Part II: Bidding, Dynamics and Competition

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Campaign Optimization
Budget Optimization (BO): Simple

- **Input:**
  - Set of keywords and a budget.
  - For each keyword, (clicks, cost) pair.
    - Same auction all day, same competitors, bids.

- **Model:**
  - Take the keyword or leave it, binary decision.
  - Maximize the number of clicks, subject to the budget.

- **Output:**
  - Subset of keywords.
Well-known Knapsack problem.

- Each KW is an *item*, cost = *weight*, clicks = *value*. Total budget = weight knapsack can carry.

NP hard in general.

Algorithm:

- Repeatedly take item largest value/weight (clicks/cost), or lowest cost per click. Last item will be fractional. Provably optimal.

- Undergrad algorithms: Sort by density=clicks/cost and be greedy.
BO: Multiple Slots

- **Input:**
  - For each keyword, multiple (clicks, cost) pairs.

- **Generalized Knapsack:**
  - Same item can be picked in different combinations.
  - NP hard in general.
Multiple Slots BO: Some Observations

- **Convex Hull.** Taking convex combination will dominate other points.

- Can treat each delta segment separately.
Consider each delta segment separately.

Solve standard Knapsack as before.

- Feasible since taken in order of decreasing clicks/cost.
- Provably optimal.

Message:

- Algorithm produces $x$
- Taking all delta segments (marginal) with cost-per-click $\leq x$ is the optimal solution.
Profit Optimization (PO)

- For each keyword (clicks, cost):
  \[ \text{profit} = \text{number of clicks} \times \text{value} - \text{total cost}. \]

- Profit Optimization: Maximize total profit.

- Take all profitable keywords. Optimal algorithm. No fractional issues.

- This algorithm targets marginal \( \text{cpc} = \text{value}. \)
PO with Budget

- Say budget $B$.
- Solve PO without $B$.
  - If spend $< B$, done.
  - Else, you will spend $B$. Then solve the BO problem given this $B$.

- [Homework] n KWs, $k$ versions per KW. Preprocess them. Query is $(V,B)$ or only $V$ or only $B$. Solve BO or PO problems.
- Can be done in $O(\log (nk))$ time. This data structure is landscapes.
Conversion Optimization.

- Given (conversions, cost), same algorithmics as above with cpc control knob.

Maximize ROI = value/cost.

- Get the 1 cheapest click!

Improve ROI:

- Bidding smartly
- Improve the creative.
- Change KW set,…
Target Positions

- Why?
- How?
  - Auction by auction.
  - Proxy bidding to average position target.
- BO/PO with Position Preference.
  - Simple: BO. Given budget B, for each KW, expected position < k.
Homework

Given \( n \) keywords with \( k \) versions each find bids for keywords such that overall average CPC is at most \( x \), and the number of clicks is maximized.

Hint:
- Algorithm will still proceed in increasing order of marginal CPCs.

Formally,
- Take increasing order of \( \frac{\text{DeltaCost}_i}{\text{DeltaClick}_i} \).
- Claim: \( \frac{\text{sumDeltaCost}_i}{\text{sumDeltaClick}_i} \) is also increasing. Hence stop when you get target average CPC.
3 Examples:
- Keyword Interaction
- Stochastic Information
- Broad Match
Keyword Interaction, BO Reexamined

- Keyword’s interact.
  - World is more complex.
    - Competitors drop in and out.
    - Multipliers change, traffic prediction is hard, …
  - Landscape functions are now complicated.
Let $C$ be the number of clicks obtained by an Omniscent bidder.

- there exists a bid $b$ such that $\text{clicks}(\text{uniform}(b)) \geq C/2$.

- There exists a distribution $d$ over two bids such that $\text{clicks}(\text{uniform}(d)) \geq (1-1/e) C$.

Better in practice and a very useful heuristic.

Feldman, Muthu, Pal, Stein. EC 07.
Bid $h(r)$ on each query and

- get $\geq r$ clicks.
- spend $\leq r h(r)$.

With some work, $r$ clicks at cost $rh(r)$. 
Bid $h(C/2)$ on each query and
• get $C/2$ clicks.
• spend $C/2 \cdot h(C/2) \leq \text{Budget}$
distribution:  \( \alpha_1 + \alpha_2 = 1 \)

budget = \( \alpha_1 b_1 f(b_1) + \alpha_2 b_2 f(b_2) \)

max. clicks = \( \alpha_1 b_1 + \alpha_2 b_2 \)
We can make up examples, so no profit approximation.

Theorem: Say we can get profit $P$ with value per click of $V$. Consider an uniform bidder with value $eV/(e-1)$, gets profit at least $P$.

Proof.

- $cl_o, co_o$ is what OPT gets and gives $P_o$.
- Uniform theorem says there exists $cl_u=(e-1)/e$ $cl_o$ and $co_u < co_opt$.
- Thus, if someone has value $Ve/(e-1)$ then, $profit_u = V e/(e-1) cl_u - co_u = V cl_o - co_o = profit_o$.

Open:

- Position, Average CPC, etc. bidding when keywords have interaction.
Stochastic BO

- (click, cost) functions are random variables with dependencies.
- Three popular stochastic models:
  - Proportional
  - Independent
  - Scenario
- Variety of approximation algorithms known.

Muthu, Pal, Svitkina WINE07.
Each scenario gives (click, cost) distribution for keywords.

There is a probability distribution over scenarios.

Finding a bidding strategy to maximize expected clicks:

- scaled by how much one overshoots the budget.

Polylog approx, log hardness of approx.


Dasgupta, Muthu 09.
Advertisers have to choose how to bid **Exact** or **Broad**.

- Because of impedance mismatch between user queries and bidding language for advertisers.

**Key technical difficulty in BO with broad match.**

- Bid on query/keyword $q$ applies implicitly to keywords eg., $q'$.
- While value from $q$ may be large, value from $q'$ may be even negative!
Bidding Broad

- Pick subset of queries to bid broad to maximize profit.
  - Polynomial time algorithms, even for budgeted versions.
- Bid on exact or broad on keywords to maximize profit.
  - Hard to even approximate (independent set).
  - $O(1)$ approx if profit >>> cost.

Even-Dar, Mansour, Mirrokni, Muthu, Nedev WWW 09.
More general problem is to combine
- Keyword and match type choice
- Target ad delivery and scheduling metrics
- Learn CTRs
- Optimize clicks, conversions, profit, brand effectiveness, ...
- For given budget.

Alternatively, think at higher level of abstraction of supply curve: (cost, value).
- The knobs like max cpc bids are just implementations.
- For each budget, Auctioneer can run BO, PO, etc.
- Advertiser needs to just pick a point.
Advertisers have to optimize across channels.
- Across search engines.
  - YMGA problem.
- Across search and display.
- Across online and offline.

Formal models will be useful.
Dynamics
Bidding Dynamics

How should advertisers bid?

- Vickrey-Clarke-Groves (VCG), Truthfully.
- Reality:
  - Other auctions (e.g., Generalized Second Price, or GSP) and strategies in repeated auctions.
  - Portfolio of auctions.
- Dynamics becomes important.
GSP: Static Game

- There exists an GSP equilibrium that has prices identical to VCG. It is the cheapest envy-free equilibrium.


- GSP with bidder-specific reserve prices. There exists an envy-free equilibrium, even though we don’t have local envy-free property.

E. Even-Dar, J. Feldman, Y. Mansour and Muthu, WINE08.
GSP: Dynamic Game

- Balanced Bidding (BB): Target the slot which maximizes the utility, and choose bid so you don’t regret getting the higher slot at bid value.

- If all bidders follow BB, there exists a unique fixed point. Then revenue is VCG equilibrium revenue.


- Asynchronous, random bidders with BB converges to this fixed point with prob. 1 in poly \( (k^{2^k}, \max v_i, n) \) steps.

FP, GSP Dynamics: Multiple Keywords

- Budget limited bidders with multiple keywords.
- Bidding such that the marginal return on investment is same for all keywords.
- Equilibrium analysis
  - To avoid cycling, need perturbation of bids.
  - With first price and uniform bidding, prices, utilities and revenue converge to Arrow-Debreu market equilibrium.

Competition

- A lot of auction design really deals with competitive behavior.

- Advertisers seem to ask about individual competitors.
  - Monitor for bids, quality, brand words,
  - Who are the competitors?
    - Micro competitors.
  - Why?
    - Relative bidding
    - Malicious bidding.

Y. Zhou and R. Lukose, WSAA06.

Summary

- [Jon] The Knobs.
- [Muthu] Controling the knobs wrt bidding.
  - Optimization: BO, PO, XO, …
  - Dynamics
  - Competition
- Rest
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