Design and Implementation of Combinatorial Prediction Markets

Tutorial
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Sébastien Lahaie and Miroslav Dudík
Microsoft Research, NYC
New York, NY 10011
{slahaie,mdudik}@microsoft.com

Description

In a prediction market, traders buy and sell securities which pay out according to the outcomes of future events, leading to forecasts of the various event probabilities via the market prices. Prediction markets have been successfully used to obtain forecasts in a variety of domains, including project performance within companies, box office performance for movies, and the outcomes of political election races.

A *combinatorial* prediction market allows participants to bet on combinations of events (e.g., whether a presidential candidate will win both Florida and Ohio). These kinds of markets offer the potential for more refined and accurate forecasts, including estimates of conditional event probabilities (e.g., the probability that a sports team wins a tournament given that it reaches the quarter finals). The algorithmic challenge is to price the exponential number of combinations, guarding against loss due to arbitrage opportunities, and in the process provide good forecasts for any event of interest.

In recent years there has been substantial progress in our understanding of how to implement and run combinatorial prediction markets. For instance, the authors of this proposal ran a market for the US Elections in 2012 that comprised an outcome space on the order of 10^{33}, and allowed participants to bet on outcomes and combinations of outcomes in presidential, senatorial, and gubernatorial elections in 50 states. Additional pub-

licly deployed instances include combinatorial markets for various sports events, such as the March Madness basketball tournament involving 64 teams. These markets were able to achieve high predictive accuracy on a wide variety of interrelated events despite the computational challenges involved.

In this tutorial, we will explain how the design of combinatorial prediction markets can be approached via convex optimization, drawing on closely related ideas in machine learning. We will begin with the connection between ways to quantify risk (from finance) and ways to quantify predictive accuracy (from statistics), leading to a principled way to price securities. We will then turn to the fundamental algorithmic problem, which is to eliminate arbitrage opportunities that lead to guaranteed profits. We will survey different arbitrage removal strategies, including domain-specific strategies and general-purpose approaches. The applications to elections and sports tournament mentioned previously will be used as running examples throughout the tutorial. Upon completion, participants will be able to systematically design prediction markets for other domains that might interest them. The tutorial will also cover several open research questions on how to run prediction markets in practice (both simple and combinatorial), including liquidity adjustment schemes, automatic market closing, and distributed implementations.

Although the material will build up to general combinatorial markets, no background on prediction markets or convex optimization will be assumed. The emphasis will be on the intuitive, economic interpretations of the market elements and algorithmic steps. The technical material will be punctuated by discussions on practical matters, including the accuracy of markets that have been deployed, evaluation methodology, and available datasets for analysis and testing.

Tutors

Sébastien Lahaie is currently a senior researcher at Microsoft Research, NYC. He received his PhD in Computer Science from Harvard in 2007 and was previously a research scientist at Yahoo. His research focuses on computational aspects of marketplace design, including sponsored search and display advertising, combinatorial auctions, and prediction markets. He is interested in designing market algorithms that scale well and properly anticipate user behavior. He has served as a co-editor for Economic Inquiry and was previously a program chair for the conference on Auctions, Market Mechanisms, and their Applications.

Miro Dudík is a senior researcher at Microsoft Research, NYC. His research focuses on combining theoretical and applied aspects of machine learning, statistics, convex optimization and algorithms. Most recently he has worked on contextual bandits, large-scale learning and tractable pricing of prediction markets. He received his PhD from Princeton in 2007. He is a co-creator of the MaxEnt package for modeling species distributions, which is used by biologists around the world to design national parks, model impacts of climate change, and discover new species.

References

[1] Jacob Abernethy, Yiling Chen, and Jennifer Wortman Vaughan. Efficient market making via convex optimization, and a connection to online learning. *ACM Transactions on Economics and Computation*, 1(2):12, 2013.

[2] Jacob Abernethy, Sindhu Kutty, Sébastien Lahaie, and Rahul Sami. Information aggregation in exponential family markets. In *Proceedings of the 15th ACM conference on Economics and computation*, pages 395–412. ACM, 2014.

[3] Shipra Agrawal, Erick Delage, Mark Peters, Zizhuo Wang, and Yinyu Ye. A unified framework for dynamic pari-mutuel information market design. In *Proceedings of the 10th ACM conference on Electronic Commerce*, pages 255–264. ACM, 2009.

[4] Yiling Chen. A utility framework for bounded-loss market makers. In *In Proceedings of the 23rd Conference on Uncertainty in Artificial Intelligence*, 2007.

[5] Miroslav Dudik, Sébastien Lahaie, and David M Pennock. A tractable combinatorial market maker using constraint generation. In *Proceedings of the 13th ACM Conference on Electronic Commerce*, pages 459–476. ACM, 2012.

[6] Miroslav Dudik, Sébastien Lahaie, David M Pennock, and David Rothschild. A combinatorial prediction market for the us elections. In *Proceedings of the 14th ACM conference on Electronic commerce*, pages 341–358. ACM, 2013.

[7] Hoda Heidari, Sébastien Lahaie, David M Pennock, and Jennifer Wortman Vaughan. Integrating market makers, limit orders, and continuous trade in prediction markets. In *Proceedings of the 16th ACM Conference on Economics and Computation*, pages 583–600. ACM, 2015.

[8] Christian Kroer, Miroslav Dudík, Sébastien Lahaie, and Sivaraman Balakrishnan. Arbitrage-free combinatorial market making via integer programming. Working paper, February 2016.