

# Contingency Exigency

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Beeminder

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Lawyers and debt collectors are classic examples of people you hire on contingency. The idea is to hire someone with payment contingent on the fruits of their labor. You don't pay them if you don't win, i.e., if you yourself don't get paid. And if you luck out, they share in the windfall.

The agency problem here is insurmountable. If you pay them hourly then they have incentive to drag things out. If you pay them a percentage of the winnings then, unless the percentage is unreasonably high, you have the opposite problem: they'll have incentive to skimp on the time they put in.

So we'll ignore the incentives and assume that this agent<sup>1</sup> will conscientiously put in exactly as much time as they would if they were you.

The question: What's a fair payment function? We'll take fairness to mean that your agent earns in expectation exactly what they would've earned had you paid a straight hourly rate for their time. Assume that you and your agent agree on what that non-contingency rate,  $r$ , would be, and also that you agree on the probability distribution,  $F$ , of the payout (e.g., the lawsuit settlement, or the debt collected). So we seek a function,  $\omega(t, X)$  that takes the number of hours the agent spent,  $t$ , and the eventual realized payout,  $X$ , and returns the amount you should pay the agent, parametrized by  $r$  and  $F$ .

The fairness desideratum can be expressed as  $E_F[\omega(t, X)] = r \cdot t$ . We also impose two other reasonable constraints:  $\omega(t, 0) = 0$  for all  $t$ . No payout means no wages. And  $\omega(t, X)$  is linear in  $t$ . Putting in twice the hours means twice the wages. Finally, assume  $r \cdot t < E_F[X]$ , namely, the agent doesn't do more work than the payout is, in expectation, worth.

Find  $\omega$ . How would you relax the first constraint, to ensure a bare minimum payment,  $m$ , or  $m \cdot t$ , regardless of payout?

Warning: Do not overthink this. This puzzle sacrifices difficulty for practicality. If you wanted to work out a fair payment arrangement with a trusted friend as your agent — and the uncertainty of both the time requirement and the payout precluded the simplest options, like a flat percentage or a straight hourly payment — what would you actually do?

Send solutions to the puzzle editor at [dreeves@beeminder.com](mailto:dreeves@beeminder.com) with subject: `conex`. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish it in the next issue of the Exchanges (ties broken in favor of earlier submissions). To make the solutions accessible to a wide audience, please try to

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<sup>1</sup>I'm using the term "agent" in the legal (and colloquial) sense of someone who acts on another's behalf.

minimize technical jargon. Until the winner is chosen the editor will not give any hints or feedback.