EC'16 Tutorial on Practical Computation in Finite Games using Gambit, Game Theory Explorer, and SageMath

General description

This tutorial will survey the state of the field of doing computation in the theory of games with finite numbers of players and strategies. Interest in methods for computing the Nash equilibria of finite games dates at least to the formulation in 1963 by Dantzig of the minimax solution for two-player constant-sum games as a linear programming problem. There today exist a number of tools to facilitate computation in finite games, suitable for by researchers and practitioners. Equally, there remain many opportunities for improvements in existing implementations, and new implementations to expand the the classes of games which can be studied computationally. The continued development of tools for analysis of games will support the use of reproducible computation in applications of game theory.

The main software package for computing in finite games is Gambit (http://www.gambit-project.org). Since the early 1990s, Gambit has provided a suite of tools to represent finite games in extensive or strategic form. It also provides implementations of many algorithms for analysing the set of Nash equilibria of games. Gambit is developed and released under the open-source GNU General Public License. It is fully-cross platform, with Linux/Unix, Mac OS X, and Microsoft Windows implementations supported.

We will begin with an introduction to the Gambit library. The core library is written in C++, and each implementation of a method for computing the equilibria of games is contained in an independent module. These methods are also exposed as standalone command-line tools for interaction via shell scripts or calling from other tools without requiring linkage to the C++ libraries.

There are several graphical front-ends available to interact with the library, which are useful especially for preliminary manipulation of smaller games. The Gambit package itself provides a desktop application on all the supported platforms for the construction and analysis of games. Game Theory Explorer (http://www.gametheoryexplorer.org) is a browser-based interface for investigating games, without requiring a download and local installation of the full Gambit packages.

There is a set of Python bindings to the Gambit library and analysis algorithms. These bindings facilitate interfacing Gambit to the well-developed ecosystem of libraries in Python for scientific computation. In particular, Gambit is now included in SageMath (http://www.sagemath.org), an Open Source system for doing computational mathematics.

We will illustrate the use of Gambit in Python and SageMath through a series of short examples. These will include examples of how the set of equilibria changes as

a function of payoff parameters of the game, using a standard "one-card" poker game as the starting point. We will also replicate the results in Bagwell's (1993) analysis of commitment in games, showing how the set of equilibria changes when commitment is only observed probabilistically.

We will then present a few examples illustrating the possibilities for computation in games at a scale suitable for research projects and real applications, including:

- The estimation of quantal response equilibrium (QRE), and its use in estimating payoff parameters using data from laboratory experiments;
- The analysis of Colonel-Blotto like games, such as the apex game model of political lobbying appearing in Montero et al (2016, Economic Theory);
- Optimal patrol strategies for real-world security applications, including metro systems and water-based transport;
- Modeling the "position auction" for online advertising allocation using action graph games.

Beyond the core currently implemented in Gambit, there are many opportunities for improving the state of tools for computing in finite games. The second part of the tutorial will be devoted to outlining these opportunities.

While the computation of equilibria in two-player games is quite well settled, the situation for games with three or more players is more open. There are several algorithms which have been implemented, but their practical performance has not been studied in detail or optimised. There are other algorithms which have been proposed, but not yet implemented. Of particular interest will be the study of games which have equilibrium sets with positive-dimensional components.

Most algorithms to date apply generically to any game. While it is known that in the worst case the complexity of computing equilibria scales poorly, many games of practical interest have structure which may be exploited in characterising equilibria. We will introduce Gambit's new support for the Action Graph Games representation and illustrate the potential advantages of alternate representations for being able to solve larger games through a series of case studies.

Another direction of extension is the study of other solution concepts for games. Among Nash equilibria, despite theoretical interest in refinements dating to the 1970s, there are few available methods for working with refinement concepts. Alternatively, the complexity of computing Nash equilibria, combined with evidence from laboratory experimentation, suggest that other solution concepts may be behaviourally more plausible predictions for how some games would be played.

Previous presentations

The Gambit software has been exhibited at various conferences in game theory, economics, and computing science throughout its existence, as has the Game Theory Explorer facility. The techniques for estimating quantal response equilibria were first demonstrated at a workshop in behavioural game theory at University of East Anglia in the summer of 2015; this has been a much-requested demonstration especially among economics researchers. The forward-looking discussion of opportunities will be drawn from various research presentations done by the presenters, but this will be the first instance in which it will be organised as a whole. This tutorial is intended to complement a number of activities intended to encourage interaction between social scientists and computing scientists to develop the state of practical computation in games, including presentations at the GW4 conference in Bristol to occur earlier in the summer of 2016, and a workshop/hackathon to be held in conjunction with PyCon UK in Cardiff in September 2016, among others.

Presenters

Prof Theodore Turocy (organiser) is Professor of Economics and Director of the Centre for Behavioural and Experimental Social Science at University of East Anglia. He has been Director of the Gambit Project for computing in finite games since 2002, and lead software engineer since 1994. Several of his lines of research involve the use of computational and numerical techniques in the development and application of game theory.

Dr Albert Xin Jiang is Assistant Professor in the Department of Computer Science at Trinity University. Much of his research is addressing computational problems arising in game theory, including the efficient computation of solution concepts and applications of game-theoretic computation to real-world domains such as large-scale infrastructure security and electronic commerce.

Dr Vincent Knight a lecturer of Mathematics at Cardiff University with research interests in applied and computational game theory. Some of his work involves building and solving game theoretic models related to healthcare settings. He is involved in developing the facilities for computation in games in SageMath.

Prof Kevin Leyton-Brown is Professor of Computer Science at University of British Columbia. He studies the intersection of computer science and microeconomics, addressing computational problems in economic contexts and incentive issues in multiagent systems. He also applies machine learning to the automated design and analysis of algorithms for solving hard computational problems.

Dr Rahul Savani leads the Economics and Computation Group in the Department of Computer Science at the University of Liverpool. He has worked extensively on computation of exact and approximate equilibria for game-theoretic models of strategic interaction. He is co-developer of Game Theory Explorer.

Prof Bernhard von Stengel is Professor of Mathematics at the London School of Economics. He has been communications officer of the Game Theory Society, and is is program chair of the GAMES 2016 congress. He is interested in mathematical questions of game theory, including the geometry and computation of Nash equilibria. He is co-developer of Game Theory Explorer.