\neq j}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{l} \mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi^*]|\theta_j^t,\theta_{-j}^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi^*]|\theta^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \end{array}\right) + K_i
\end{aligned}
$$

- Can we prevent public deviations (="quitting") for any history?
  - Can think of this as joint IC-IR constraints

Implement the Balanced Team Mechanism

- When no publicly observed deviation, make payments

$$
\begin{aligned}
z_{i,j,t} &= \frac{1}{I-1}\gamma_{j,t}(\theta_j^t, \theta_{-j}^{t-1}) + K_i \\
&= \frac{1}{I-1}\sum_{k\neq j}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{l} \mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi^*]|\theta_j^t,\theta_{-j}^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi^*]|\theta^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \end{array}\right) + K_i
\end{aligned}
$$

- Can we prevent public deviations (="quitting") for any history?
    - Can think of this as joint IC-IR constraints
- Problem: transfers may be unbounded as $\delta \to 1$.

Implement the Balanced Team Mechanism

- When no publicly observed deviation, make payments

$$
\begin{aligned}
z_{i,j,t} &= \frac{1}{I-1}\gamma_{j,t}(\theta_j^t, \theta_{-j}^{t-1}) + K_i \\
&= \frac{1}{I-1}\sum_{k\neq j}\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left(\begin{array}{c}\mathbb{E}_{\tilde{\theta}}^{\mu_t^j[\chi^*]|\theta_j^t,\theta_{-j}^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right] \\ -\mathbb{E}_{\tilde{\theta}}^{\mu_t[\chi^*]|\theta^{t-1}}\left[\bar{u}_k\left(\chi^*\left(\tilde{\theta}_\tau\right),\tilde{\theta}_\tau\right)\right]\end{array}\right) + K_i
\end{aligned}
$$

- Can we prevent public deviations (="quitting") for any history?
    - Can think of this as joint IC-IR constraints
- Problem: transfers may be unbounded as $\delta \to 1$.
- But: with limited persistence of $\tilde{\theta}$, the two expectations may be close as $\tau \to \infty$

Sustaining Efficiency

Theorem

Take the Markov game with independent private values, which has a zero-payoff belief-free static NE. Suppose that a Blackwell policy χ^* *induces a Markov process with a unique ergodic set (and a possibly empty transient set), and that the ergodic distribution gives a positive expected total surplus. Then for* δ *large enough,* χ^* *can be sustained in a PBE using Balanced Team Transfers.*

- Dynamic Games
  - In decentralized games, actions and transfers have to be self-enforcing; not commitment mechanism is available to the agents
  - In many games, transfers are not available
  - What is the relationship between the outcomes that can be attained WITH commitment and transfers, and what can be attained without?
  - When can efficiency be sustained as an eqm?
  - What do equilibria look like for different discount factors?
  - Efficiency includes BB

- Literature in Microeconomics on Dynamic Games and Contracts
  - Collusion: Athey and Bagwell (series of papers)
  - Repeated Trade: Athey and Miller
  - Relational Contracts: Levin, Rayo
  - Continuous time models, principal agent: Sannikov and coauthors
  - Cost of ex post as opposed to Bayesian equilibrium: Miller
- Literature in Dynamic Public Finance, Macro
  - Amador, Angeletos, and Werning; Tsyvinski; Athey, Atkeson, and Kehoe; others

- Focus Today: Hidden Information
  - Hidden actions impt, techniques and applications often different
  - Auctions, collusion, bilateral or multilateral trade, public good provision, resource allocation, favor-trading in relationships, mutual insurance
- Contracts, Games, and Games as Contracts

- Mechanism Design Approach to Dynamic Games
  - In static theory, we are familiar with mechanism design approach to analyzing games such as auctions
  - Use tools such as envelope theorem, revenue equivalence, etc. to characterize equilibria
  - Analyze constraints
  - Take this approach to dynamic games
  - Combine dynamic programming and mechanism design tools
  - Frontier of current research: fully dynamic games (not repeated)

A Toolkit for Analyzing Dynamic Games and Contracts

- Abreu-Pearce-Stacchetti and dynamic programming
- The mechanism design approach to repeated games with hidden information
- Sustaining efficiency with transfers
- The folk theorem without transfers
- Dynamic Programming for Dynamic Games

Analyzing Repeated and Dynamic Games with Hidden Information

- Model the game/contract in extensive form
  - Dynamic games–see Battiglini (2005), Athey and Segal (2007)
  - Cumbersome to specify full strategy space and optimize over it
- Use APS/Mechanism Design combination
  - Applicability of results with the right assumptions
  - Can apply body of knowledge for hidden info games

A Dynamic Game with Time-Varying Hidden Information

- Players $i = 1, .., I$
- Time $t = 1, .., T$ (special cases: $T = 1, T = \infty$)
- Superscript/subscript notation: given $((y_{i,t})_{t=1}^{T})_{i=1}^{I}$,

$$y_t = (y_{i,t})_{i=1}^{I}, \quad y_i = (y_{i,t})_{t=1}^{T}, \ y^t = (y_{t'})_{t'=1}^{t}.$$

- Type spaces $\Theta_{i,t} \subseteq \mathbf{R}^n$, random variables $\tilde{\theta}_{i,t}$ with realizations $\theta_{i,t}$.
- Communication amoung players: $m_{i,t} \in \mathcal{M}_{i,t}$
- Decisions $X_{i,t} \subseteq \mathbf{R}^n$.
- Transfer from player j to player i :

  $y_{j,i,t} \geq 0$, let $y_{i,t} = \sum_j y_{j,i,t} - y_{i,j,t}$.

  - Some models rule out transfers, e.g. collusion

- History has two components:
  - Public history $h^{t-1} = (x^{t-1}, m^{t-1}, y^{t-1})$, private histories θ^{t-1}
- Timeline in period t:
  - Types realized (θ_t)
    * History potentially affects distributions: $F_t(\theta_t; x^{t-1}, \theta^{t-1})$.
  - Players communicate (m_t)
  - Players simultaneously make decisions (x_t) and send transfers (y_t)
- Note: can consider models without communication in this framework
  - Messages can be contentless
  - Athey-Bagwell (2001) show this can relax incentive constraints

Approach: Model Game with Mechanism Design Tools

- Define a recursive (direct revelation) mechanism
  - Replace mapping from types to actions with reporting strategy
  - Many games of interest have single crossing property, already restricted to monotone strategies
- Specify appropriate constraints
  - "On-schedule" and "off-schedule" deviations
  - Comparison between decentralized game and recursive mechanism
    * Game has add'l constraints, action space unrestricted
    * With patience, these can be satisfied
    * Game without transfers must deal with restrictions on continuation values

- The role of patience
  - Static mechanism that satisfies BIC, EPBB, IR may *not* be eqm in decentralized game with low patience
    - Mechanism provides commitment
  - Static mechanism that satisfies BIC, EPBB, *fails* IR may be eqm in game with high patience
    - Future gain from relationship relaxes participation constraints
- Independent (over time) types or perfectly persistent types
  - Use static tools
- More general dynamics
  - Contingent, multi-stage deviations
  - Transfers and continuation equilibria not perfect substitutes

Approach Here: Recursive Mechanisms

- Athey and Bagwell (2001), Athey, Bagwell, and Sanchirico (2004)
  - Miller (2005) sets out approach for general model
- Idea: use APS approach together with mechanism design tools
- Start by focusing on stationary (repeated) games
  - For appropriately selected constraints, a “self-generating” recursive mechanism will be a PPE
  - A PPE can be written as a recursive mechanism
- Apply tools from static mechanism design theory

The Recursive Mechanism

- Stage Mechanism
  - Action plan for each player: $\chi : \Theta_t \to X$
  - Transfer plan from i to j, $\psi_{i,j} : \Theta_t \to \mathbf{R}^+$, $\psi_i = \sum_j \psi_{j,i} - \psi_{i,j}$
  - Continuation value function $w : \Theta_t \to \mathbf{R}^I$.
  - Let $\gamma = (\chi, \psi, w)$

  Ex post utility: $u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \pi_i(\chi(\hat{\theta}_t), \theta_{i,t}) + \psi_i(\hat{\theta}_t) + \delta w_i(\hat{\theta}_t)$

  Interim utility: $\bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) = \mathbb{E}_{\tilde{\theta}_{-i,t}}[u_i((\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}), \theta_{i,t}; \gamma)]$

- Recursive Mechanism: $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$
  - A set V An initial condition $v_0 \in V$
  - A set of stage mechanisms $\{\gamma(v)\}_{v \in V}$

Constraints

- (Bayesian, Interim) IC:

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \geq \ \bar{u}_i(\hat{\theta}_{i,t}, \theta_{i,t}; \gamma) \text{ for all } \hat{\theta}_{i,t} \in \Theta_{i,t}$$

- IR(p_0)
  - "Outside option": punishment equilibrium with payoffs p_0.
  - Could be static Nash, "Nonparticipation."
  - For simplicity, assume informative communication.

$$\bar{u}_i(\theta_{i,t}, \theta_{i,t}; \gamma) \ \geq \sup_{\hat{\theta}_{i,t}} \left\{ \mathbb{E}_{\tilde{\theta}_{-i,t}} \left[\sup_{x_i} \left(\begin{array}{c} \pi_i(x_i, \chi_{-i}(\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}), \theta_{i,t}) \\ + \sum_j \psi_{j,i}(\hat{\theta}_{i,t}, \tilde{\theta}_{-i,t}) \end{array} \right) \right] \right\} + \delta p_{0,i}.$$

    * More generally, take expectations given messages. See Athey and Bagwell (2001) for more discussion of alternative IRs.
    * Note assn about transfers and actions simultaneous.

Self-Generating Recursive Mechanism

- Define the set of attainable payoffs to be

$$\mathcal{V} = co \left\{ v \in \mathbf{R}^I : \exists \gamma \text{ s.t. } \sum_i v_i = \sum_i \frac{\mathbb{E}_{\tilde{\theta}_t}\left[u_i(\tilde{\theta}_t, \tilde{\theta}_{i,t}; \gamma) \right]}{1-\delta} \right\}.$$

- For $V \subset \mathcal{V}$, $p_0 \in \mathbf{R}^I$, define $T(V; p_0)$ to be the set of $v \in \mathbb{R}^I$ for which there exist stage mechanisms $\gamma(v) = (\chi, \psi, w)(v)$ whereby

1. *Promise-keeping*: $\mathbb{E}_{\tilde{\theta}_t}\left[u_i(\tilde{\theta}_t, \tilde{\theta}_{i,t}; \gamma(v))\right] = v_i$.
2. *Coherence*: $w(v) : \Theta_t \to V$.
3. *Best response*: $\gamma(v)$ satisfies IC and IR(p_0).

- V is *self-generating relative to* p_0 if $V \subseteq T(V; p_0)$.
  - Note: full set is $V \cup p_0$. Worst eqm not our focus; can extend to address this.

- $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is *self-generating relative to* p_0 (SGRM(p_0)) if:
  V is self-generating relative to p_0 and,
  for each $v \in V$, (1)-(3) hold for $\gamma(v)$ and p_0.

Recursive Mechanism as a Tool for Analyzing Decentralized PPE

Proposition 1 *Fix δ. Suppose p_0 is a PPE and consider $V >> p_0$.*
(i) If V is a set of PPE payoffs with informative communication, then there exists $v_0 \in V$, $\{\gamma(v)\}_{v \in V}$ such that $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is a SGRM(p_0).
(ii) Suppose that $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ is a SGRM(p_0). Then V is in the set of PPE payoffs.

- Proof: See Miller (2005) (does folk theorem; adapt arguments). Analogous to APS. Have to verify that constraints deter relevant deviations.
- If interested in set V of PPE payoffs w/o informative communication, modify IRs to get corresponding result.
- IR constraints imply that deviating "off-schedule" is not desirable.

Transforming to a Static Problem: The Case with Transfers

- Recall

$$u_i(\hat{\theta}_t, \theta_{i,t}; \gamma) = \pi_i(\chi(\hat{\theta}_t), \theta_{i,t}) + \psi_i(\hat{\theta}_t) + \delta w_i(\hat{\theta}_t).$$

  - With independent types, value for future play is the same for all types
  - Transfers and continuation values completely fungible
- WLOG, can consider stationary mechanisms (Levin, 2003)
- Then, consider static mechanism design problem with bounds on transfers imposed by IR

Folk Theorem with Transfers

Proposition 2 *Given χ, suppose there exist EPBB, uniformly bounded, IC transfers for χ, and that*

$$\sum_i \mathbb{E}[\pi_i(\chi(\theta_t), \theta_{i,t})] > \sum p_{0,i}.$$

Then for δ sufficiently large, there exists a SGRM(p), $\langle V, \{\gamma(v)\}_{v \in V}, v_0 \rangle$ that is stationary, where

$$\sum_i v_{0,i} = \sum_i \mathbb{E}[\pi_i(\chi(\theta_t), \theta_{i,t})].$$

- Result says that if policy can be implemented with commitment, it can be self-enforcing for sufficiently patient agents
- See Cremer, d'Aspremont, Gerard-Varet (2003) for sufficient conditions; see also Miller (2005).

- As δ grows, value of future eventually outweighs transfers. Independent future key.

Transforming to a Static Problem: The Case without Transfers

- Continuation values can mimic role of transfers, but for fixed δ, Pareto frontier of V is not in general linear
- Tradeoff between using variation in continuation values to provide incentives, and Pareto efficient continuation values
  - "Efficiency today v. efficiency tomorrow"
  - Finding: Sacrifice efficiency today

- Details of model determine shape of frontier of V
  - Multiplicity of efficient outcomes: partial linearity
- Approach (see Athey and Bagwell (2001)): start with large V, characterize $T(V)$
  - Analogous to static problem with restricted transfers

Folk Theorem without Transfers

- Fudenberg, Levine and Maskin (1994), Miller (2005)
  - Small changes in future per-period utility mimic transfers
  - FLM make unnecessary assumptions: independent, finite types
    - They focus on hidden action models and so don't look for most general conditions
  - Miller (2005) generalizes to continuous types, correlated values
- Key elements of argument
  - Angle of supporting hyperplanes doesn't matter generically
  - Average period payoffs (outside set) and hyperplane (inside set)
  - As $\delta \to 1$, length of hyperplane shrinks fast enough
  - Nothing about what to do for fixed δ

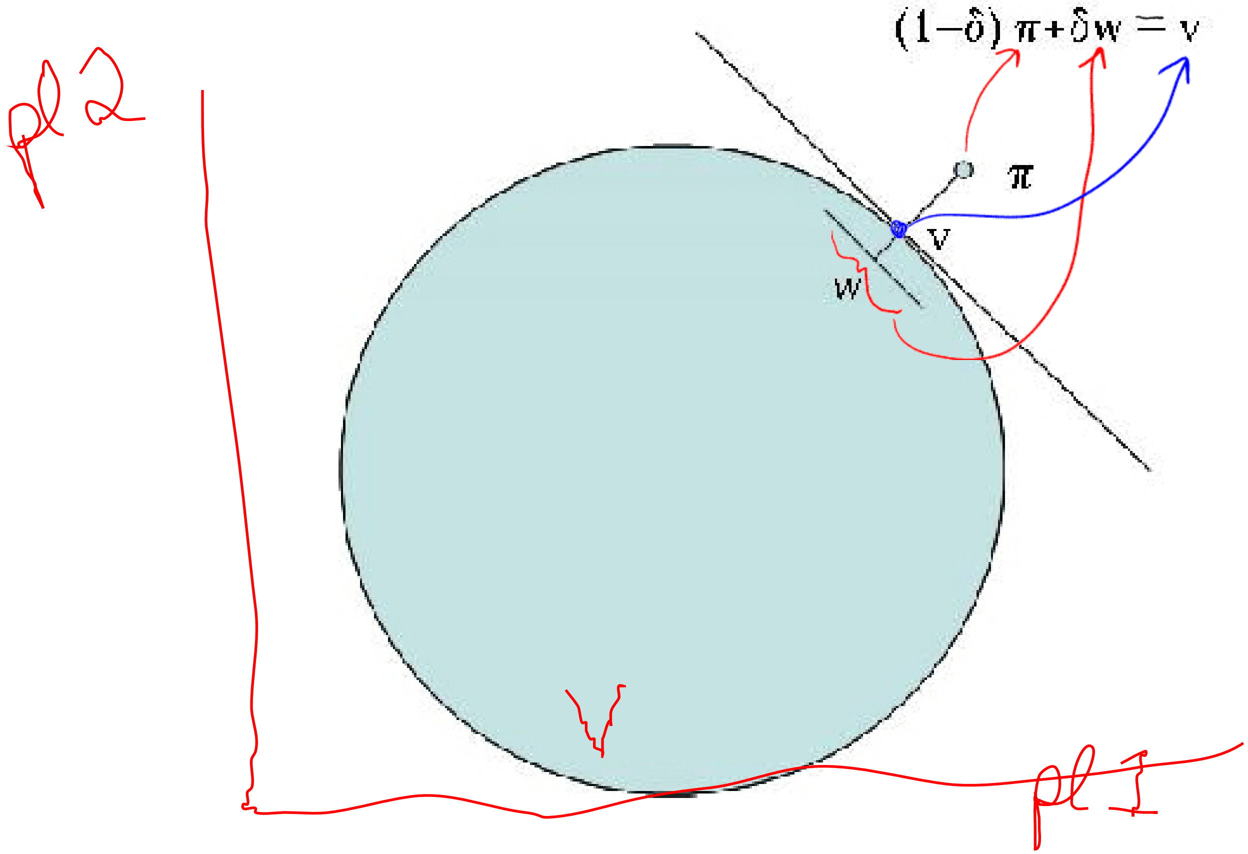


FIGURE: Supporting Hyperplanes

Applications

- Ongoing Relationships
  - Time-varying individual costs and benefits to acting, i.i.d. private information
  - Restrictions on monetary transfers
- Examples
  - Colluding firms, i.i.d. cost/inventory shocks
  - Public good provision
    - Families/villages   Organizations
    - Legislatures   Academic departments
  - Policy games (government is privately informed)

- Questions about Collusion
  - Response of collusive behavior to institutional setting
  - Effects of anti-trust policy (Restrictions on communication, side-payments)
  - Market design: info. about indiv. bids and identities
  - Institutional design: industry assoc., smoke-filled rooms
- Central Tradeoffs
  - Productive efficiency requires low-cost firm serves market
  - Firms like market-share, incentive to mimic low-cost firm
  - Need low prices or future "punishment" with high market-share
  - Future price wars v. "future market-share favors"

Asymmetric Collusion

- Setup
  - 2 firms produce perfect substitutes
  - Unit mass of consumers, reservation price r
  - 2 cost types: $\theta^i \in \{\theta_L, \theta_H\}$, $\Pr(\theta^i = \theta_j) = \eta_j$.
    Case: $\eta_L > 1/2$.
- Firms...
  - may split the market unevenly; details not imp't.
  - may not charge different prices to different consumers.
  - communicate prior to producing (see Athey and Bagwell (2001) for analysis of communication)

Summary of Ideas for Asymmetric Eq'a

- A first best scheme, always price at r
  - Eqm described by two "states"
  - Each period, announce types
  - State x: low cost firm serves market, but firm 2 serves most of market if firms have same cost
    - If (H, L), switch to state y, oth. return to x
  - State y: low cost firm serves market, but firm 1 serves most of market if firms have same cost
    - If (L, H), switch to state x, oth. return to y
- Paper: shows that first-best scheme can work if patient enough that diff. betw. x and y provides suff. incentives; if less patient shows similar schemes with partial prod. eff. are optimal.

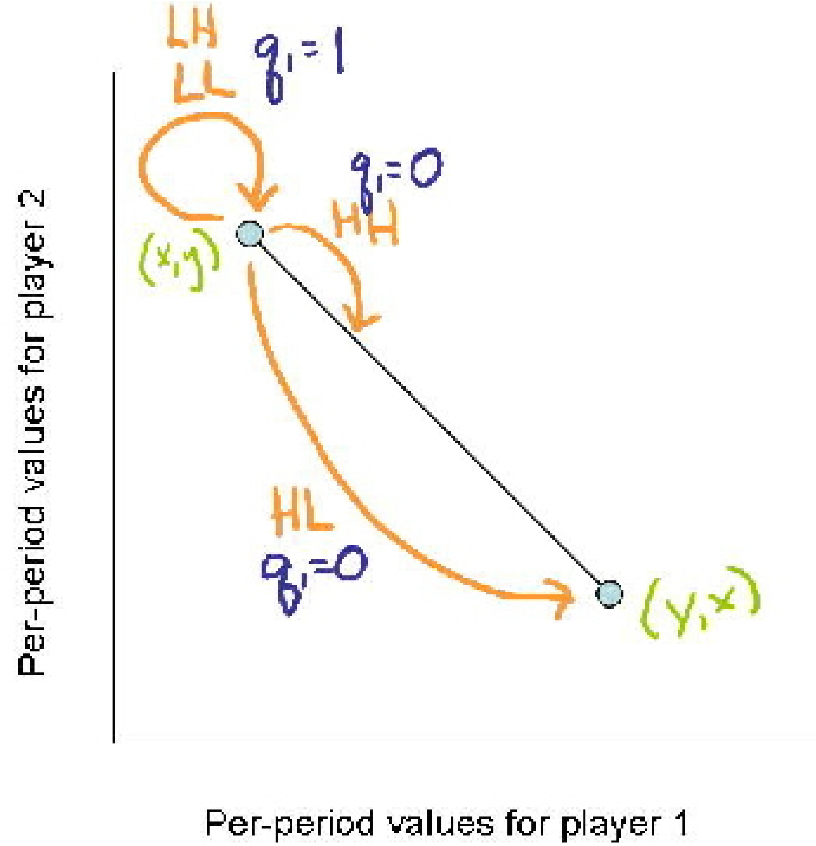
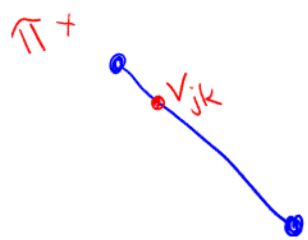


Illustration of First-Best equilibrium

A Linear Self-Generating Set with First-Best Profits

- Goal: Compute a critical discount factor above which first-best profits can be attained in every period.
  - Requires linear, "self-generating" set with slope -1 :

$$[(x, y), (y, x)]$$



  - Two parts.
  - "Adding Up": First, ignore IC-Off. Is it possible to have linear self-generating set with full efficiency?
    - Need to implement (x, y) using $v_{jk} \in [(x, y), (y, x)]$.
    - Future looks brighter than today for firm 1, and enough brighter when firm 1 has high cost to satisfy IC-On.

* Does it all "add up"?

– Second, when are IC-Off's cleared?.

Proposition 3 *Suppose that* $r - \theta_H < \theta_H - \theta_L$. *Then, for all* $\delta \in (\delta^{FB}, 1]$, *there exist values* $y > x > 0$ *such that* $x + y = 2\pi^{FB}/(1 - \delta)$, *and the line segment* $[(x, y), (y, x)]$ *is "self-generating" and in the set of PPE values,* V^*.

Persistent Types

- See Cole and Kocherlakota, Athey and Bagwell on persistent types and extending recursive mechanism design approach
- Two-period sophisticated rotation
  - Produce today, give up market share tomorrow
  - Not very effective with persistent types
- First-best example
  - Extends to persistent types
  - Keep track of beliefs as state variables
  - In a fully revealing equilibrium, all that matters is last period's state

- As persistence grows relative to patience, rigid pricing approximately optimal with log-concavity
  - Cannot do efficient transfers, so pooling is optimal

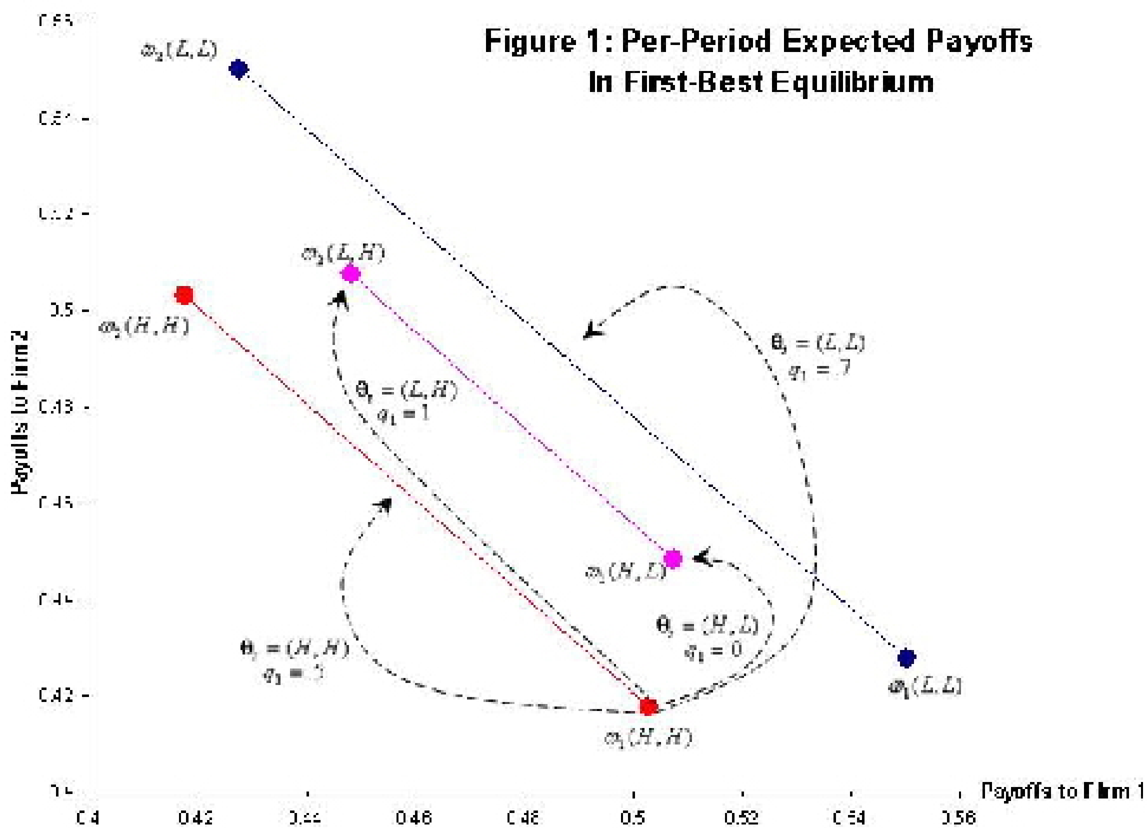


FIGURE: First-Best Equilibrium with Persistent Types

Summing Up Dynamic Games

- Bring together mechanism design and dynamic programming to analyze repeated and dynamic games
- Apply tools from static literature
- Generalize to incorporate interesting dynamics
  - Today: Serial correlation
  - Learning-by-doing, experimentation, information gathering (Athey-Segal)
  - Maintaining budget account (Athey-Miller)
- Efficiency possible in wide range of circumstances
- Pooling is optimal for agents when limited instruments for providing incentives