Computational Social Choice

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EC-12 Tutorial

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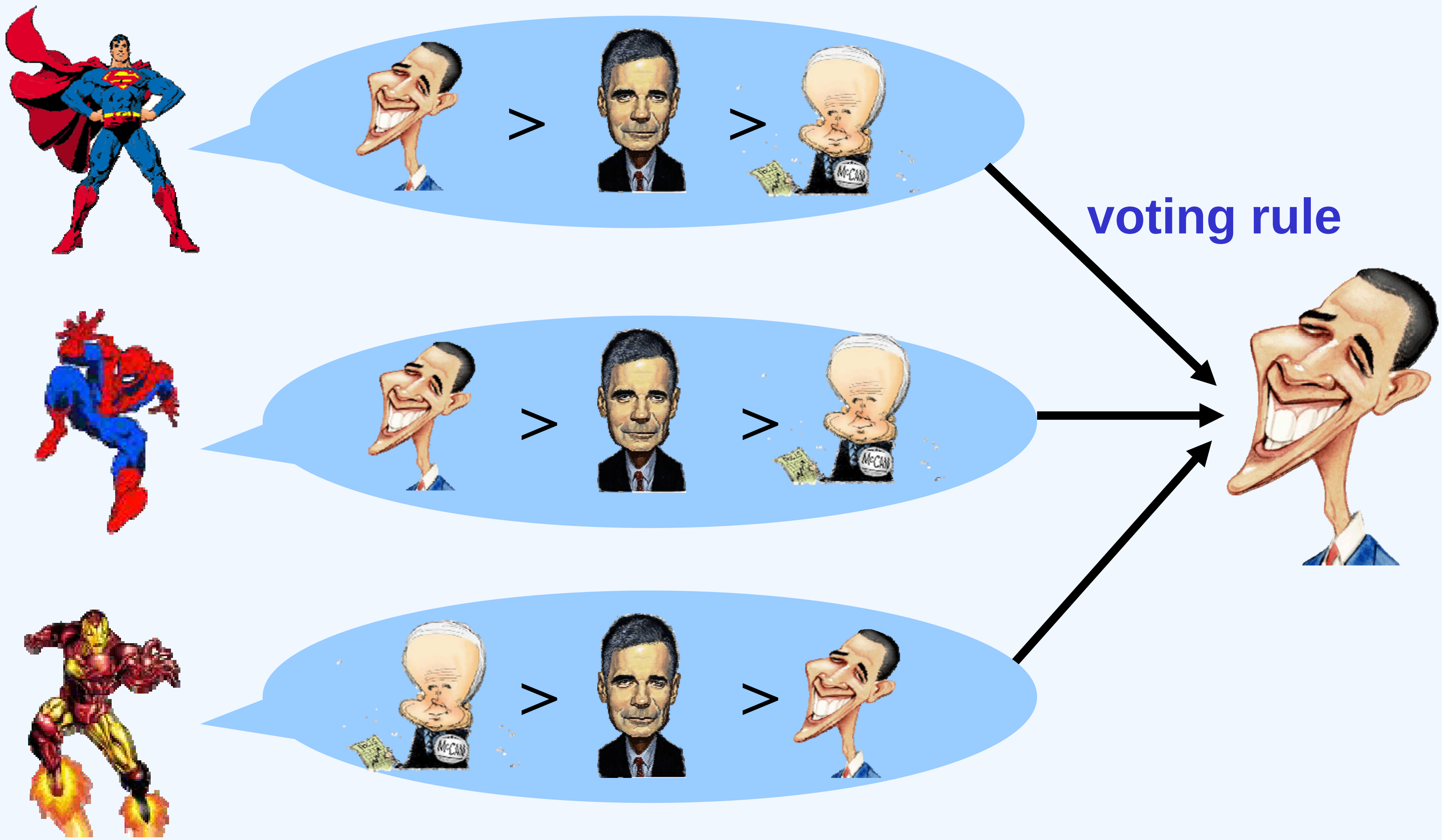
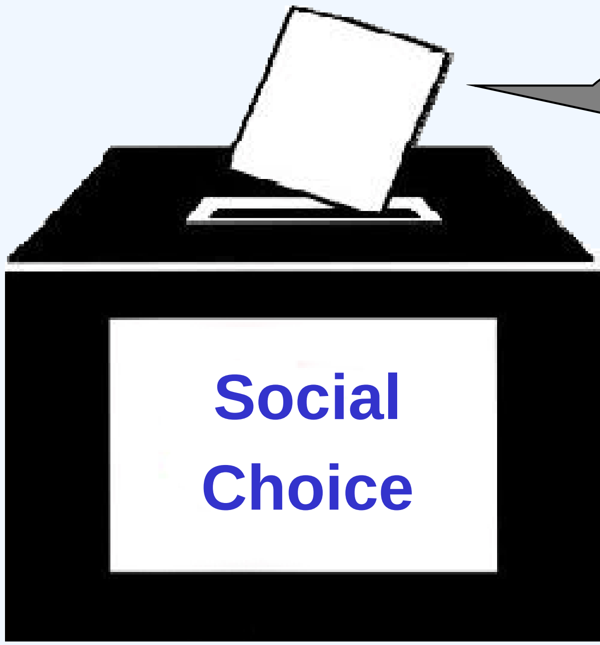
Preference Aggregation: Social Choice

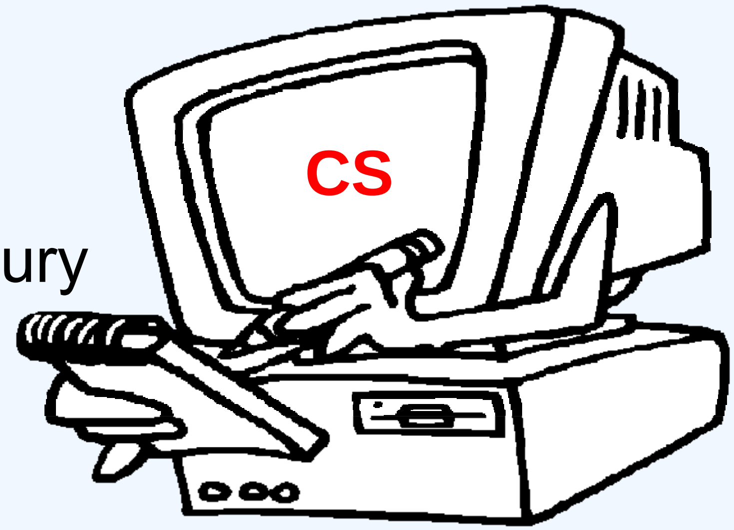


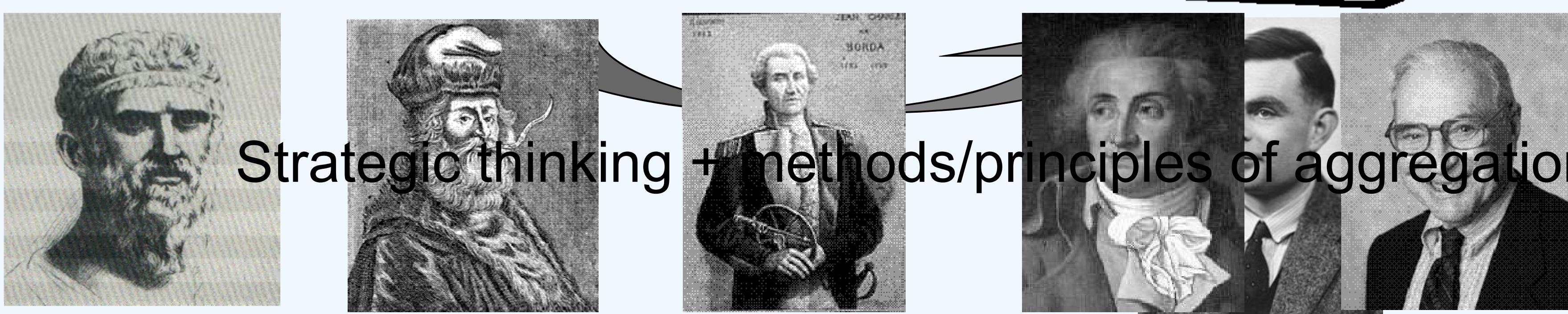
Social Choice and Computer Science

Computational thinking + optimization algorithms



21th Century





Strategic thinking + methods/principles of aggregation

PLATO 4thC. B.C.

LULL 13thC.

BORDA 18thC.

CONDORCET 18thC.

ARROW 20thC.

PLATO et al. 4thC. B.C.---20thC.

TURING et al. 20thC.

Many applications

- People/agents often have conflicting preferences, yet they have to make a joint decision

U.S.News & WORLD REPORT BEST GRADUATE SCHOOLS 2011

NETFLIX BEST BUY facebook Like amazonmechanical turk Artificial Artificial Intelligence

Applications

- Multi-agent systems [Ephrati and Rosenschein 91]
- Recommendation systems [Ghosh et al. 99]
- Meta-search engines [Dwork et al. 01]
- Belief merging [Everaere et al. 07]
- Human computation (crowdsourcing)
- etc.

A burgeoning area

- Recently has been drawing a lot of attention
  - IJCAI-11: 15 papers, best paper
  - AAAI-11: 6 papers, best paper
  - AAMAS-11: 10 full papers, best paper runner-up
  - AAMAS-12 9 full papers, best student paper
  - EC-12: 3 papers
- Workshop: COMSOC Workshop 06, 08, 10, 12
- Courses taught at Technical University Munich (Felix Brandt), Harvard (Yiling Chen), U. of Amsterdam (Ulle Endriss)

Flavor of this tutorial

- High-level objectives for
  - design
  - evaluation
  - logic flow among research topics

"Give a man a fish and you feed him for a day.

Teach a man to fish and you feed him for a lifetime."

-----Chinese proverb

- Plus some concrete examples of research directions

Outline

30 min **1. Traditional Social Choice**

45 min **2. Game-theoretic aspects**

45 min **3. Combinatorial voting**

45 min **4. MLE approaches**

Outline

1. Traditional Social Choice

2. Game-theoretic aspects

3. Combinatorial voting

4. MLE approaches

How to design a good social choice (voting) rule?

What is "good"?

Objectives of social choice rules

- **OBJ1:** Compromise among subjective preferences

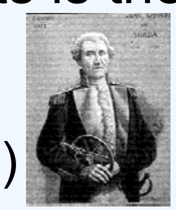
1. Traditional Social Choice

- **OBJ2:** Reveal the "truth"

4. MLE approaches

Common voting rules

(what has been done in the past two centuries)

- Mathematically, a voting rule is a mapping from {All profiles} to {outcomes}
  - an outcome is usually a winner, a set of winners, or a ranking
  - m : number of alternatives (candidates)
  - n : number of voters
- Positional scoring rules
  - A score vector $s_1,\ldots,s_m$
  - For each vote V, the alternative ranked in the i-th position gets s_i points
  - The alternative with the most total points is the winner
  - Special cases
    - Borda, with score vector $(m\text{-}1, m\text{-}2, \ldots,0)$ 
    - Plurality, with score vector $(1,0,\ldots,0)$ [Used in the US]

An example

- Three alternatives $\{c_1, c_2, c_3\}$
- Score vector $(2,1,0)$ (=Borda)
- 3 votes,

$$\underset{2}{c_1} > \underset{1}{c_2} > \underset{0}{c_3} \qquad \underset{2}{c_2} > \underset{1}{c_1} > \underset{0}{c_3} \qquad \underset{2}{c_3} > \underset{1}{c_1} > \underset{0}{c_2}$$

- c_1 gets 2+1+1=4, c_2 gets 1+2+0=3,

  c_3 gets 0+0+2=2
- The winner is c_1

Plurality with runoff

- The election has two rounds
  - In the first round, all alternatives except the two with the highest plurality score drop out
  - In the second round, the alternative that is preferred by more voters wins
- [used in North Carolina State]

$a > b > c > d$	$d > a > b > c$	$c > d > a > b$	$b > c > d > a$
10	7	6	3

 d

Single transferable vote (STV)

- Also called instant run-off voting or alternative vote
- The election has m-1 rounds, in each round,
  - The alternative with the lowest plurality score drops out, and is removed from all of the votes
  - The last-remaining alternative is the winner
- [used in Australia and Ireland]

$a > b > c > d$	$d > a > b > c$	$c > d > a > b$	$b > c > d > a$
10	7	6	3


a

Kemeny

- Kendall's tau distance
  - $K(V,W)$= # {different pairwise comparisons}

$$K(\, b \succ c \succ a \,,\, a \succ b \succ c \,) = ?$$

- Kemeny(P)=argmin$_W$ $K(P,W)$=argmin$_W$ $\Sigma_{V\in P} K(P,W)$
- [has an MLE interpretation]

…and many others

- Approval, Baldwin, Black, Bucklin, Coombs, Copeland, Dodgson, maximin, Nanson, Range voting, Schulze, Slater, ranked pairs, etc…

- Q: How to evaluate rules in terms of compromising subjective preferences?
- A: Axiomatic approach
  - Preferences are ordinal and utilities might not be transferable

Axiomatic approach

(what has been done in the past 50 years)

- Anonymity: names of the voters do not matter
  - Fairness for the voters
- Non-dictatorship: there is no dictator, whose top-ranked alternative is always the winner
  - Fairness for the voters

- Neutrality: names of the alternatives do not matter
  - Fairness for the alternatives
- Condorcet consistency: if there exists a *Condorcet winner*, then it must win
  - A Condorcet winner beats all other alternatives in pairwise elections
- Consistency: if $r(P_1)\cap r(P_2)\neq \phi$, then $r(P_1\cup P_2)=r(P_1)\cap r(P_2)$
- Strategy-proofness: no voter can cast a false vote to improve the outcome of election
- Easy to compute: winner determination is in P
  - Computational efficiency of preference aggregation
- Hard to manipulate: computing a beneficial false vote is hard
  - More details in the next section

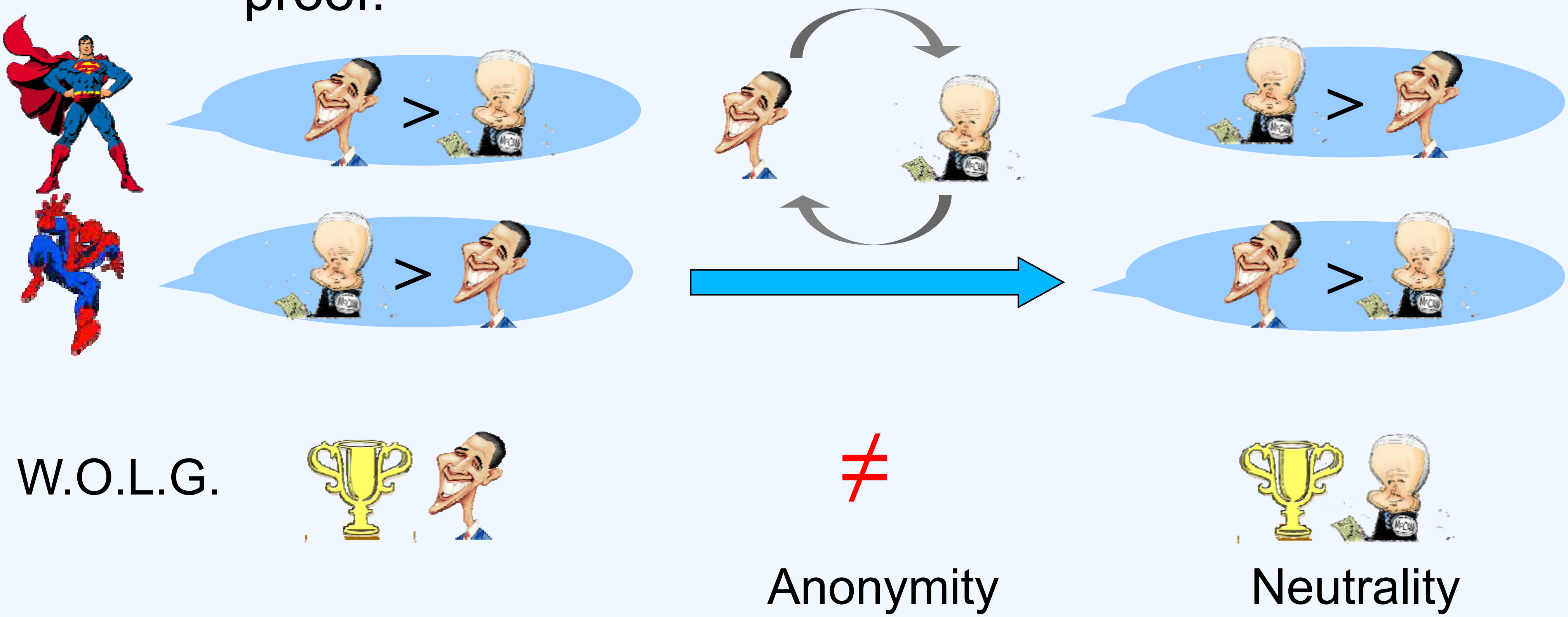
Which axiom is more important?

	Condorcet consistency	Consistency	Polynomial-time computable
Positional scoring rules	N	Y	Y
plurality with runoff	N	N	Y
STV	N	N	Y
Kemeny	Y	N	N
Ranked pairs	Y	N	Y

- Some of them are not compatible with each other

An easy fact

- Thm. For voting rules that selects a single winner, anonymity is not compatible with neutrality
  - proof:



Another easy fact
[Fishburn APSR-74]

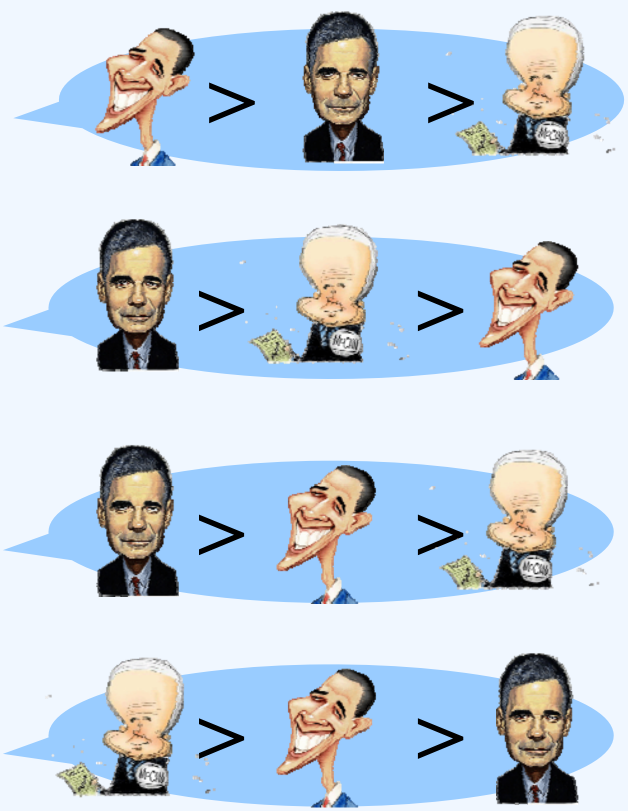
- Thm. No positional scoring rule is Condorcet consistent:
  - suppose $s_1 > s_2 > s_3$

3 Voters

2 Voters

1 Voter

1 Voter



 is the Condorcet winner

 : $3s_1 + 2s_2 + 2s_3$

$\wedge$

 : $3s_1 + 3s_2 + 1s_3$

CONTRADICTION

Not-So-Easy facts

- Arrow's impossibility theorem
  - Google it!
- Gibbard-Satterthwaite theorem
  - Next section
- Axiomatic characterization
  - Template: A voting rule satisfies axioms A1, A2, A2 if and only if it is rule X
  - If you believe in A1 A2 A3 altogether then X is optimal

Food for thought

- Can we quantify a voting rule's satisfiability of these axiomatic properties?
  - Tradeoffs between satisfiability of axioms
  - Use computational techniques to design new voting rules
    - CSP to prove or discover new impossibility theorems [Tang&Lin AIJ-09]

Outline

1. Traditional Social Choice

15 min 

2. Game-theoretic aspects

3. Combinatorial voting

4. MLE approaches

Outline

1. Traditional Social Choice

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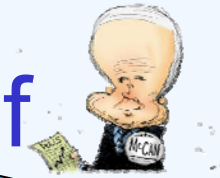
Strategic behavior (of the voters)

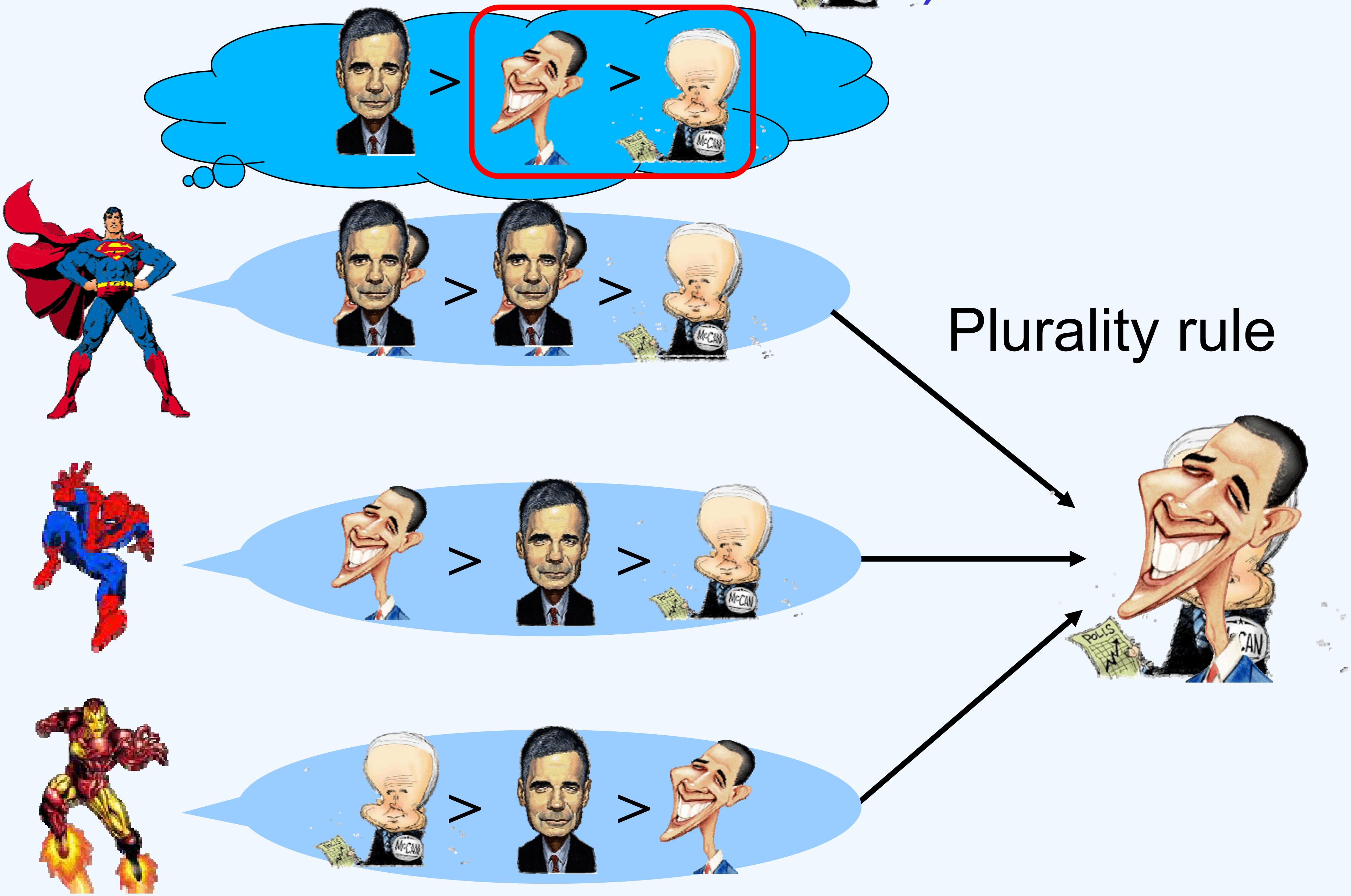
- In most of work before 1970's it was assumed that voters are truthful
- However, sometimes a voter has incentive to lie, to make the winner more preferable
  - according to her true preferences

Strategic behavior

- Manipulation: a voter (manipulator) casts a vote that does not represent her true preferences, to make herself better off
- A voting rule is strategy-proof if there is never a (beneficial) manipulation under this rule
- How important strategy-proofness is as an desired axiomatic property?
  - compared to other axiomatic properties

Manipulation under plurality rule

(ties are broken in favor of )



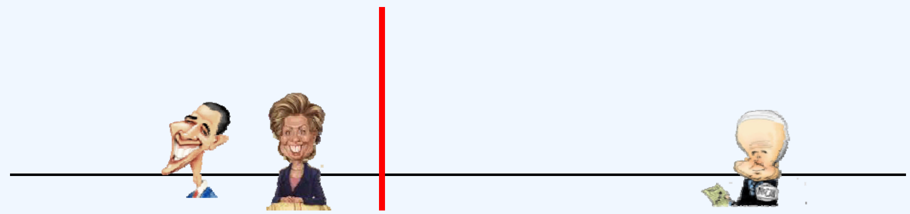
Any strategy-proof voting rule?

No reasonable voting rule is strategyproof

- Gibbard-Satterthwaite Theorem [Gibbard Econometrica-73, Satterthwaite JET-75]: When there are at least three alternatives, no voting rules except dictatorships satisfy
  - non-imposition: every alternative wins for some profile
  - unrestricted domain: voters can use any linear order as their votes
  - strategy-proofness
- Axiomatic characterization for dictatorships!

A few ways out

- Relax non-dictatorship: use a dictatorship
- Restrict the number of alternatives to be 2
- Relax unrestricted domain: mainly pursued by economists



  - Single-peaked preferences:
  - Range voting: A voter submit any natural number between 0 and 10 for each alternative
  - Approval voting: A voter submit 0 or 1 for each alternative

Computational ways out

- Use a voting rule that is too complicated so that nobody can easily figure out who will be the winner
  - Dodgson: computing the winner is Θ_2^p-complete [Hemaspaandra, Hemaspaandra, &Rothe JACM-97]
  - Kemeny: computing the winner is NP-hard [Bartholdi, Tovey, &Trick SCW-89] and Θ_2^p-complete [Hemaspaandra, Spakowski, & Vogel TCS-05]
  - The randomized voting rule used in Venice Republic for more than 500 years [Walsh&Xia AAMAS-12]
- We want a voting rule where
  - Winner determination is easy
  - Manipulation is hard

Overview

Manipulation is inevitable
(Gibbard-Satterthwaite Theorem)

Can we use computational complexity as a barrier?

Yes

Is it a strong barrier?

No

Other barriers?

Limited information
Limited communication

Why prevent manipulation?

May lead to very undesirable outcomes

How often?

Seems not very often

Manipulation: A computational complexity perspective

- If it is computationally too hard for a manipulator to compute a manipulation, she is best off voting truthfully
  - Similar as in cryptography

- For which common voting rules manipulation is computationally hard?

Computing a manipulation

- Study initiated by [Bartholdi, Tovey, &Trick SCW-89b]
- Votes are weighted or unweighted
- Bounded number of alternatives [Conitzer, Sandholm, &Lang JACM-07]
  - Unweighted manipulation is easy for most common rules
  - Weighted manipulation depends on the number of manipulators
- Unbounded number of alternatives (next few slides)
- Assuming the manipulators have complete information!

Unweighted coalitional manipulation (UCM) problem

- Given
  - The voting rule r
  - The non-manipulators' profile P^{NM}
  - The number of manipulators n'
  - The alternative c preferred by the manipulators
- We are asked whether or not there exists a profile P^M (of the manipulators) such that c is the winner of $P^{NM} \cup P^M$ under r

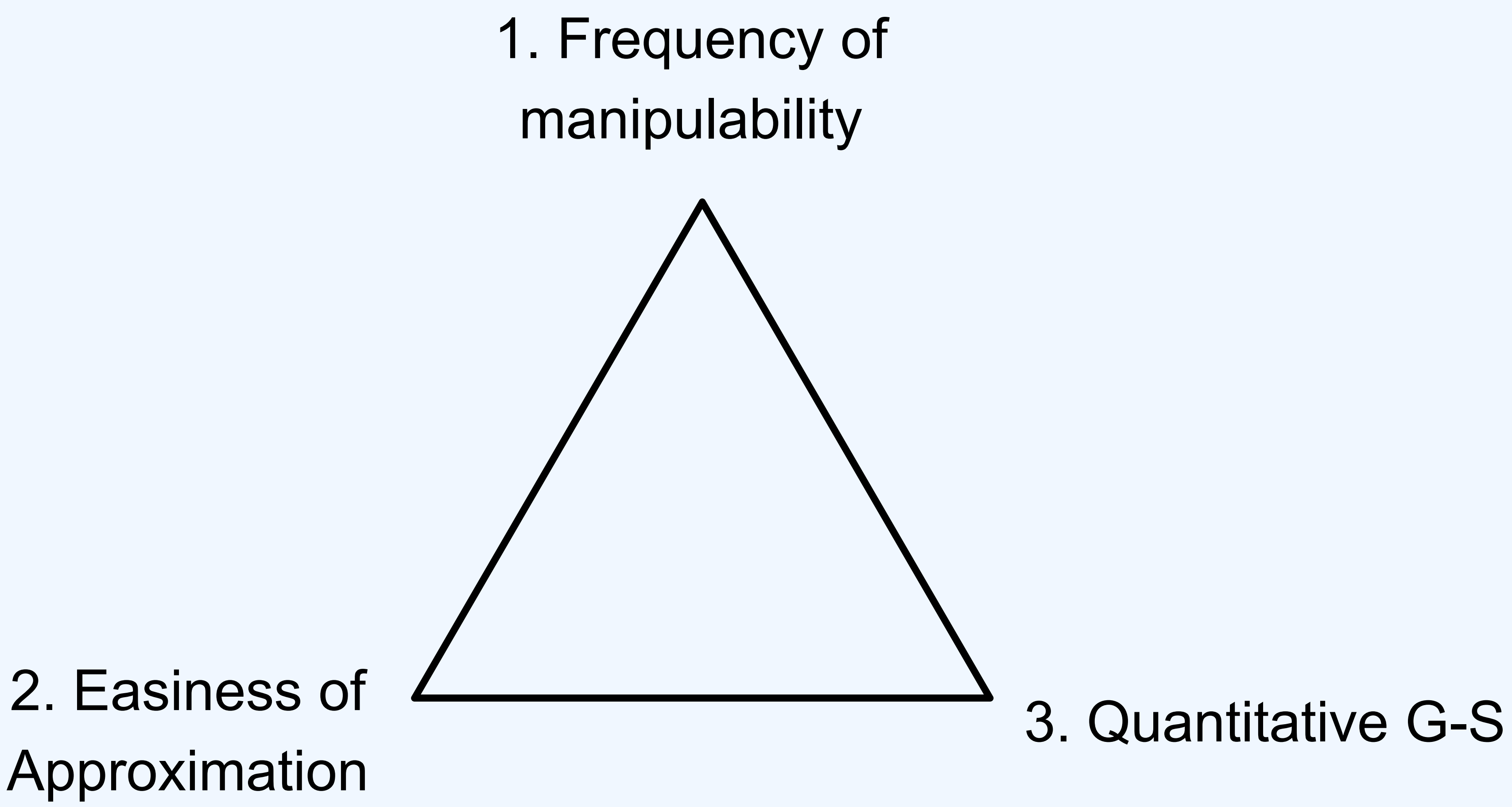
The stunningly big table for UCM

#manipulators	One manipulator	At least two
Copeland	P [BTT SCW-89b]	NPC [FHS AAMAS-08,10]
STV	NPC [BO SCW-91]	NPC [BO SCW-91]
Veto	P [ZPR AIJ-09]	P [ZPR AIJ-09]
Plurality with runoff	P [ZPR AIJ-09]	P [ZPR AIJ-09]
Cup	P [CSL JACM-07]	P [CSL JACM-07]
Borda	P [BTT SCW-89b]	NPC [DKN+ AAAI-11] [BNW IJCAI-11]
Maximin	P [BTT SCW-89b]	NPC [XZP+ IJCAI-09]
Ranked pairs	NPC [XZP+ IJCAI-09]	NPC [XZP+ IJCAI-09]
Bucklin	P [XZP+ IJCAI-09]	P [XZP+ IJCAI-09]
Nanson’s rule	NPC [NWX AAA-11]	NPC [NWX AAA-11]
Baldwin’s rule	NPC [NWX AAA-11]	NPC [NWX AAA-11]

What can we conclude?

- For some common voting rules, computational complexity provides some protection against manipulation
- Is computational complexity a strong barrier?
  - NP-hardness is a worst-case concept

Probably NOT a strong barrier



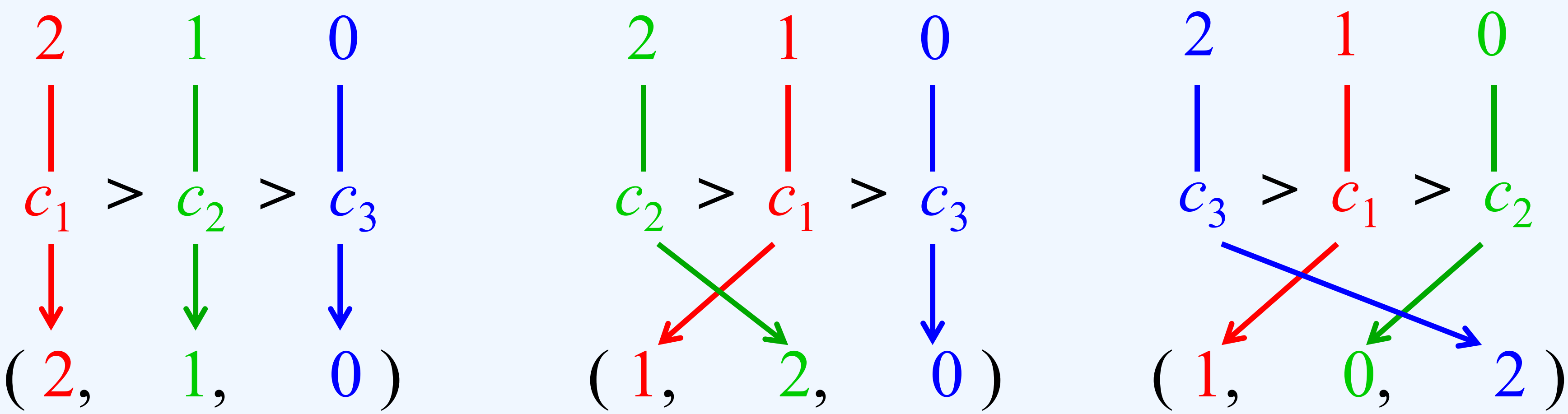
A first angle: frequency of manipulability

- Non-manipulators' votes are drawn i.i.d.
  – E.g. i.i.d. uniformly over all linear orders (the impartial culture assumption)
- How often can the manipulators make c win?
  – Specific voting rules [Peleg T&D-79, Baharad&Neeman RED-02, Slinko T&D-02, Slinko MSS-04, Procaccia and Rosenschein AAMAS-07]

General results?

A slightly different way of thinking about positional scoring rules

- Map each vote to 3 real numbers, such that the i-th component is the score that alternative c_i obtains in this vote.



- Summing up the vectors to get the total score vector:

$$(2, 1, 0) + (1, 2, 0) + (1, 0, 2) = (4, 3, 2)$$

- Comparing the components, we have $1^{st} > 2^{nd} > 3^{rd}$, so the winner is c_1

Generalized scoring rules (GSRs)

[Xia&Conitzer EC-08]

- For any $k \in \mathbf{N}$, a generalized scoring rule $GS(f,g)$ of order k is composed of two functions:
  - $f: L(C) \rightarrow \mathbf{R}^k$
    - Assigns to each linear order a vector of k real numbers, called a generalized score vector (GSV)
  - g: {weak orders over k components} $\rightarrow C$

$$P = (V_1, \quad \ldots \quad, V_n)$$

$$\downarrow$$

$$f(V_1) + \cdots + f(V_n)$$

$$\downarrow$$

$$f(P) = \sum_{i=1}^{n} f(V_i) \longrightarrow \text{Order}\{f(P)\}$$

Weak order over the k components

$$\uparrow$$

$$g(\text{Order}\{f(P)\})$$

STV as a generalized scoring rule

- The components are indexed by (c, S)
  - c is an alternative and S is a subset of other alternatives
  - the value of (c, S) is the plurality score of c given that exactly S has been eliminated from the election

	$(c_1,\emptyset)$	$(c_2,\emptyset)$	$(c_3,\emptyset)$	$(c_2,\{c_1\})$	$(c_3,\{c_1\})$	$(c_1,\{c_2\})$	$(c_3,\{c_2\})$	$(c_1,\{c_3\})$	$(c_2,\{c_3\})$
$c_1 \succ c_2 \succ c_3$	1	0	0	1	0	1	0	1	0
$c_1 \succ c_3 \succ c_2$	1	0	0	0	1	1	0	1	0
$c_3 \succ c_2 \succ c_1$	0	0	1	0	1	0	1	0	1
Sum	**2**	**0**	**1**	**1**	**2**	**2**	**1**	**2**	**1**

- First round: $\arg\min_j (f(P)_{(\emptyset,j)}) = 2,\ S_1 = \{c_2\}$
- Second round: $\arg\min_j (f(P)_{(S_1,j)}) = 3,\ S_2 = S_1 \cup \{c_3\} = \{c_2, c_3\}$
- Therefore, the winner is $A \setminus S_2 = \{c_1\}$

Characterizing frequency of manipulability [Xia&Conitzer EC-08a]

- **Theorem.** For any generalized scoring rule
  - Including many common voting rules

$$\#\text{ manipulators}\quad \frac{\text{All-powerful}}{\text{No power}}\quad \Theta(\sqrt{n})$$

- Computational complexity is not a strong barrier against manipulation
  - UCM as a decision problem is easy to compute in most cases
  - Does NOT mean that it is easy for the manipulators to succeed
  - The case of $\Theta(\sqrt{n})$ has been studied experimentally in [Walsh IJCAI-09]

Idea behind part of the proof

- For any pair of components of the total generalized score vector, with high probability the difference between them is $\omega(\sqrt{n})$
  - Central Limit Theorem
  - $o(\sqrt{n})$ manipulators cannot change the order between any pair of components
    - so they cannot change the winner

Characterizing GSRs

[Xia&Conitzer IJCAI-09]

- **Theorem.** A voting rule is a generalized scoring rule if and only if it satisfies
  - Anonymity
  - Homogeneity
  - Finite local consistency
- Dodgson's rule does not satisfy homogeneity [Brandt MLQ09]
  - Therefore, it is not a GSR

A second angle: approximation

- Unweighted coalitional optimization (UCO): compute the smallest number of manipulators that can make c win
  - A greedy algorithm has additive error no more than 1 for Borda [Zuckerman, Procaccia, &Rosenschein AIJ-09]

An approximation algorithm for positional scoring rules[Xia,Conitzer,& Procaccia EC-10]

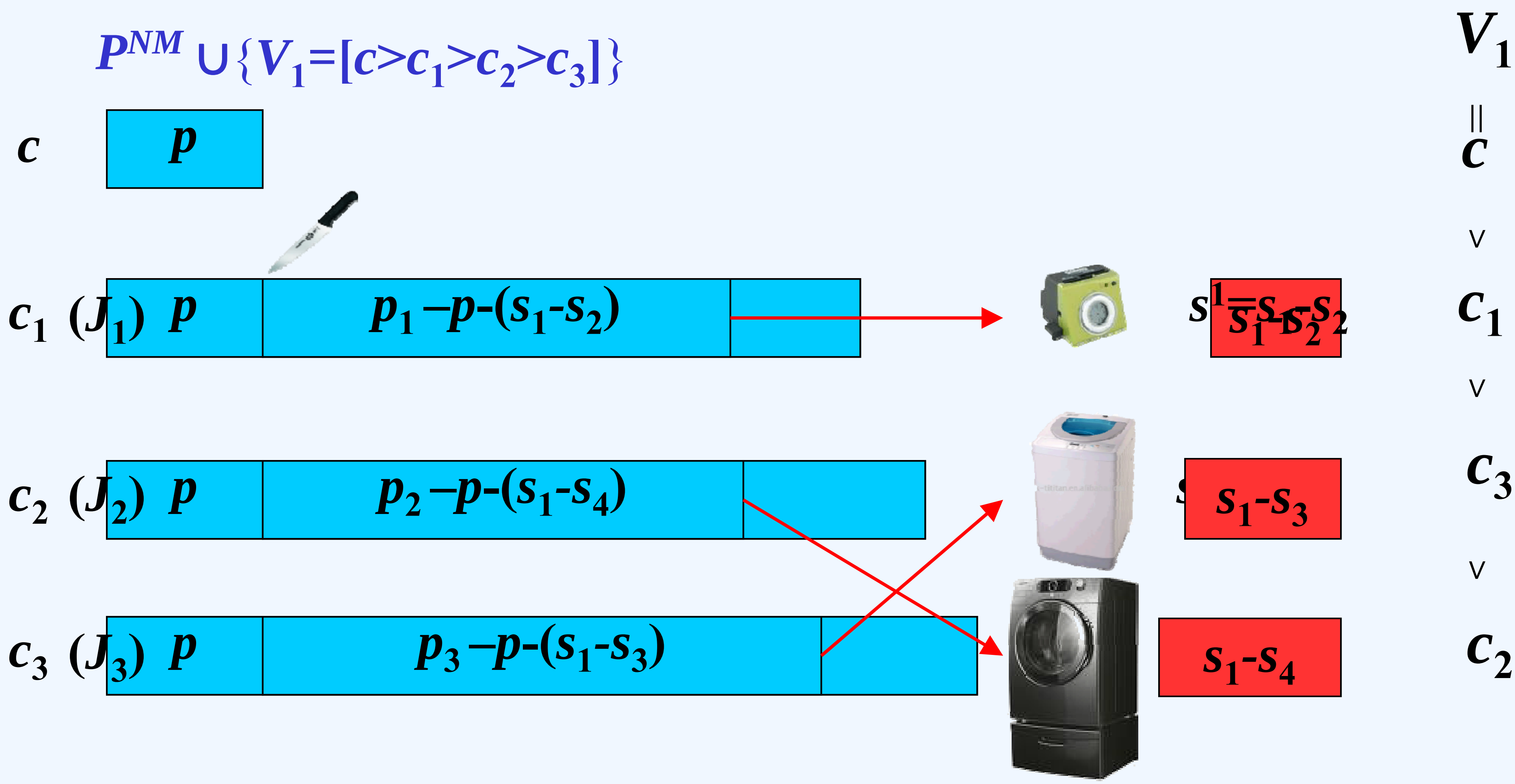
- A polynomial-time approximation algorithm that works for all positional scoring rules
  - Additive error is no more than m-2
  - Based on a new connection between UCO for positional scoring rules and a class of scheduling problems
- Computational complexity is not a strong barrier against manipulation
  - The cost of successful manipulation can be easily approximated (for some rules)

The scheduling problems $Q|pmtn|C_{max}$

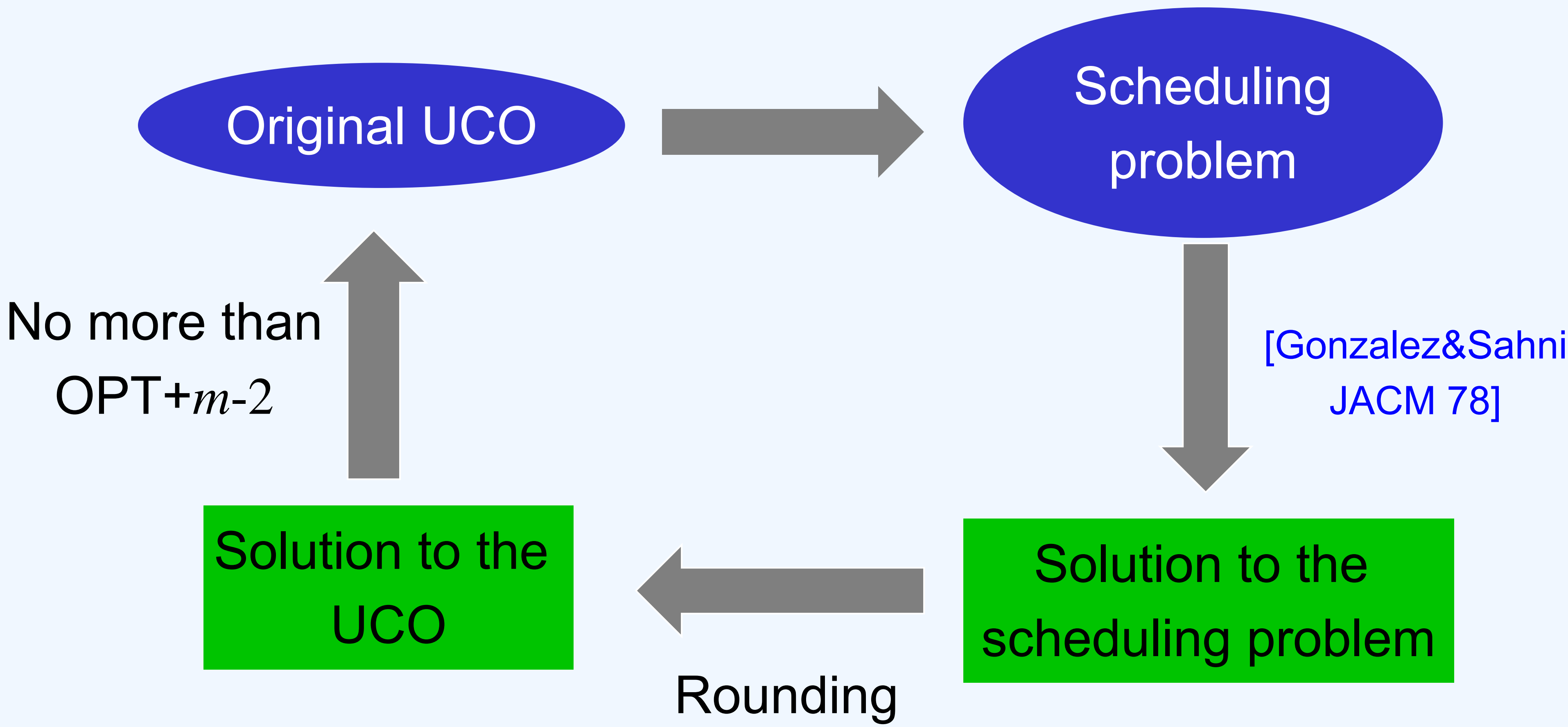
- m^* parallel uniform machines $M_1,\ldots,M_{m^*}$
  - Machine i's speed is s^i (the amount of work done in unit time)
- n^* jobs $J_1,\ldots,J_{n^*}$
- preemption: jobs are allowed to be interrupted (and resume later maybe on another machine)
- We are asked to compute the minimum makespan
  - the minimum time to complete all jobs

Thinking about UCO_{pos}

- Let $p, p_1, \ldots, p_{m-1}$ be the total points that $c, c_1, \ldots, c_{m-1}$ obtain in the non-manipulators' profile



The algorithm in a nutshell



Helps to prove complexity of UCM for Borda

- Manipulation of positional scoring rules = scheduling (preemptions only allowed at integer time points)
  - Borda manipulation corresponds to scheduling where the machines speeds are m-1, m-2, …, 0
    - NP-hard [Yu, Hoogeveen, & Lenstra J.Scheduling 2004]
  - UCM for Borda is NP-C for two manipulators
    - [Davies et al. AAAI-11 best paper]
    - [Betzler, Niedermeier, & Woeginger IJCAI-11 best paper]

A third angle: quantitative G-S

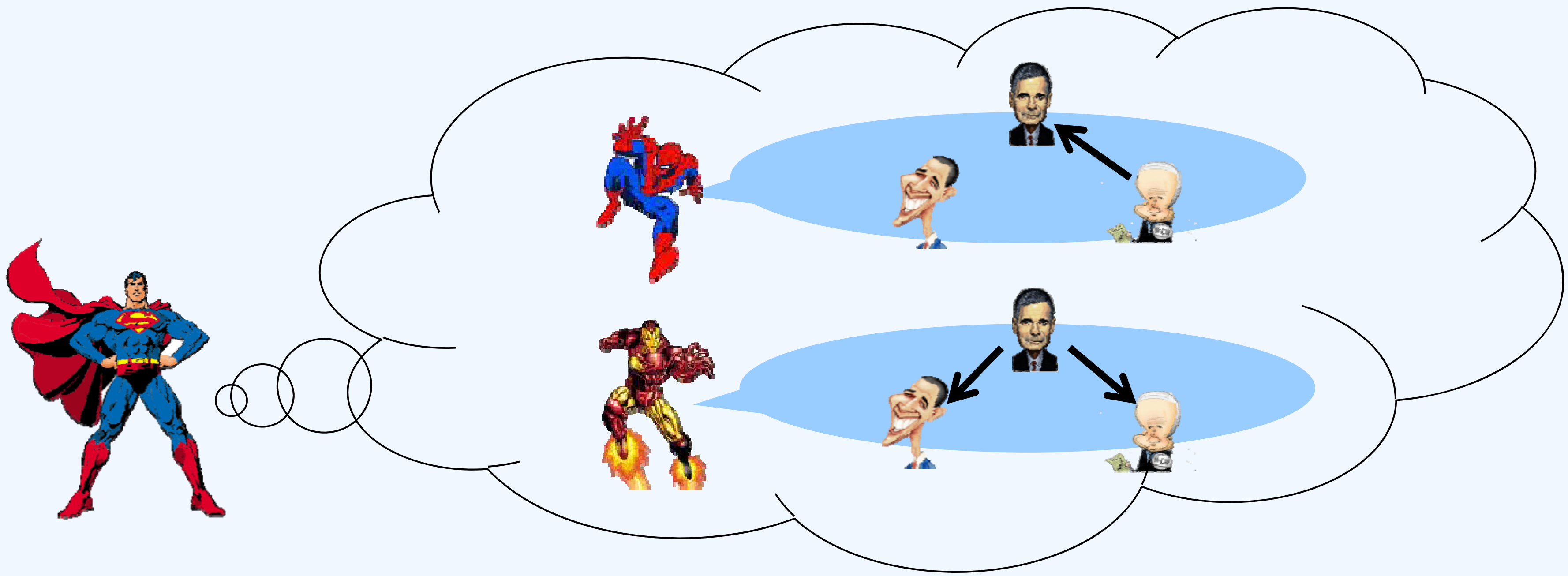
- G-S theorem: for any reasonable voting rule there exists a manipulation
- Quantitative G-S: for any voting rule that is "far away" from dictatorships, the number of manipulable situations is non-negligible
  - First work: 3 alternatives, neutral rule [Friedgut, Kalai, &Nisan FOCS-08]
  - Extensions: [Dobzinski&Procaccia WINE-08, Xia&Conitzer EC-08b, Isaksson,Kindler,&Mossel FOCS-10]
  - Finally solved: [Mossel&Racz STOC-12]

Next step

- The first attempt seems to fail
- Can we obtain positive results for a restricted setting?
  - The manipulators has complete information about the non-manipulators' votes
  - The manipulators can perfectly discuss their strategies

Information constraints
[Conitzer,Walsh,&Xia AAAI-11]

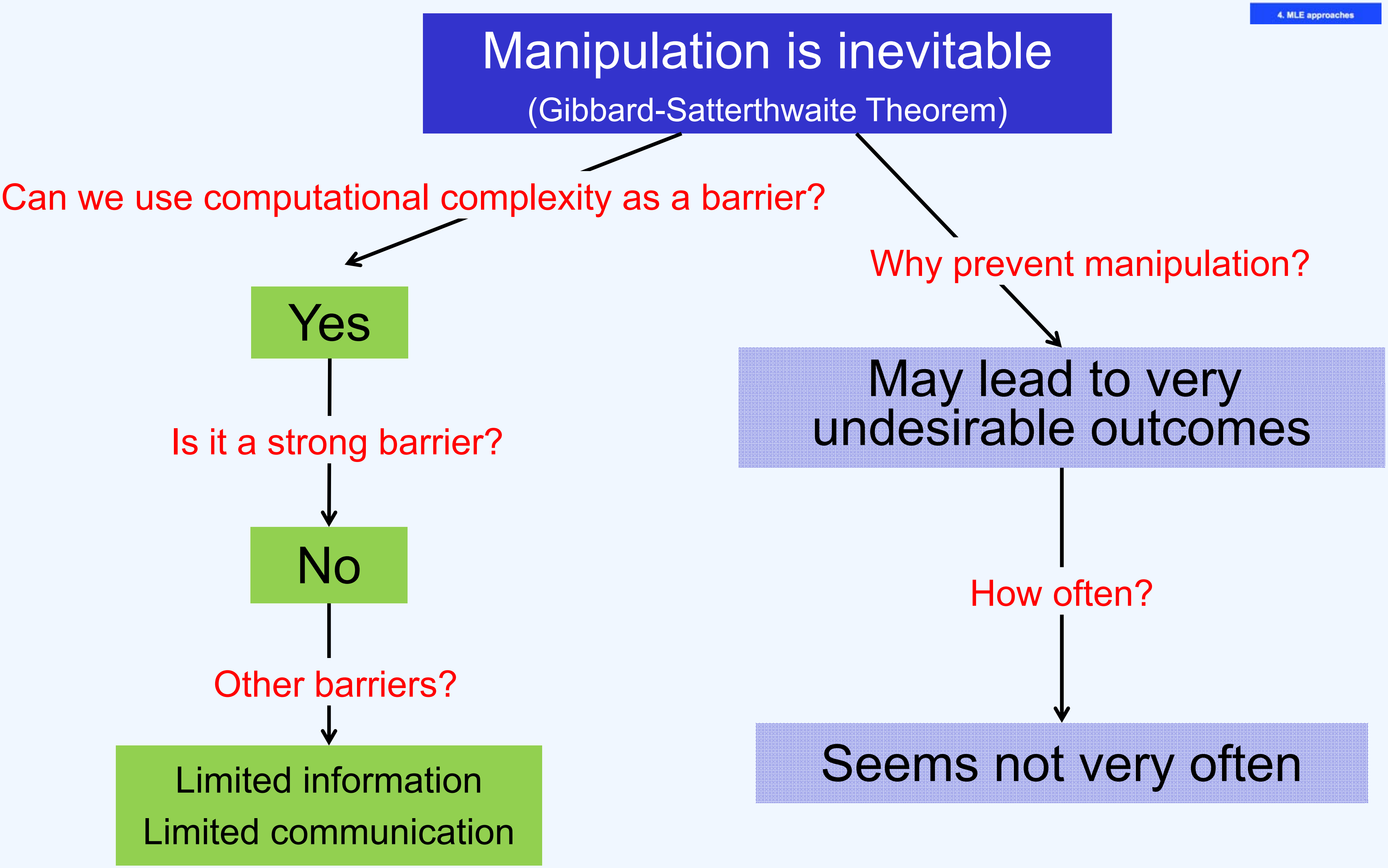
- Limiting the manipulator's information can make dominating manipulation computationally harder, or even impossible



Imperfect communication among manipulators

- The leader-follower model
  - The leader broadcast a vote W, and the potential followers decide whether to cast W or not
    - The leader and followers have the same preferences
  - Safe manipulation [Slinko&White COMSOC-08]: a vote W that
    - No matter how many followers there are, the leader/potential followers are not worse off
    - Sometimes they are better off
  - Complexity: [Hazon&Elkind SAGT-10, Ianovski et al. IJCAI-11]

Overview


Manipulation is inevitable
(Gibbard-Satterthwaite Theorem)
Can we use computational complexity as a barrier?
Yes
Is it a strong barrier?
No
Other barriers?
Limited information
Limited communication
Why prevent manipulation?
May lead to very undesirable outcomes
How often?
Seems not very often

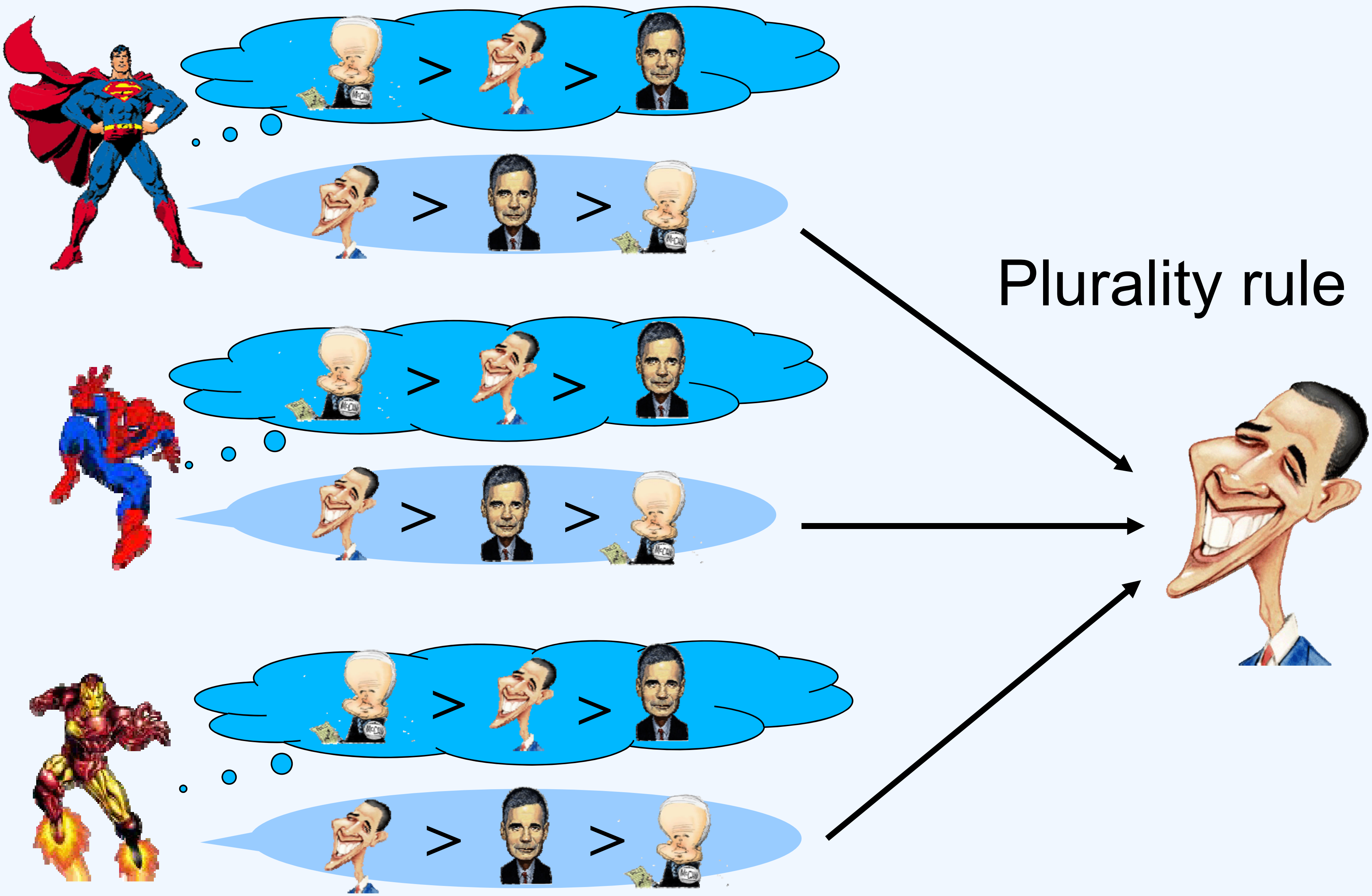
Research questions

- How to predict the outcome?
  - Game theory
- How to evaluate the outcome?
- Price of anarchy [Koutsoupias&Papadimitriou STACS-99]
  - $$\frac{\text{Optimal welfare when agents are truthful}}{\text{Worst welfare when agents are fully strategic}}$$
  - Not very applicable in the social choice setting
    - Equilibrium selection problem
    - Social welfare is not well defined

Simultaneous-move voting games

- Players: Voters $1,\ldots,n$
- Strategies / reports: Linear orders over alternatives
- Preferences: Linear orders over alternatives
- Rule: $r(P')$, where P' is the reported profile

Equilibrium selection problem



Stackelberg voting games

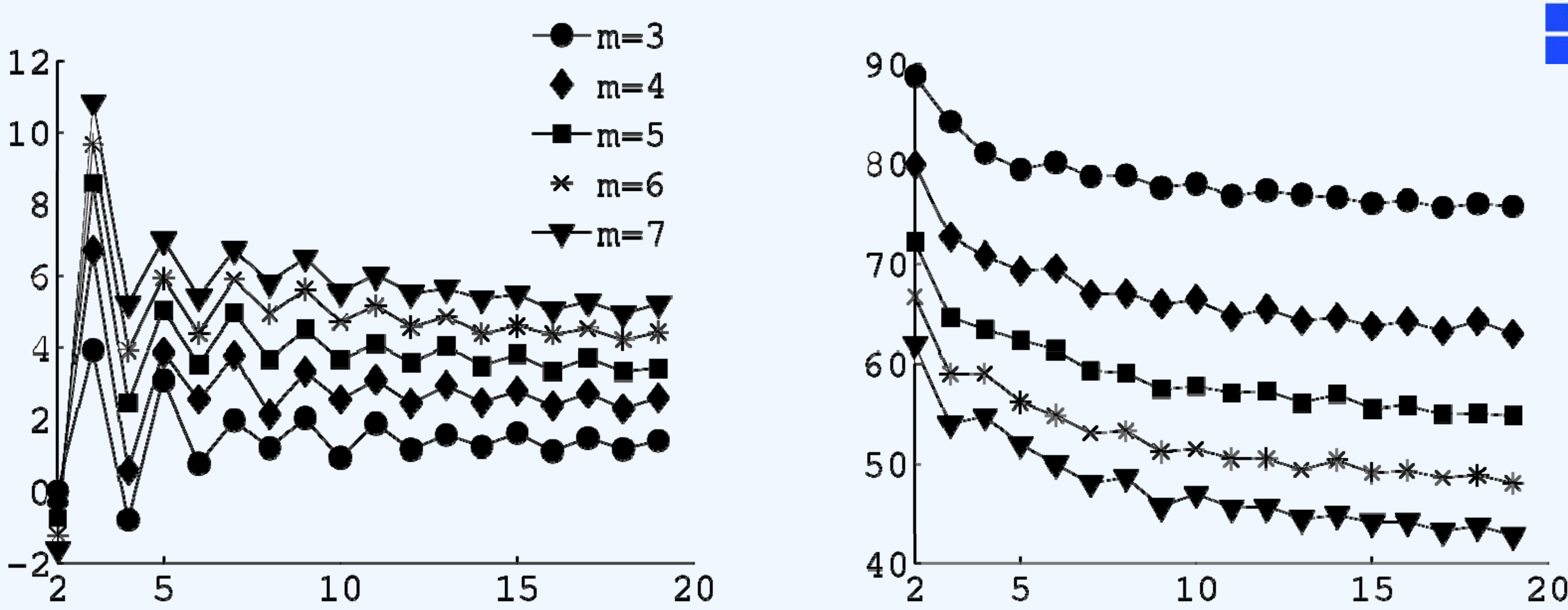
[Xia&Conitzer AAAI-10]

- Voters vote sequentially and strategically
  - voter $1 \rightarrow$ voter $2 \rightarrow$ voter $3 \rightarrow \ldots \rightarrow$ voter n
  - any terminal state is associated with the winner under rule r
- At any stage, the current voter knows
  - the order of voters
  - previous voters' votes
  - true preferences of the later voters (complete information)
  - rule r used in the end to select the winner
- Called a Stackelberg voting game
  - Unique winner in SPNE (not unique SPNE)
  - Similar setting in [Desmedt&Elkind EC-10]

General paradoxes (ordinal PoA)

- **Theorem.** For any voting rule r that satisfies majority consistency and any n, there exists an n-profile P such that:
  - (*many voters are miserable*) $SG_r(P)$ is ranked somewhere in the bottom two positions in the true preferences of n-2 voters
  - (*almost Condorcet loser*) $SG_r(P)$ loses to all but one alternative in pairwise elections
- Strategic behavior of the voters is extremely harmful in the worst case

Simulation results



(a) (b)

- Simulations for the plurality rule (25000 profiles uniformly at random)
  - x: #voters, y: percentage of voters
  - (a) percentage of voters who prefer SPNE winner to the truthful winner minus those who prefer truthful winner to the SPNE winner
  - (b) percentage of profiles where SPNE winner is the truthful winner
- SPNE winner is preferred to the truthful r winner by more voters than vice versa

Other types of strategic behavior (of the chairperson)

- Procedure control by
  - {adding, deleting} × {voters, alternatives}
  - partitioning voters/alternatives
  - introducing clones of alternatives
  - changing the agenda of voting
  - [Bartholdi, Tovey, &Trick MCM-92, Tideman SCW-07, Conitzer,Lang,&Xia IJCAI-09]
- Bribery [Faliszewski, Hemaspaandra, &Hemaspaandra JAIR-09]
- See [Faliszewski, Hemaspaandra, &Hemaspaandra CACM-10] for a survey on their computational complexity
- See [Xia Axriv-12] for a framework for studying many of these for generalized scoring rules

Food for thought

- The problem is still open!
  - Shown to be connected to integer factorization
    [Hemaspaandra, Hemaspaandra, & Menton Arxiv-12]
- What is the role of computational complexity in analyzing human/self-interested agents' behavior?
  - NP-hardness might not be a good answer, but it can be seen as a desired "axiomatic" property
  - Explore information assumption
  - In general, why do we want to prevent strategic behavior?
- Practical ways to protect election

Outline

1. Traditional Social Choice

2. Game-theoretic aspects

10 min

3. Combinatorial voting

4. MLE approaches

Outline

1. Traditional Social Choice

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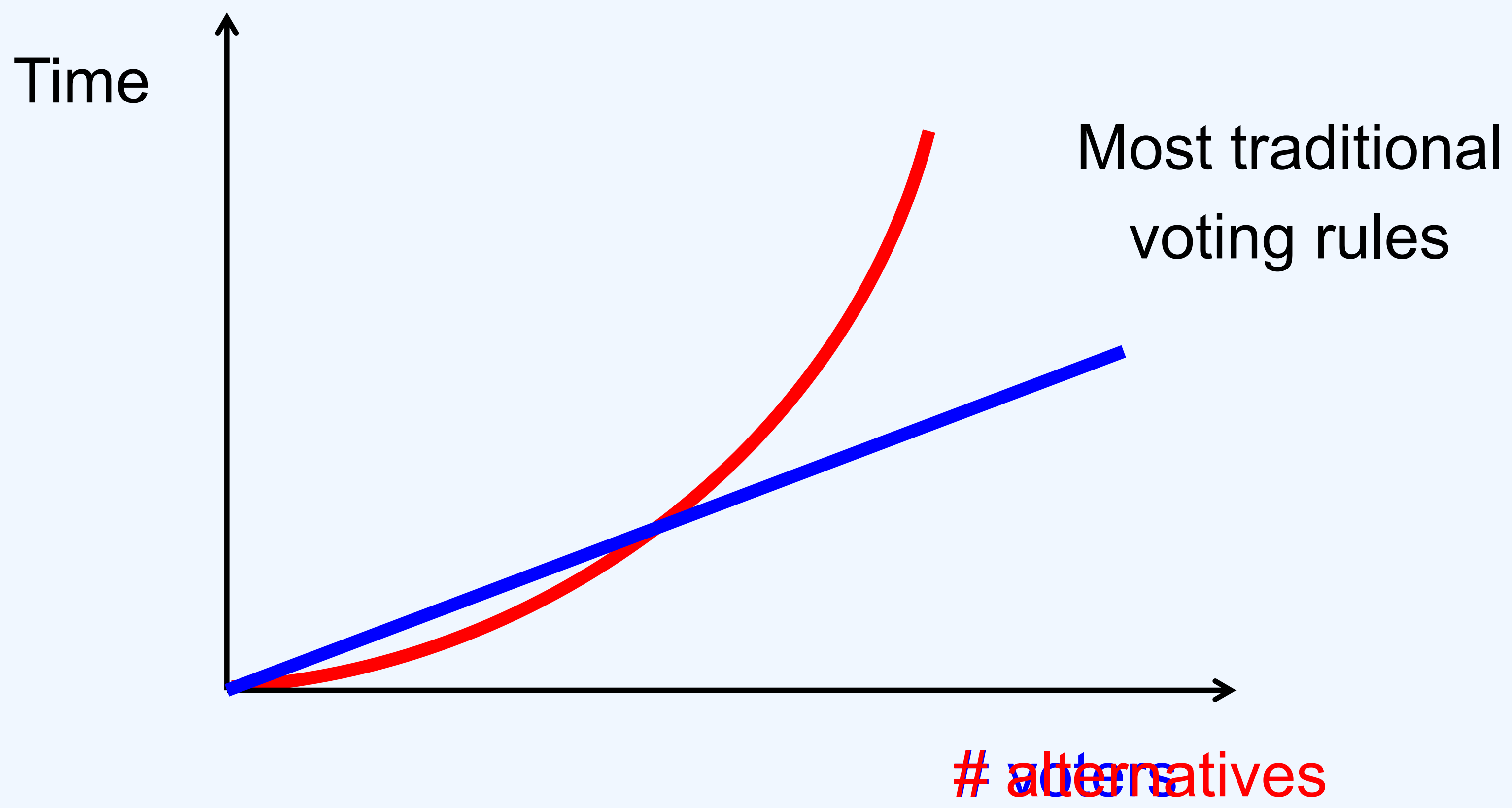
Winner determination for traditional voting rules



Settings with exponentially many alternatives

- Representation/communication: How do voters communicate their preferences?

- Computation: How do we efficiently compute the outcome given the votes?

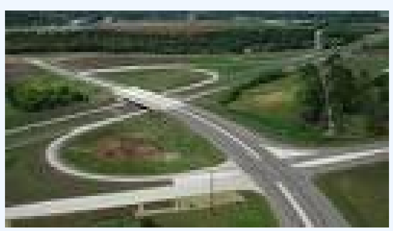
Combinatorial domains (Multi-issue domains)

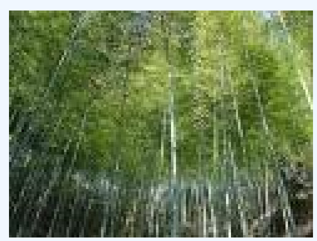
- The set of alternatives can be uniquely characterized by multiple issues
- Let $I=\{x_1,\ldots,x_p\}$ be the set of p issues
- Let D_i be the set of values that the i-th issue can take, then $C=D_1\times\ldots\times D_p$
- Example:
  - Issues={ Main course, Wine }
  - Alternatives={   } ×{   }

Example: joint plan

[Brams, Kilgour & Zwicker SCW 98]

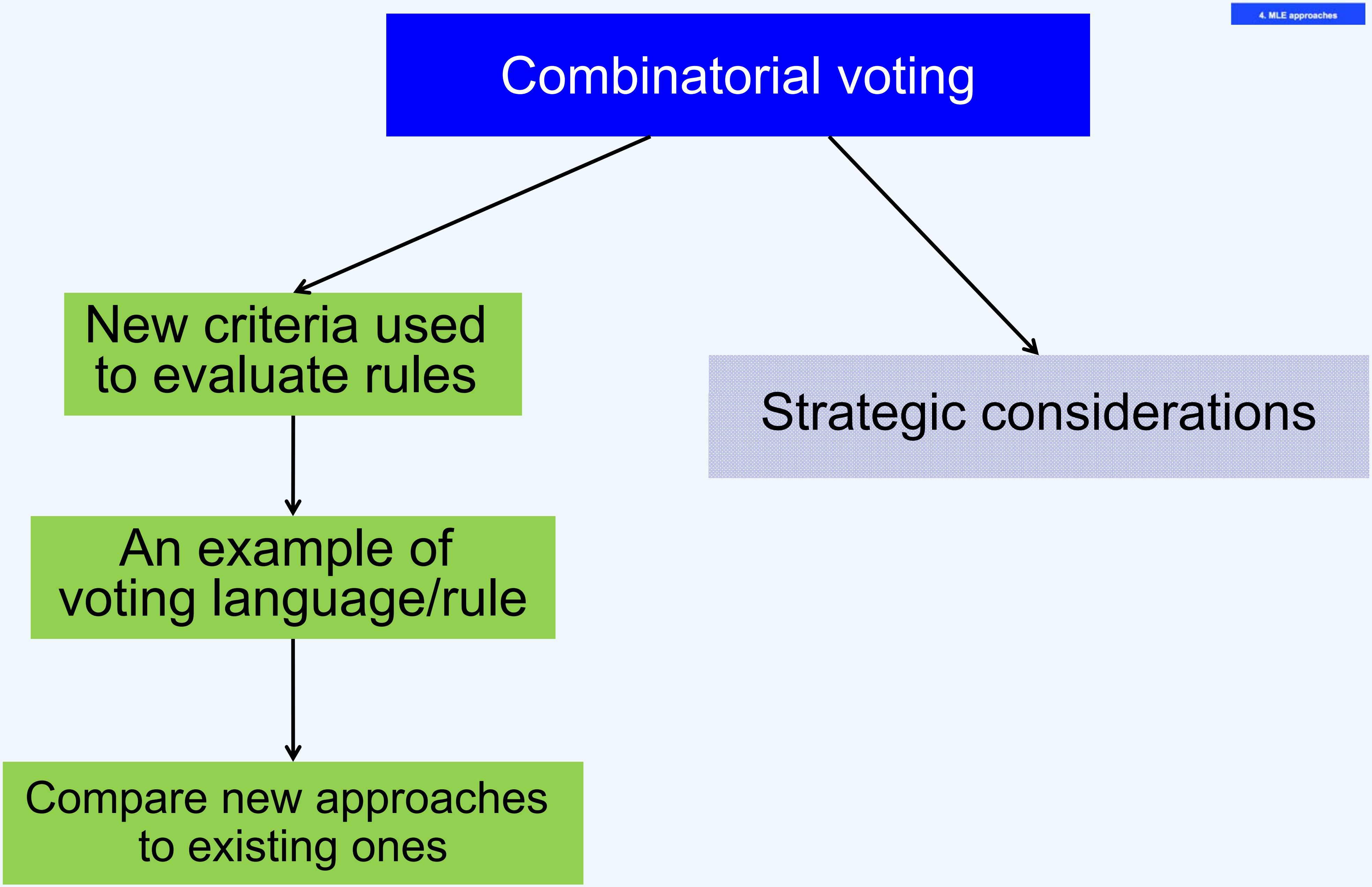
- The citizens of LA county vote to directly determine a government plan
- Plan composed of multiple sub-plans for several issues
  - E.g.,





- # of alternatives is exponential in the # of issues

Overview


Combinatorial voting
New criteria used to evaluate rules
An example of voting language/rule
Compare new approaches to existing ones
Strategic considerations

Criteria for combinatorial voting

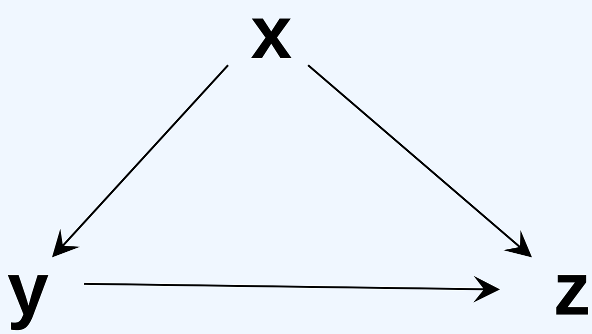
- Criteria for the voting language
  - Compactness
  - Expressiveness
    - Usability: how comfortable voters are about it
    - Informativeness: how much information is contained
- Criteria for the voting rule
  - Computational efficiency
  - Whether it satisfies desirable axiomatic properties

CP-net [Boutilier et al. JAIR-04]

- An CP-net consists of
  - A set of variables $x_1,\ldots,x_p$, taking values on $D_1,\ldots,D_p$
  - A directed graph G over $x_1,\ldots,x_p$
  - Conditional preference tables (CPTs) indicating the conditional preferences over x_i, given the values of its parents in G
- c.f. Bayesian network
  - Conditional probability tables
  - A BN models a probability distribution, a CP-net models a partial order

CP-nets: An example

Variables: x,y,z. $D_x = \{x, \bar{x}\}, D_y = \{y, \bar{y}\}, D_z = \{z, \bar{z}\}$.

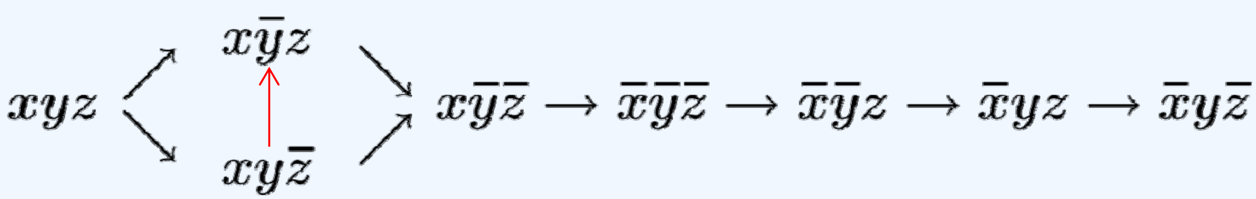


Graph

CPT(x)	CPT(y)	CPT(z)
$x \succ \bar{x}$	$x : y \succ \bar{y}$	$xy : z \succ \bar{z}$
	$\bar{x} : \bar{y} \succ y$	$x\bar{y} : z \succ \bar{z}$
		$\bar{x}y : z \succ \bar{z}$
		$\bar{x}\bar{y} : \bar{z} \succ z$

CPTs

This CP-net encodes the following partial order:



Lexicographic extension w.r.t. x>y>z

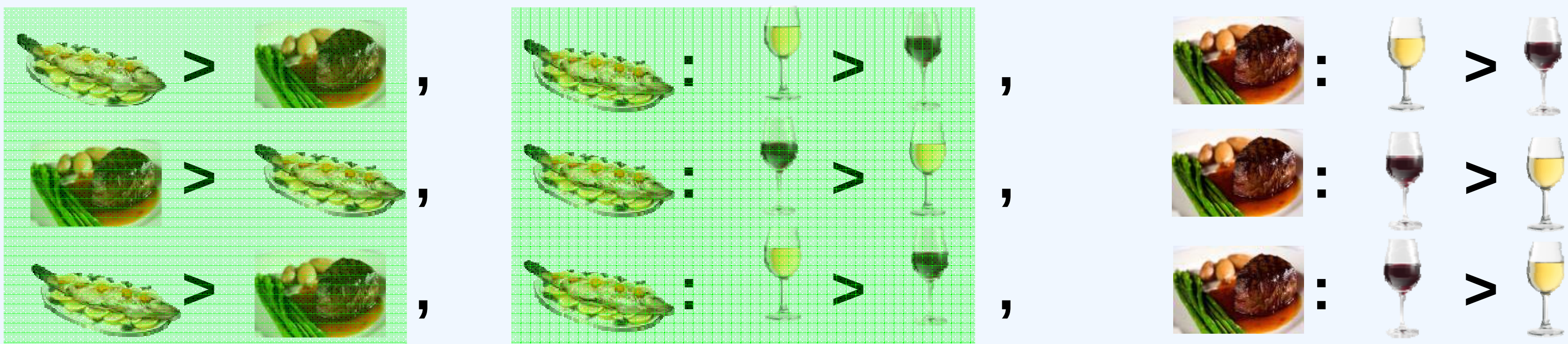
Inference in CP-nets

- The dominance problem: decide where an alternative a is preferred to alternative b
- NP-complete for acyclic CP-nets [Boutilier et al. JAIR-04]
  - P for some special cases
- PSPACE-hard for cyclic CP-nets [Goldsmith et al. JAIR-08]

Sequential voting rules

[Lang IJCAI-07, Lang&Xia MSS-09]

- Issues: main course, wine
- Order: main course > wine
- Local rules are majority rules
- V_1:
- V_2:
- V_3:



- **Step 1:** 
- **Step 2: given**  **,**  **is the winner for wine**
- **Winner: (**  **,**  **)**

Axiomatic property of sequential voting [Lang&Xia MSS-09]

Axiomatic property	Global to local	Local to global
Anonymity	Y	Y
Neutrality	Y	N
Monotonicity	Only last local rule	Only last local rule
Consistency	Y	Y
Participation	Y	N
Pareto Efficiency	Y	N
Strong monotonicity	Y	Y

Quantifying the criteria for the voting language

- Compactness
  - number of bits used to encode the elements in the language
- Expressiveness
  - Usability
    - Suppose a voter's preferences are a linear order over all 2^p alternatives
    - We say that a voter is comfortable if she can find at least one element in the language that is consistent with her preferences

$$\frac{\text{\# linear orders that are consistent with some element in the language}}{\text{\# all linear orders}}$$

  - Informativeness: $\frac{\text{\# Pairwise comparisons encoded by an element}}{2^p(2^p-1)/2}$
- Mainly used to evaluate languages that encodes partial orders

Previous approaches

Voting rule	Computational efficiency	Compactness	Expressiveness	
			Usability	Informativeness
Plurality	High	High	High	~~Low~~
Borda, etc.	~~Low~~	~~Low~~	High	High
Issue-by-issue	High	High	~~Low~~	Medium

We want a balanced rule!

Sequential voting vs. issue-by-issue voting

Voting rule	Computational efficiency	Compactness	Expressiveness	
			Usability	Informativeness
Plurality	High	High	High	Low
Borda, etc.	Low	Low	High	High
Issue-by-issue	High	High	Low	Medium
Sequential voting	High	Usually high	Medium	Medium

Acyclic CP-nets
(compatible with the same ordering)

Usability of acyclic CP-nets

[Xia, Conitzer, &Lang AAAI-08]

- **Theorem**

$$\frac{\text{\# linear orders compatible with acyclic CP-nets}}{\text{\# all linear orders}}$$

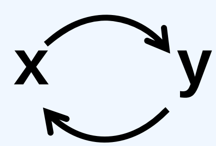
is **exponentially** small (in 2^p)

- Acyclic CP-nets are still too restrictive

Generalization

- Cyclic CP-net + local rules
- Why?
  - Any linear order is consistent with a (possibly) cyclic CP-net
    - CP-nets with a complete graph (each edge has both directions)
    - Cyclic CP-nets has high usability

$$xy \succ \bar{x}\bar{y} \succ x\bar{y} \succ \bar{x}y$$



CPT(y)	CPT(x)
$x : y \succ \bar{y}$	$y : x \succ \bar{x}$
$\bar{x} : \bar{y} \succ y$	$\bar{y} : \bar{x} \succ x$

  - CP-nets encode "localized" preferential information

H-composition

[Xia, Conitzer, &Lang AAAI-08]

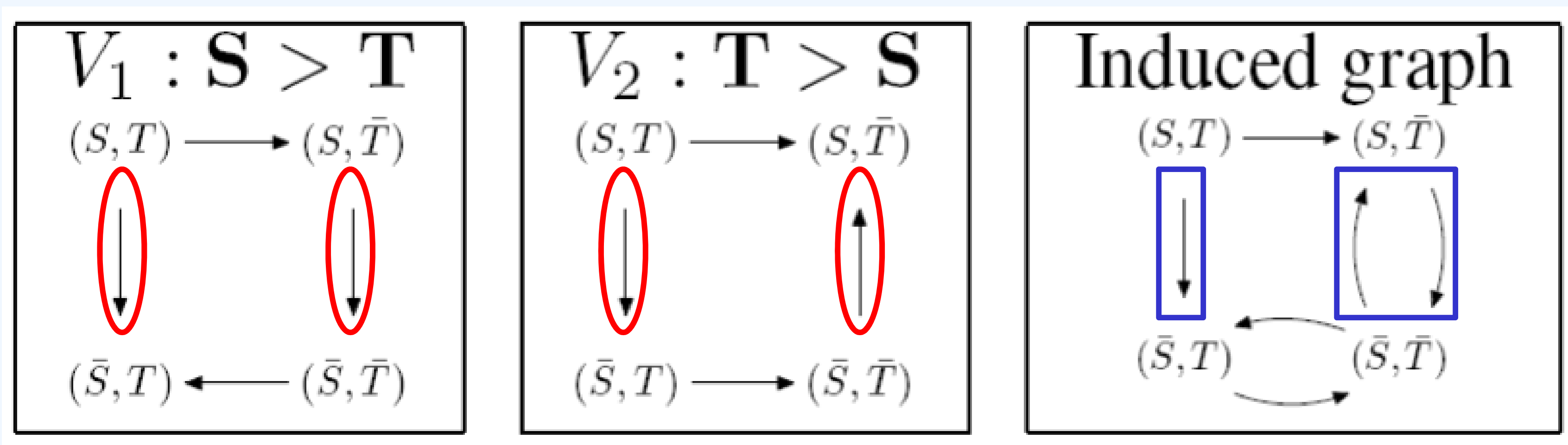
- For any variable x_i and any valuation of the other variables (context), use r_i to select the winners in this context
- In the induced graph, draw an edge from any winner to any other candidates in the same context.
- Use a choice set function to select the global winner based on this graph

H-composition: an example

- Local rules: majority rules
- Choice set: Schwartz set
  - The set of “top” nodes

S

T



$$H_{Schwartz}(V_1, V_2) = \{(S,T)\}$$

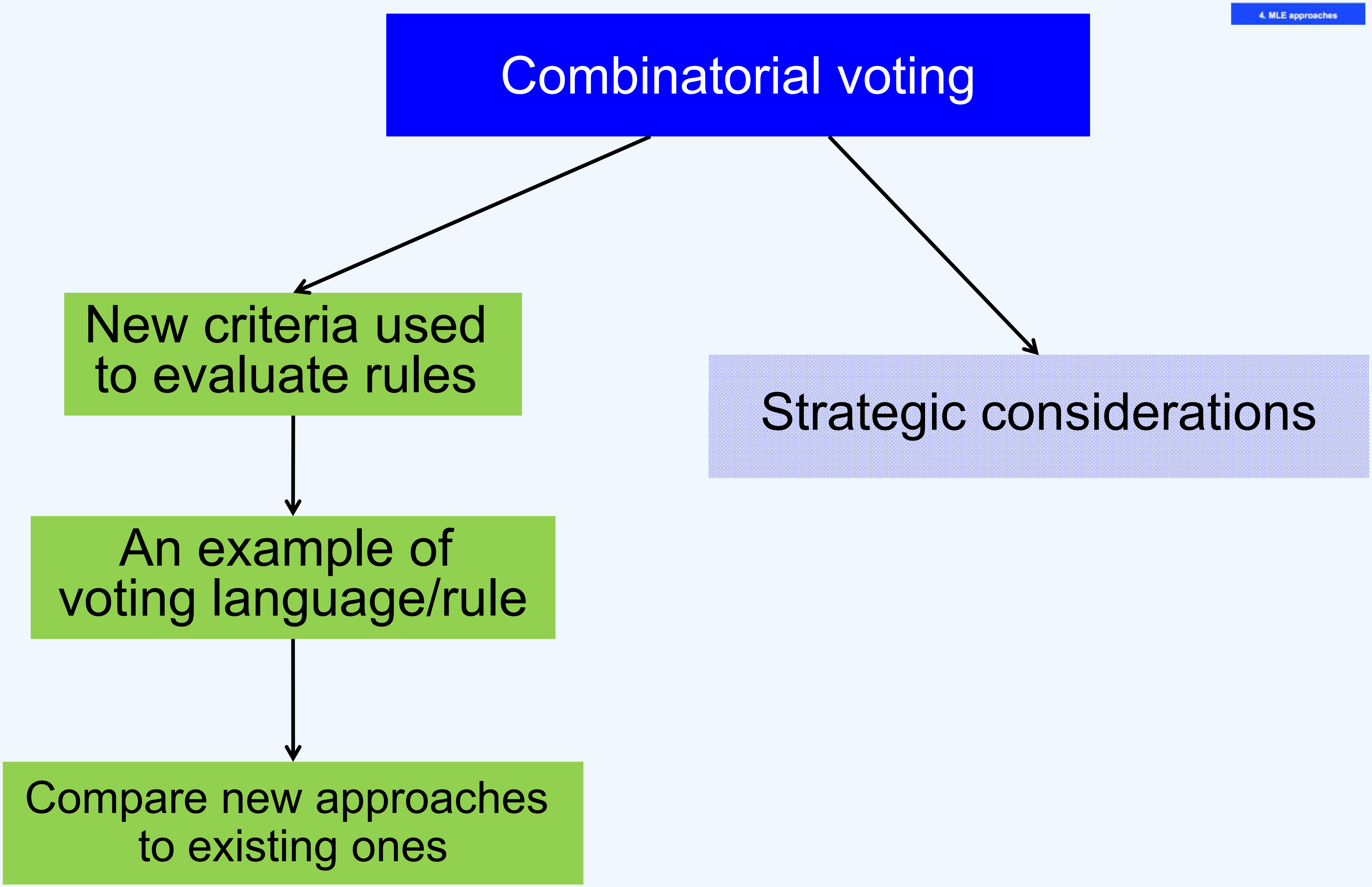
H-composition vs.
Yet another approach
Sequential rules

Voting rule	Computational efficiency	Compactness	Expressiveness	
			Usability	Informativeness
Plurality	High	High	High	~~Low~~
Borda, etc.	~~Low~~	~~Low~~	High	High
Issue-by-issue	High	High	~~Low~~	Medium
Sequential voting	High	Usually high	Medium	Medium
H-composition [Xia, Conitzer, &Lang AAAI-08]	Low-High	Usually high	High	Medium
MLE approach [Xia , Conitzer, & LangAAAMAS-10]	Low-High	Usually high	High	Medium

AI may help!

- Computing local/global Condorcet winner
  - CSP with cardinality constraints [Li, Vo, & Kowalczyk AAMAS-11]
- Applying common voting rules (including Borda) to preferences represented by lexicographic preference trees
  - Weighted MAXSAT solver [Lang, Mengin, & Xia CP-12]

Overview


Combinatorial voting
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Strategic consideration

- So far we have examined combinatorial voting from
  - axiomatic viewpoints
  - computational considerations
- With strategic voters
  - how to evaluate the harm?
  - how to prevent strategic behavior?

Strategic sequential voting

[Xia,Conitzer,&Lang EC-11]

- What if we want to apply sequential rules anyway?
  – Often done in real life
  – Ignore usability concerns
  – Voters vote strategically

Example

S **T**

$V_1 : st > \bar{s}t > s\bar{t} > \bar{s}\bar{t}$

$V_2 : s\bar{t} > st > \bar{s}t > \bar{s}\bar{t}$

$V_3 : \bar{s}t > \bar{s}\bar{t} > s\bar{t} > st$

- In the first stage, the voters vote simultaneously to determine **S**; then, in the second stage, the voters vote simultaneously to determine **T**
- If **S** is built, then in the second step $t > \bar{t},\ \bar{t} > t,\ \bar{t} > t$ so the winner is $s\bar{t}$
- If **S** is **not** built, then in the 2nd step $t > \bar{t},\ t > \bar{t},\ t > \bar{t}$ so the winner is $\bar{s}t$
- In the first step, the voters are effectively comparing $s\bar{t}$ and $\bar{s}t$, so the votes are $\bar{s} > s,\ s > \bar{s},\ \bar{s} > s$, and the final winner is $\bar{s}t$

Strategic sequential voting (SSP)

- Binary issues (two possible values each)
- Voters vote simultaneously on issues, one issue after another
- For each issue, the majority rule is used to determine the value of that issue
- No equilibrium selection problem
  – Unique SSP winner

Multiple-election paradoxes for SSP (ordinal PoA)

- **Main theorem** (informally). For any $p \geq 2$, there exists a profile such that the SSP winner is
  - ranked almost at the bottom by every voter
  - Pareto dominated by almost every other alternative
  - an almost Condorcet loser
- Known as multiple-election paradoxes [Brams, Kilgour & Zwicker SCW-98, Scarsini SCW-98, Lacy&Niou JTP-00, Saari&Sieberg APSR-01], [Lang&Xia MSS-09]
- Strategic behavior of the voters is extremely harmful in the worst case

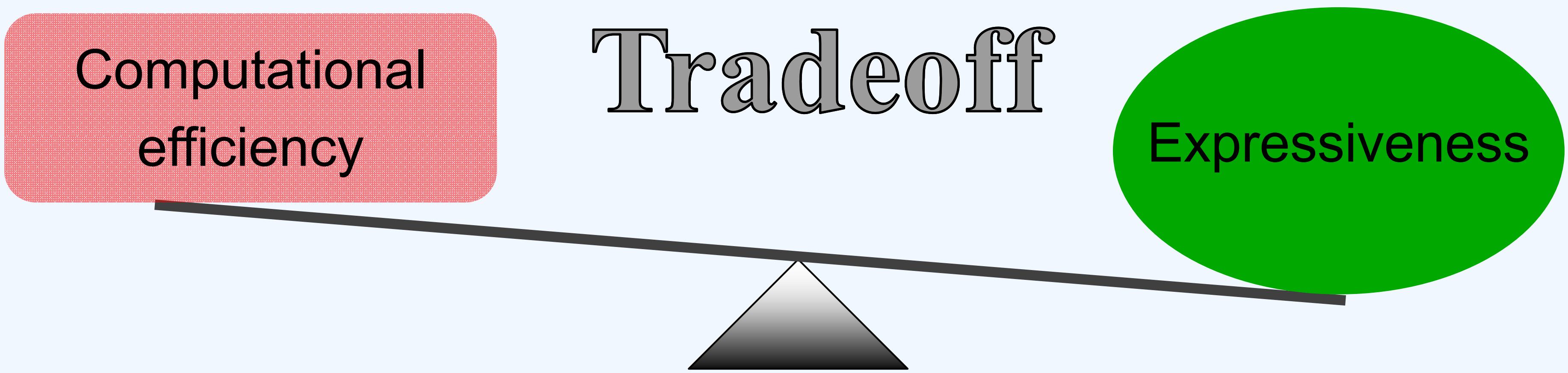
Any better choice of the order?

- **Theorem** (informally). At least some of the paradoxes cannot be avoided by a better choice of the order over issues

Preventing manipulation by domain restrictions

- Relax the unrestricted domain property in Gibbard-Satterthwaite
- A concise characterization for all strategy-proof voting rules for separable preferences [LeBreton&Sen Econometrica-99]
- A concise characterization for all strategy-proof voting rules for lexicographic preferences [Xia&Conitzer WINE-10]

Food for thought



Outline

1. Traditional Social Choice

2. Game-theoretic aspects

3. Combinatorial voting

10 min 

4. MLE approaches

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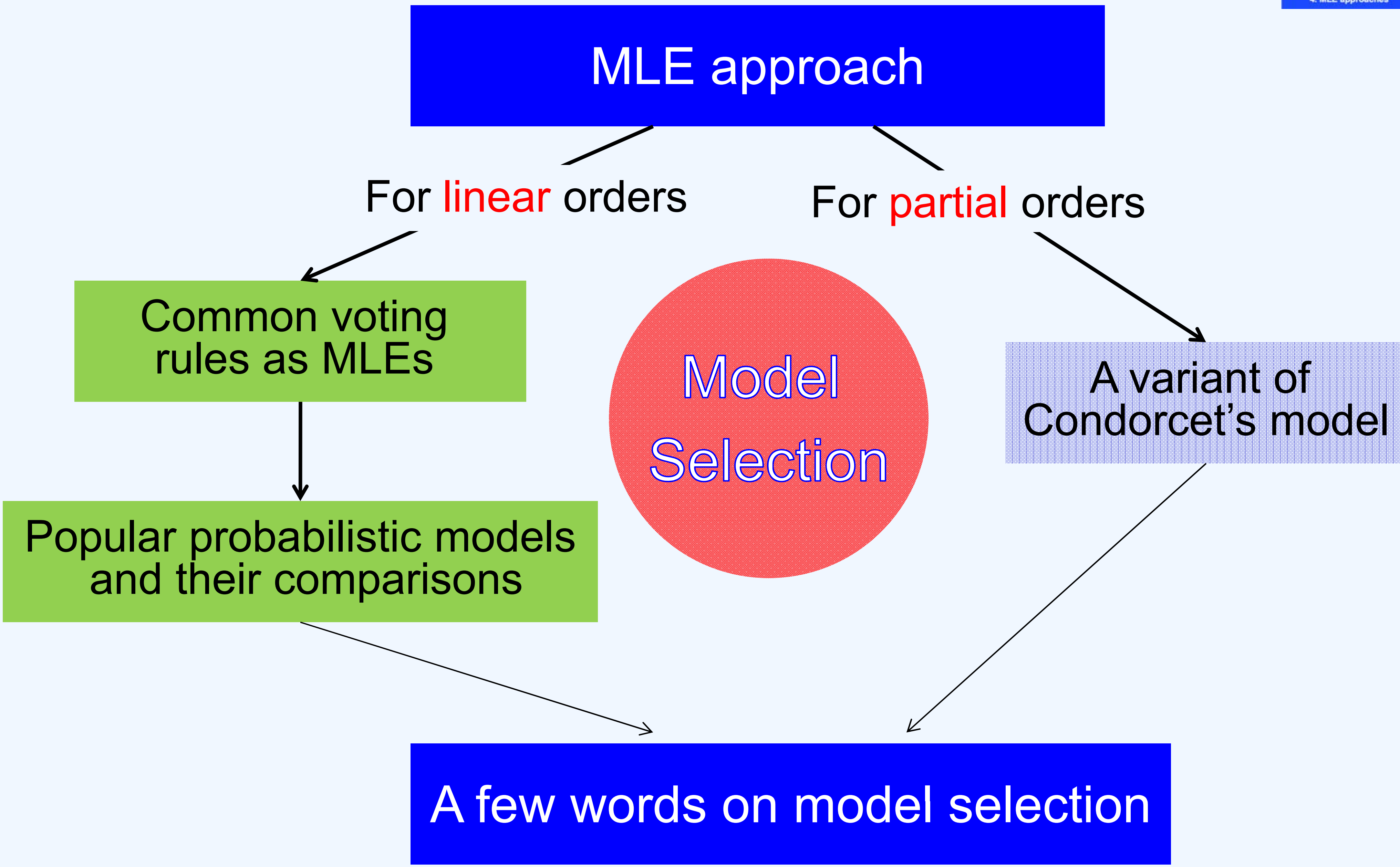
Overview


MLE approach
For linear orders
For partial orders
Common voting rules as MLEs
Model Selection
A variant of Condorcet's model
Popular probabilistic models and their comparisons
A few words on model selection

Objectives of designing social choice rules

- **OBJ1:** Compromise among subjective preferences

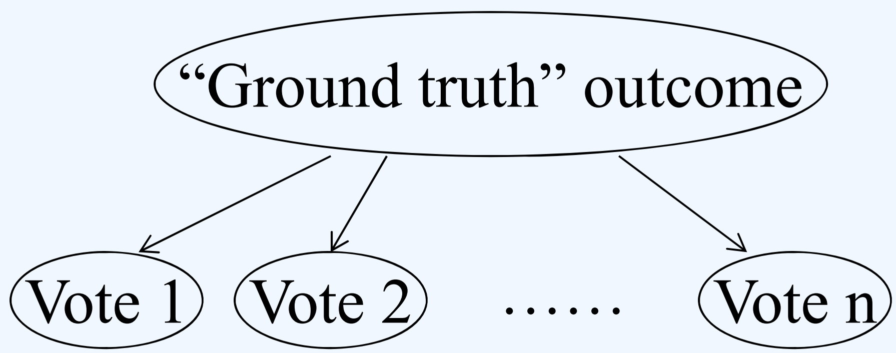
- **OBJ2:** Reveal the "truth"

Evaluation

- Most importantly: the ability to reveal the ground truth
- Do we care about satisfiability of axiomatic properties?
  - Consistency: if $r(P_1)\cap r(P_2)\neq\phi$, then $r(P_1\cup P_2)=r(P_1)\cap r(P_2)$
  - Monotonicity: the current winner c still wins if some voters raise c (while keeping other positions relatively unchanged)
  - Neutrality?
    - Yes for MLE
  - Anonymity?
    - Probably no, informed voters should have heavier weights

The MLE approach to voting

- The generative epistemic model: given a "groundtruth outcome" o
  - each vote is drawn conditionally independently given o, according to $\Pr(V|o)$
  - o can be a winning ranking or a winning alternatives



- The MLE rule: For any profile P,
  - The likelihood of P given o: $L(P|o)=\Pr(P|o)=\prod_{V\in P}\Pr(V|o)$
  - The MLE as rule is defined as

$$\text{MLE}_{\Pr}(P)=\text{argmax}_o\prod_{V\in P}\Pr(V|o)$$

  - Defines a correspondence (that selects multiple outcomes)

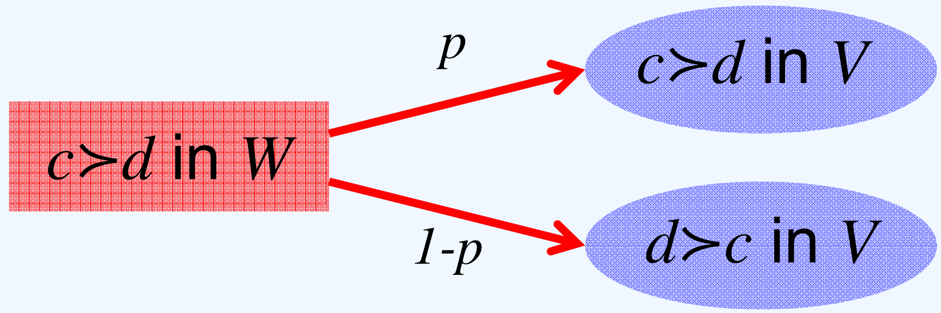
Assuming independence among the voters

- If we allow arbitrary correlation among voters, then any voting rule is the MLE of some probabilistic model [Conitzer&Sandhom UAI-05]
- Choice theory may help!
  - Adopt (random) utility theory

Condorcet's MLE model

[Condorcet 1785]

- Ground truth (outcome) is a ranking
- Given a "ground truth" ranking W and $p>1/2$, generate each pairwise comparison in V independently as follows (Suppose $c \succ d$ in W)



$$\Pr(\ b \succ c \succ a \mid a \succ b \succ c\ ) = ?\ p(1-p)^2$$

- The MLE is equivalent to the Kemeny rule [Young JEP-95]
  - $\Pr(P|W) = p^{nm(m-1)/2-K(P,W)}(1-p)^{K(P,W)} = p^{nm(m-1)/2}\left(\frac{1-p}{p}\right)^{K(P,W)}$ — $p^{nm(m-1)/2}$: Constant; $\frac{1-p}{p}$: <1
  - The winning rankings are insensitive to the choice of p (>1/2)

Criticisms on Condorcet's model

- Too much independence among pairwise comparisons
  - May lead to cycles in V
  - Not a problem to apply the MLE method: we allow inputs to have possibly cyclic preferences
- MLE (Kemeny) is too hard to compute:
  - NP-hard to compute [Bartholdi, Tovey, & Trick SCW-89a]
  - Practical ILP formulation [Conitzer, Davenport, & Kalagnanam AAAI-06]
  - Approximation [Ailon, Charikar, & Newman STOC-05]
  - Fixed-parameter analysis [Betzler et al. TCS-09]

Which common voting rules are MLEs? [Conitzer&Sandholm UAI-05]

- When the outcomes are winning alternatives
  - MLE rules must satisfy consistency: if $r(P_1) \cap r(P_2) \neq \phi$, then $r(P_1 \cup P_2) = r(P_1) \cap r(P_2)$
  - All common voting rules except positional scoring rules are NOT MLEs
- Positional scoring rules are MLEs
  - Score vector $s_1, \ldots, s_m$
  - For any alternative c and any linear order V, let $\Pr(V|c) \propto 2^{s_i}$, where i is the rank of c in V
  - $L(P|c) \propto 2^{\text{Total score of } c}$
- This is NOT a coincidence!
  - Positional scoring rules are the only voting rules that satisfy anonymity, neutrality, and consistency! [Young SIAMAM-75]

Which common voting rules are MLEs? [Conitzer&Sandholm UAI-05]

- When the outcomes are winning rankings
  - MLE rules must satisfy reinforcement (the counterpart of consistency for rankings)
  - All common voting rules except positional scoring rules and Kemeny are NOT MLEs
- This is not a coincidence!
  - Kemeny is the only preference function (that outputs rankings) that satisfies neutrality, reinforcement, and Condorcet consistency [Young&Levenglick SIAMAM-78]

Designing new MLE rules

- How can we choose the generative model?
- How can we compute the MLE efficiently?

Mallows Model

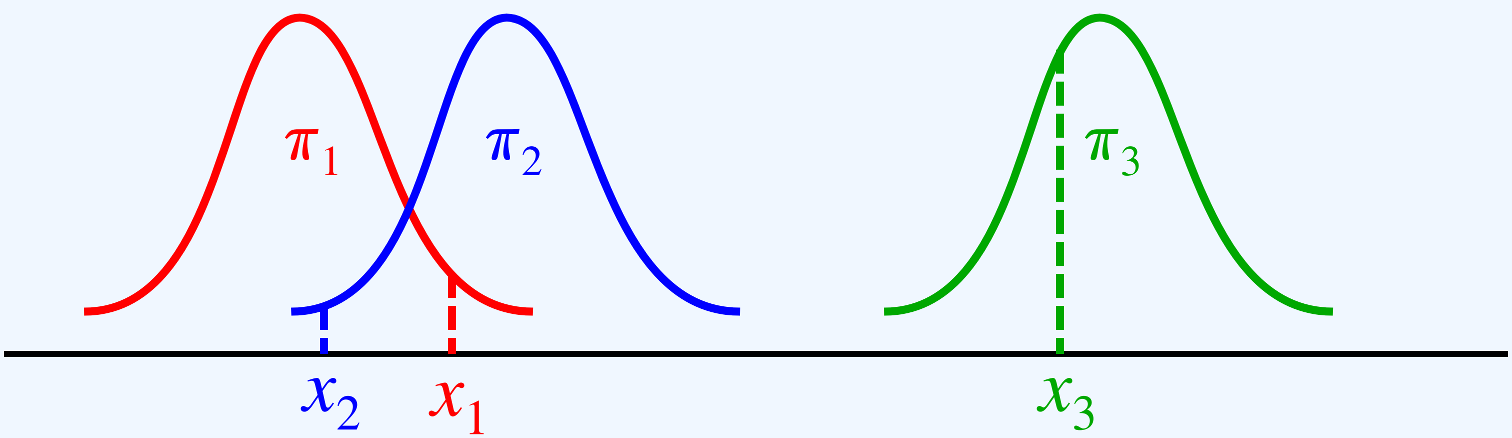
[Mallows Biometrika-57]

- Ground truth (outcome) is a ranking
- Parameterized by $\phi > 1$
  - $\Pr(V|W) = \phi^{K(V,W)} / Z$ normalization factor
- MLE is equivalent to Kemeny when profiles only contain linear orders
  - Let $\phi = \dfrac{p}{1-p}$

Random utility model (RUM)

[Thurstone-27, McFadden 74]

- Ground truth is $\pi_1, \ldots, \pi_m$
  - Represent the "utility distributions" of alternatives
- Voters rank alternatives according to their stochastic utilities
  - $\Pr(c_2 \succ c_1 \succ c_3 \mid \pi_1, \pi_2, \pi_3) = \Pr_{x_i \approx \pi_i}(x_2 \succ x_1 \succ x_3)$



Plackett-Luce Model

[Luce 59, Plackett 75]

- Ground truth is $\lambda_1,\ldots,\lambda_m$
  - Represent the "utilities" of alternatives

$$\Pr(c_1 \succ c_2 \succ \cdots \succ c_m \mid \lambda_1 \cdots \lambda_m) = \frac{\lambda_1}{\lambda_1 + \cdots + \lambda_m} \times \frac{\lambda_2}{\lambda_2 + \cdots + \lambda_m} \times \cdots \times \frac{\lambda_{m-1}}{\lambda_{m-1} + \lambda_m}$$

The quality of c_1 is the largest among $\{c_1,\ldots,c_m\}$

The quality of c_2 is the largest among $\{c_2,\ldots,c_m\}$

The quality of c_{m-1} is larger than the quality of c_m

RUMs with double exponential distributions

- All $\pi_1, \ldots, \pi_m$ are shifts of the same distribution
  - The alternatives are parameterized by the means of distributions
- π's are double-exponential (Gumbel) distributions
  - Gives us the Plackett-Luce model [Block&Marschak 60]
  - The only distribution that give us P-L [McFadden 74, Yellott 77]

Pros:

  - Computationally tractable (gradient descent, EM etc)
    - Widely applied in Economics [McFadden 74] and "learning to rank" [Liu 11]
    - Also in elections [Gormley&Murphy 06,07,08,09]
  - Justified by Luce's Choice Axiom [Luce 59]

Cons: the model is not a very natural RUM

A more natural RUM

- π's are normal distributions
  - Thurstone's Case V [Thurstone 27]

Pros: very natural model

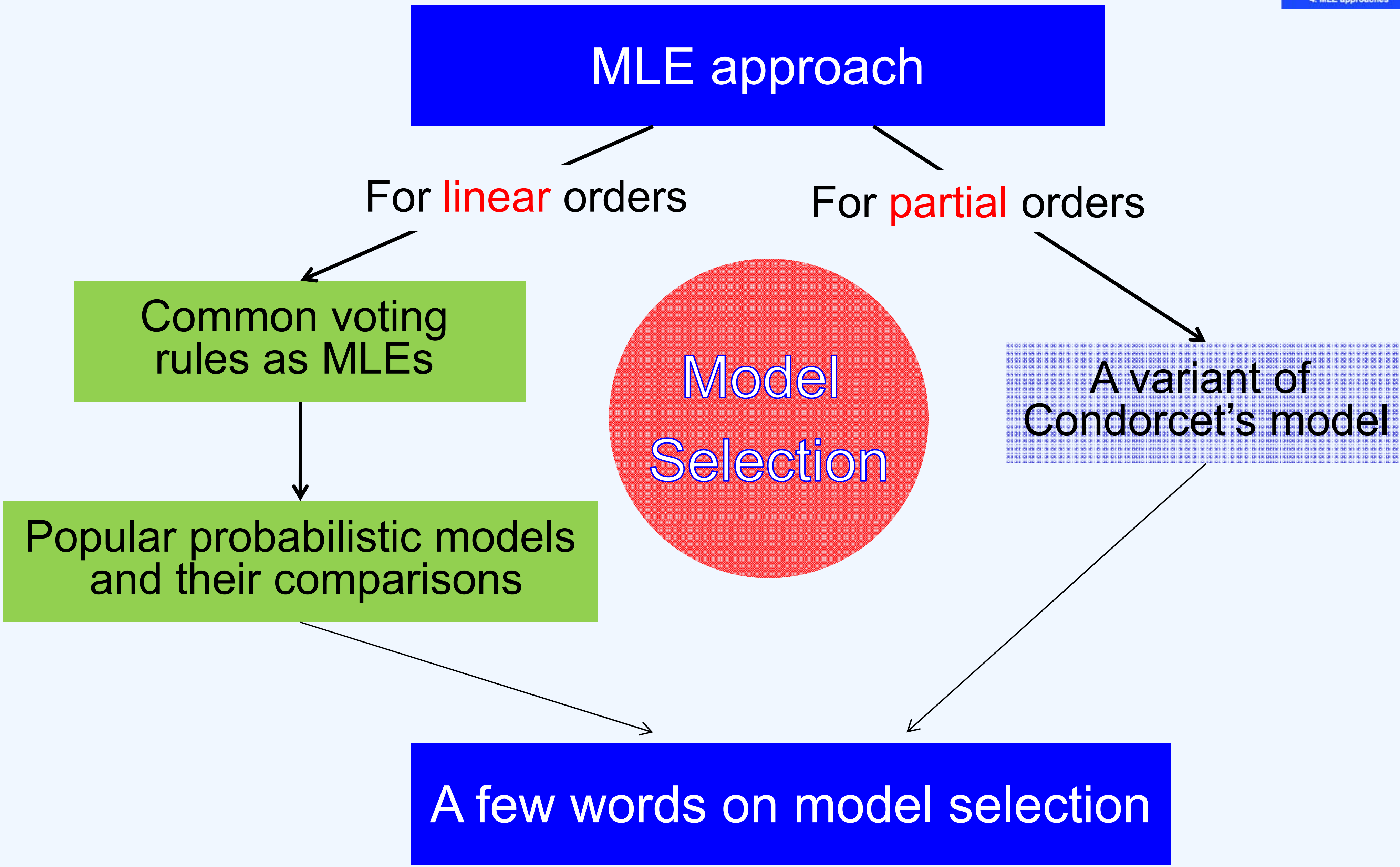
Cons: computationally intractable

  - No closed-form formula for the likelihood function $\Pr(V \mid \pi)$ is known

Comparing Condorcet (Mallows) and RUMs

	Condorcet (Mallows)	RUMs
Ground truth	A ranking	Distribution of the utilities of alternatives
Likelihood function	Has a simple form	Usually do not have a closed-form formula
Hardness of computation	Enumeration of $m!$ ground truth rankings	

Overview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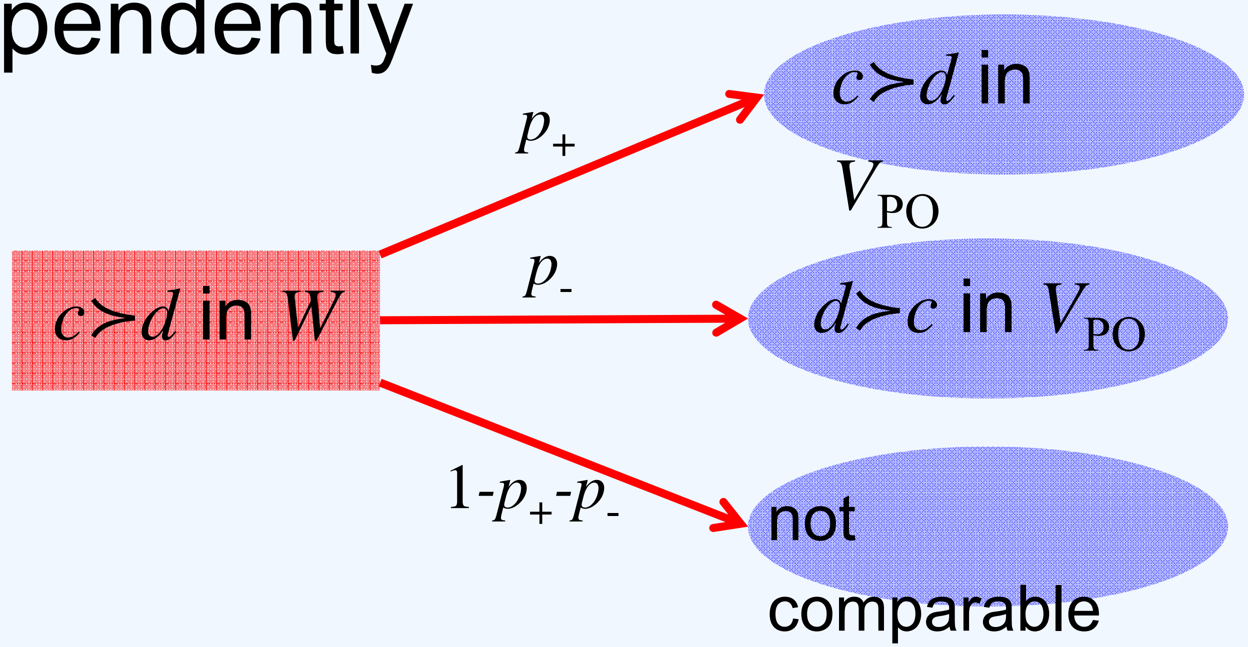
Aggregating partial orders

- Extending existing model by marginalization
  - $\Pr(V_{\text{PO}}|o) = \Sigma_{V \text{ extends } V_{\text{PO}}} \Pr(V\,|o)$
    - V_{PO} : a partial order over C
    - o is a ground truth outcome
  - RUMs [Gormley&Murphy 06,07,08,09]
  - Mallows [Lebanon&Mao JMLR-08, Lu&Boutilier ICML-11]
  - Condorcet model: $\Pr(V_{\text{PO}}|W)=(1\text{-}p)^{\text{K}(V_{\text{PO}}|W)}(p)^{\text{T-K}(V_{\text{PO}}|W)}$
    - T: the number of pairwise comparisons in V_{PO}
    - Different from Mallows!

A variant of Condorcet's model

[Xia&Conitzer IJCAI-11]

- Parameterized by $p_+ > p_- \geq 0 \quad (p_+ + p_- \leq 1)$
- Given the "correct" ranking W, generate pairwise comparisons in a vote V_{PO} independently



How many different MLE models? [Xia&Conitzer IJCAI-11]

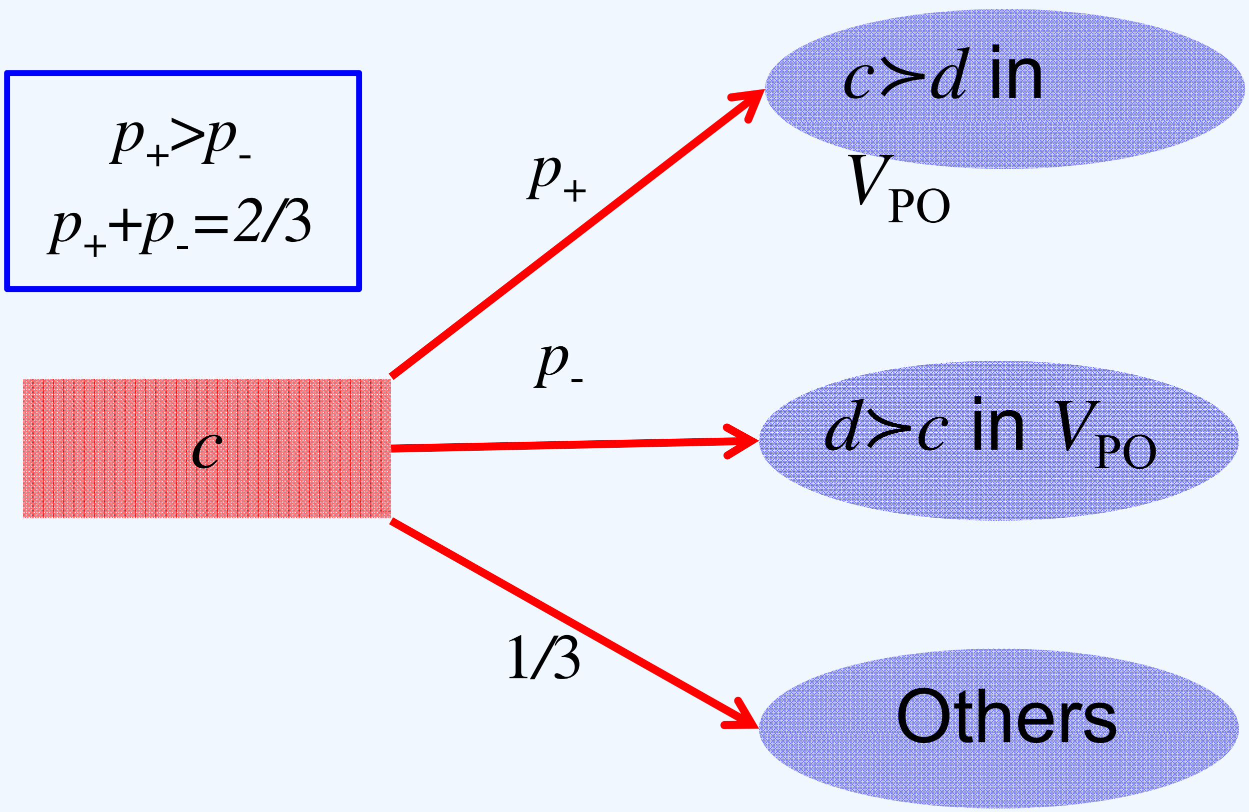
- Recall that Kemeny is indifferent to the choice of p
- In the variant to Condorcet's model
  - Let T denote the number of pairwise comparisons in P_{PO}
  - $\Pr(P_{\text{PO}}|W) = (p_+)^{T-\mathrm{K}(P_{\text{PO}},W)} (p_-)^{\mathrm{K}(P_{\text{PO}},W)} (1-p_+-p_-)^{nm(m-1)/2-T}$

    = Constant <1

  - The winner is $\operatorname{argmin}_W \mathrm{K}(P_{\text{PO}},W)$
  - Equivalent to the marginalization approach
  - Being used in Duke CS to rank Ph.D. Candidates

Choosing a winning alternative

- Ground truth is a winning alternative c (as opposed to a ranking)



MLE is equivalent to Borda when the profile only contains linear orders

A general framework

[Xia&Conitzer IJCAI-11]

- Let O denote the set of outcomes
  - O={All rankings over C}
  - O=C
- The model is parameterized by π $(\cdot|o)$, where $o \in O$
- Key idea: explicitly model the probability of "no comparison" in a randomly generated V_{PO}
  - $d \succ d'$ in V_{PO} w.p. $\pi(d \succ d' |o)$
  - $d' \succ d$ in V_{PO} w.p. $\pi(d' \succ d |o)$
  - $d' \sim d$ in V_{PO} w.p. $\pi(d' \sim d |o)$
  - $\pi(d \succ d' |o) + \pi(d' \succ d |o) + \pi(d' \sim d |o) = 1$
  - π is called a **pairwise-independent model**

Weakly neutral pairwise-independent models

- A pairwise independent model π is weakly neutral, if for any pair of outcomes o and o', there exists a permutation M over C such that for any pair of alternatives (d,d')

$$\pi(d \succ d' \mid o) = \pi(M(d) \succ M(d') \mid o')$$

Borda is the only extendable neutral rule

- **Theorem.** Let O=C. The MLE of a weakly neutral pairwise-independent model satisfies
  - The restriction r on profiles of linear orders is neutral

if and only if r is Borda

What are good generative probabilistic models?

How to evaluate a model?

- Axiomatic approaches
  - Luce’s choice axioms [Luce 59]
  - Mallows [Mallows Biometrika-57]
- Experimental studies
  - Usually hard if we do not know the ground truth
  - Sometimes we know the ground truth
    - Learning to rank, validating P-L [Cao et al. ICML-07]
    - Crowdsourcing, validating RUMs with normal distributions for pairwise comparisons [Pfeiffer et al. AAAI-12]

Food for thought

- Existing models
  - How to overcome the computational intractability of MLE inference?
  - Testing the models on different application domains
- New models
  - Captures how agents form their preferences
  - May adopt the traditional social choice axiomatic approach (on the MLE as a whole)
  - Consider correlations among voters' preferences

2. Game-theoretic aspects

- Complexity of strategic behavior

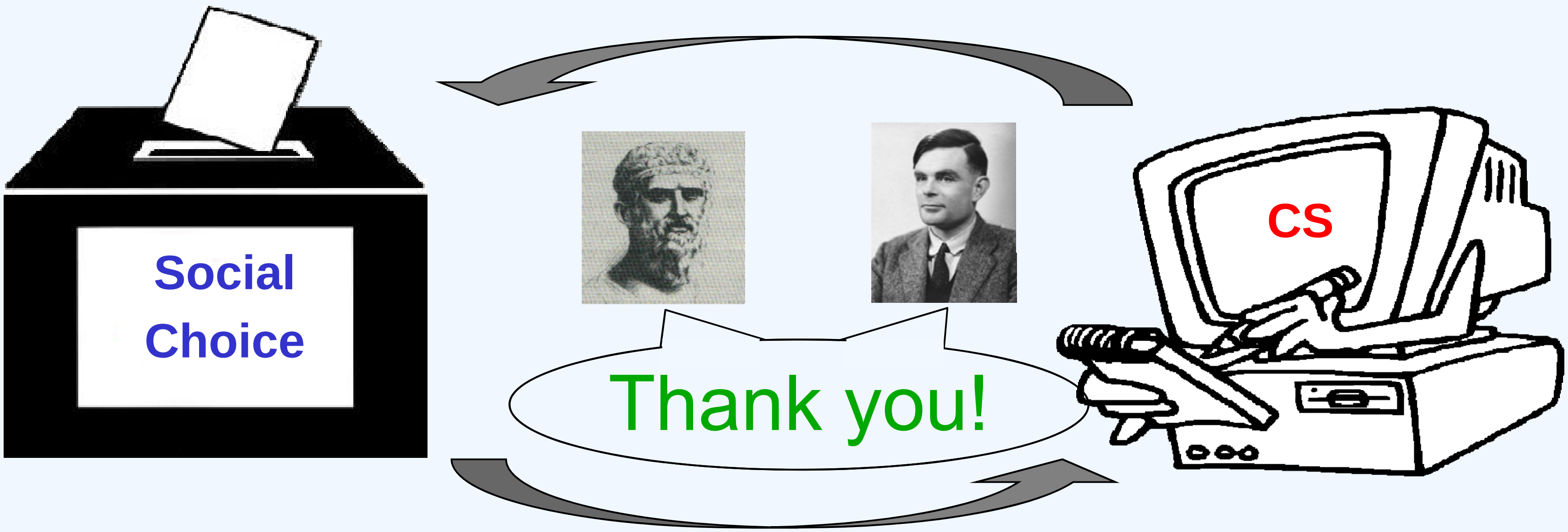
3. Combinatorial voting

- Complexity of representation and aggregation

4. MLE approaches

- Complexity of MLE inference

Computational thinking + optimization algorithms



Strategic thinking + methods/principles of aggregation

2. Game-theoretic aspects

- Stackelberg voting games

3. Combinatorial voting

- Strategic sequential voting
- Axiomatic properties

4. MLE approaches

- Axiomatic characterization

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Pairwise scoring rules

- A pairwise scoring function is a function $s:C\times C\times O\rightarrow \mathbf{R}$ that
  - Given $o\in O$, s scores each pairwise comparisons in the partial order independently, denoted by $s(d\succ d',o)$
  - $s(P_{\text{PO}},o)=\sum_{V_{\text{PO}}\in P_{\text{PO}}}\sum_{(d\succ d')\in V_{\text{PO}}} s(d\succ d',o)$
- A pairwise scoring rule r_s select the outcome that maximizes $s(P_{\text{PO}},o)$

Weakly neutral pairwise scoring functions

- A pairwise scoring function s is weakly neutral, if for any pair of outcomes o and o', there exists a permutation M over C such that for any pair of alternatives (d,d')

$$s(\, d \succ d', o) = s(\, M(d) \succ M(d'), o' \,)$$

Examples

- Kemeny

$$s(d \succ d' \mid W) = \begin{cases} 1 & \text{if } d \succ d' \text{ in } W \\ 0 & \text{if } d' \succ d \text{ in } W \end{cases}$$

- Borda: $q_+ > q_-$

$$s(d \succ d' \mid c) = \begin{cases} q_+ & \text{if } d = c \\ q_- & \text{if } d' = c \\ 0 & \text{Otherwise} \end{cases}$$

Characterizing MLE rules

- **Theorem.** [Xia&Conitzer IJCAI-11]

MLE of a weakly neutral pairwise-independent model = Pairwise scoring rule with a weakly neutral PSF