# Table of Contents

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editor’s Introduction</td>
<td>1</td>
</tr>
<tr>
<td>SHADDIN DUGHMI</td>
<td></td>
</tr>
<tr>
<td>Job Market Candidate Profiles 2016</td>
<td>2</td>
</tr>
<tr>
<td>SHADDIN DUGHMI, VASILIS GKATZELIS, and JASON HARTLINE</td>
<td></td>
</tr>
<tr>
<td>Multi-Item Auctions Defying Intuition?</td>
<td>41</td>
</tr>
<tr>
<td>CONSTANTINOS DASKALAKIS</td>
<td></td>
</tr>
<tr>
<td>Finding Any Nontrivial Coarse Correlated Equilibrium Is Hard</td>
<td>76</td>
</tr>
<tr>
<td>SIDHARTH BARMAN and KATRINA LIGETT</td>
<td></td>
</tr>
<tr>
<td>Introduction to Dynamic Belief Elicitation</td>
<td>80</td>
</tr>
<tr>
<td>CHRISTOPHER P. CHAMBERS and NICOLAS S. LAMBERT</td>
<td></td>
</tr>
<tr>
<td>Approximating the Nash Social Welfare with Indivisible Items</td>
<td>84</td>
</tr>
<tr>
<td>RICHARD COLE and VASILIS GKATZELIS</td>
<td></td>
</tr>
<tr>
<td>Behavioral Mechanism Design</td>
<td>89</td>
</tr>
<tr>
<td>DAVID EASLEY AND ARPITA GHOSH</td>
<td></td>
</tr>
<tr>
<td>A New Approach to Measure Social Capital</td>
<td>95</td>
</tr>
<tr>
<td>TOMASZ P. MICHALAK et al.</td>
<td></td>
</tr>
<tr>
<td>Optimizing From Revealed Preferences Feedback</td>
<td>101</td>
</tr>
<tr>
<td>AARON ROTH, JONATHAN ULLMAN, and ZHIWEI STEVEN WU</td>
<td></td>
</tr>
<tr>
<td>Algorithmic Game Theory and Econometrics</td>
<td>105</td>
</tr>
<tr>
<td>VASILIS SYRGKANIS</td>
<td></td>
</tr>
</tbody>
</table>
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Editor’s Introduction

SHADDIN DUGHMI
University of Southern California

Issue 14.1 of SIGecom Exchanges is not your typical issue. It includes 7 excellent research letters (rather than the usual 6), a survey/position paper on optimal multi-item auctions by Costis Daskalakis, and our newest feature: an article profiling job market candidates in the SIGecom community, co-edited by Vasilis Gkatzelis, Jason Hartline, and myself.

In his survey, Costis does a great job illustrating the intuition-defying properties of optimal auctions in multidimensional settings, before presenting a program for demystifying them. Costis’s program is two-fold, involving both improved structural understanding of optimal mechanisms, as well as their exploration through the lens of computation. If you were looking for a primer to this deep and exciting research area, this article is what you’ve been waiting for!

In the article profiling job market candidates, our goals were two-fold: to alleviate some of the informational inefficiencies in the junior faculty and postdoc market, and to present our recent graduates as members of a cohesive community. We tried to be as inclusive as possible, the only nontrivial eligibility requirement being a paper at EC or a similar conference. We hope this experiment is well received by the community, in which case it will become a yearly undertaking. Naturally, we would appreciate any feedback on whether this effort serves the interests of our community, and how it can be done better.

I hope you enjoy this issue. Please continue to volunteer letters, surveys, and position papers — in particular if you feel your area(s) of interest have not been properly represented in Exchanges of late. Moreover, please let me know what you’d like to see more or less of in Exchanges. As usual, thanks to Felix Fischer for help putting this issue together!

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ACM SIGecom Exchanges, Vol. 14, No. 1, June 2015, Page 1
In the sixteen years since the inception of EC as the flagship conference of the SIGecom community, we have learned a lot about the efficiency (or lack thereof) of markets. We have studied markets for school choice, marriage, supply chains, advertising, etc. We understand the loss in welfare from decentralized versus centralized market mechanisms. We understand that information plays an important role in market efficiency and stability. Profiled herein are thirty candidates for the 2016 junior academic job market with research areas that span the interest of the SIGecom community. Though it may not be a complete list, we hope that it will help alleviate some of the informational inefficiencies in the most important market for the community, its junior faculty job market.

A table of contents lists the candidates in alphabetical order with keywords they provided. The candidate profiles are listed with brief biographies, research summaries, and three representative papers. Following the candidate profiles is an index of the candidates by keywords.

Fig. 1. Generated using the research summaries of the candidates.

Contents

Nick Arnosti  
matching, market design, auction theory ........................................... 4

Markus Brill  
preference aggregation, tournament solutions, strategyproofness .... 5

Yun Kuen Cheung  
equilibrium computation, tatonnement, market games .................. 7

Ilan Cohen  
mechanism design, dynamic posted price schemes ...................... 8
John P. Dickerson
stochastic optimization, dynamic matching, kidney exchange 9

Aris Filos-Ratsikas
mechanism design, voting, matching, fair division 10

Sam Ganzfried
artificial intelligence, game theory, imperfect information 11

Xi (Alice) Gao
information elicitation, peer prediction, behavioural experiments 13

Nikolai Gravin
combinatorial auctions, competitive analysis, Bayesian equilibrium 14

Nima Haghpanah
mechanism design, algorithms, revenue management 15

Darrell Hoy
mechanism design, non-truthful auctions, risk-aversion 16

Mohammad Reza Khani
auction design, revenue, prior-free setting, combinatorial optimization 17

Eli Meirom
network formation, epidemic detection, social networks 18

Ilan Nehama
computational judgement aggregation, social choice, decision theory 19

Svetlana Obraztsova
computational social choice, voting games 20

Joel Oren
social networks, cascades, big data algorithms 22

Emmanouil Pountourakis
mechanism design, revenue maximization, social choice, cost sharing 23

Baharak Rastegari
mechanism design, game theory, matching, algorithms, auctions 24

Nisarg Shah
fair division, social choice, game theory, multi-agent systems 25

Or Sheffet
differential privacy, game theory, ranking, mechanism design 26

Vasilis Syrgkanis
game theory, mechanism design, econometrics, online learning . . . . . 27

Bo Tang
mechanism design, revenue maximization, Nash equilibrium . . . . . 29

Panos Toulis
experimental design, causal inference, interference, incentives . . . . . 30

Daniel Urieli
autonomous energy trading, learning agents, smart grid . . . . . . . . . 31

Angelina Vidali
mechanism design, cost-sharing, revenue maximization . . . . . . . . . 32

Elaine Wah
algorithmic trading, agent-based simulation, market design . . . . . . 33

Matt Weinberg
mechanism design, online algorithms, optimal auctions. . . . . . . . . 34

James R. Wright
game theory, behavioral models, machine learning . . . . . . . . . . . 35

Jie Zhang
mechanism design, fair division, matching, markets . . . . . . . . . . . 36

Yair Zick
game theory, overlapping coalition formation, learning, privacy . . . . 37

NICK ARNOSTI
Thesis. Frictions in Matching Markets
Advisor. Ramesh Johari, Stanford University

Brief Biography. I am a PhD candidate in the department of Management Science and Engineering at Stanford University, with a research focus on market design. I am drawn to this area because it offers the opportunity to develop deep and elegant theory that has clear practical applications. I have found that most of my projects start as conversations. I am insatiably curious, and have never found myself short of questions to ask or problems to work on. I believe that the best problems come from observing and understanding the operation of real-world marketplaces. Once I have selected a problem, the tools that I use to address it are drawn from my background in computer science, statistics, and probability. One technique that I have repeatedly found useful is to use simulations to identify interesting patterns and build intuition (though as a theoretician, I am not satisfied until I have leveraged this understanding into analytical proofs).

Research Summary. My research is in market design. I am particularly interested in studying the effects of frictions in matching markets. Frictions are barriers
that market participants face when trying to find suitable partners. They take many forms, but two common ones are screening costs (incurred when evaluating potential partners) and application costs (incurred when communicating interest to these partners).

One recent paper of mine studies the outcomes of centralized matching markets when people learn their preferences through time-consuming interviews. In such markets, agents may waste interviews on others who are unlikely to be available to them, and may go unmatched despite the presence of an unrealized mutually agreeable match. I study the magnitude of these welfare losses, under various assumptions. In a school assignment setting, my work predicts a tradeoff between two tie-breaking procedures: one matches more students to their top choice, while the other matches more students overall. Thus, the choice of procedure should depend on how the district weighs these two goals.

Of course, most matching markets are not cleared centrally. One goal of mine is to better understand the costs and benefits of using a central clearinghouse when participants are initially uncertain about their preferences. Though the benefits are well-studied, the costs less understood. One cost is that participants in centralized procedures evaluate potential partners “in advance”, and must choose between evaluating many partners (at high cost), and listing only a few (increasing the risk of going unmatched).

A theme in my research is a focus on simple mechanisms, rather than “optimal” ones. In work with Ramesh Johari and Yash Kanoria, I study congestion in decentralized matching markets. Rather than seeking a first best solution, we study the benefits that can accrue if the market operator merely limits the number of jobs for which each individual can apply.

Though simple interventions are appealing from a practical point of view, it is important to ask whether they are “adequate”. One way to do this is to use the optimal procedure as a benchmark, and seek a simple procedure that performs well against this benchmark. This is the approach taken in my work with Marissa Beck and Paul Milgrom - we introduce a simple mechanism with several appealing properties, and then demonstrate that the gains of moving to any alternative mechanism are minimal.

Representative Papers.

[1] Short Lists in Centralized Clearinghouses (SSRN)
[3] Adverse Selection and Auction Design for Internet Display Advertising (SSRN) with M. Beck and P.R. Milgrom

MARKUS BRILL


Advisor. Felix Brandt, Technische Universität München

Brief Biography. Markus Brill is a postdoc at the Department of Computer Science at Duke University. He received a Ph.D. degree in Computer Science (2012)
and diploma (2008) and B.Sc. (2006) degrees in Mathematics from Technische Universität München (TUM), Germany. Markus is a graduate of the elite graduate program TopMath. For his Ph.D. thesis, he received the dissertation award of Bund der Freunde der TUM and an honorable mention for the Artificial Intelligence Dissertation Award sponsored by ECCAI. He is also a recipient of a Research Scholarship by ParisTech and currently holds a Feodor Lynen Research Fellowship, awarded by the Alexander von Humboldt Foundation.

Research Summary. I am fascinated by the formal analysis of scenarios in which multiple agents with possibly conflicting preferences interact. As such, my research interests lie at the intersection of computer science, theoretical economics, and mathematical social sciences. More precisely, my work focusses on axiomatic and computational aspects of game theory and social choice theory.

In my Ph.D. thesis, I have studied set-valued solution concepts. The first part of my thesis focusses on solution concepts for normal-form games that are based on varying notions of dominance [1]. The framework is very general and captures a number of concepts that have been proposed in the literature. In the second part of my thesis, I studied social choice functions and the complexity of the winner determination problem. I also studied the complexity of computing possible and necessary winners for partially specified instances and the axiomatic and asymptotic properties of tournament solutions [2].

My current research interests include various randomized solution concepts such as random serial dictatorship (RSD). My work has led to the identification of a fundamental tradeoff between strategyproofness and economic efficiency, and to results on the computational complexity of RSD [3]. I’m also working on finding axiomatically desirable and computationally feasible ways to randomized tiebreaking.

Moreover, I’m interested in mechanism design settings in which the power of the game designer is limited in the sense that she can only control part of the game. A natural question is whether one can “complete” an incompletely specified game in such a way that the resulting game has certain properties.

As a final example of ongoing work, let me mention an interdisciplinary research project whose goal is to arrive at numerical tradeoffs between different kinds of socially undesirable activities. For example, can we say that using one gallon of gasoline is just as bad for society as creating $x$ bags of landfill trash? How would we arrive at a reasonable value of $x$? Such estimates would be useful to policy makers as well as well-meaning institutions and individuals. The vision is to create a system that can credibly arrive at numerical values for societal tradeoffs. A successful solution seems to require the application of techniques from a variety of research areas, such as game theory, social choice theory, mechanism design, prediction markets, etc.

Representative Papers.

   with F. Brandt

   with F. Brandt, F. Fischer, and P. Harrenstein.

   with H. Aziz and F. Brandt
YUN KUEN CHEUNG

Thesis. Analyzing Tatonnment Dynamics in Economic Markets

Advisor. Richard Cole, New York University

Brief Biography. I am a postdoctoral scholar in the Computer Science Department at the University of Vienna, working with Monika Henzinger. In 2014, I received my PhD from the Courant Institute of Mathematical Sciences, New York University, where I was advised by Richard Cole. Before my PhD, I received an MPhil in Mathematics and a BSc in Mathematics and Physics, both from the Hong Kong University of Science and Technology (HKUST). My MPhil thesis was honored with the New World Mathematics Silver Award in 2010. I was a team member of the HKUST Programming team for two years, and was in the top four in two regional contests of the ACM ICPC. I was a bronze medallist in the 2004 International Mathematical Olympiad.

Research Summary. My research interests are in computational economics and algorithmic game theory. One of my main foci is the convergence analysis of the well-known tatonnement price dynamic. I am also working on mechanism design problems and game-theoretic aspects of markets.

Arguably, tatonnement is the most well-studied price dynamic in the theory of markets; it was introduced by Walras in 1874, along with the market equilibrium concept. Scarf showed tatonnement did not always converge to an equilibrium. Consequently, a fundamental problem is to identify broad classes of markets in which it does converge. Before my work, convergence was known for markets with goods that are substitutes, but little was known w.r.t. complementary goods. We are the first to show that tatonnement converges quickly in a number of interesting market classes with complementary goods, and more generally in some markets with both substitutes and complements.

One of the results mentioned above relies on the equivalence of tatonnement and coordinate descent for many markets. Motivated by asynchronous variants of tatonnement, we have been studying asynchronous coordinate descent (ACD), which has recently drawn attention in optimization theory. We designed a novel amortized technique to analyze ACD, with a general update rule that covers most known update rules, e.g. the round-robin rule.

Recently, I have worked on mechanism design problems with conflict-based negative externalities, e.g. in an ad auction, an advertiser’s value for an ad slot drops when its rival has a better slot. We proposed a model for such externalities, and designed mechanisms with good approximation guarantees to the social welfare. One of our main results is to design a cone program (CP), which is a combination of a semi-definite program for independent set problems and the standard linear program for combinatorial auctions, and a rounding scheme for the CP, which achieves the best approximation ratio that one would expect.

I am now starting to work on game-theoretic aspects of markets. While truthful markets are efficient, individual agents can benefit by misreporting. Such manipulations are formulated by casting markets into games; thus analyses in terms of well-known effectiveness measures, e.g. the price of anarchy, are readily motivated. I am interested in how the amount of money an agent has and the similarity of her
preferences to those of other agents influences both her power to manipulate and market efficiency.

Representative Papers.


ILAN COHEN

Thesis. Online Algorithms and Game Theory

Advisor. Yossi Azar, Tel Aviv University

Brief Biography. Ilan Cohen is a PhD student at the Blavatnik School of computer science in Tel Aviv University under the supervision of Professor Yossi Azar. He holds an M.S. in computer science from Tel Aviv University and a B.S. cum laude in computer science from the Technion Institute in Haifa. His research involves online and approximation algorithms with game theoretical aspects. During the past three years, he has been a teaching assistant in the Algorithms course. Prior to his doctoral program, he worked as an algorithms developer and a programmer at IDF in the intelligence corps.

Research Summary. My research interests lie at the intersection of approximation algorithms, online algorithms and game theory. My work is divided into three parts. The first part adds game theoretical aspects to fundamental online problems. The second part involves oblivious algorithms that are motivated by designing prompt mechanisms for online bounded capacity auctions. The third part covers various subjects in online packing and covering problems.

Online algorithms deal with making irrevocable decisions while handling a sequence of events. In our scenario, the events are strategic in nature and have a private cost function, and seek to maximize their utility, i.e. minimize their private cost incurred by making a decision plus the surcharge posted on the decision by our dynamic pricing scheme. An example of this is the “parking problem” where an online sequence of cars arrive in some metric space and need to park in a vacant parking spot. Online algorithms know the next car’s destination and order it where to park, while in our setting the algorithm sets a surcharge for each parking place (without knowing the next car’s destination) and defers the decision on where to park to the car itself. This scenario is natural for problems such as: k-server, online metric matching and metrical task systems. We achieve essentially the same approximation ratio (up to a constant) as the best known online algorithms for these problems.

A bounded capacity auction is a single-item periodic auction for bidders that arrive online, where the amount of participating bidders is bounded. The algorithm decides which agents will participate and the allocation and pricing rule. We show a reduction from a simple stochastic balls and bins game to this problem. Although the algorithm for the game is oblivious (i.e., it does not receive input), we devise
a non-uniform randomized algorithm. We establish a lower bound of 1.5 and an upper bound of 1.55, which implies a 1.55 competitive ratio mechanism for this auction.

In online packing and covering problems we establish almost tight lower and upper bounds for packing multidimensional vectors into bins. In this work we give almost tight bounds on the number of bins where the competitive ratio depends on the number of dimensions and the ratio between the maximum coordinate to the bin size. Additionally, we have worked on online covering with convex objective functions, including application such as unrelated machine scheduling with startup costs.

Representative Papers.


JOHN P. DICKERSON


Advisor. Tuomas Sandholm, Carnegie Mellon University

Brief Biography. John is a Ph.D. candidate in the Computer Science Department at Carnegie Mellon University, where he works in the Electronic Marketplaces Lab with his advisor Tuomas Sandholm. John’s research is at the intersection of computer science and economics, with a focus on solving practical economic problems using stochastic optimization. He has worked extensively on theoretical and empirical approaches to kidney exchange, where his work has set policy at the UNOS nationwide exchange; game-theoretic approaches to counter-terrorism, where his models have been deployed; and computational advertising through Optimized Markets, a CMU spin-off company. With Tuomas Sandholm, he created FutureMatch, a general framework for learning to match subject to human value judgments. FutureMatch won a 2014 HPCWire Supercomputing Award and now provides sensitivity analysis for matching policies at UNOS. He is the winner of a 2012–2015 NDSEG Fellowship and a 2015–2017 Facebook Fellowship.

Research Summary. The exchange of indivisible goods without money addresses a variety of constrained markets where a medium of exchange—such as money—is considered inappropriate. Participants are either matched directly with another participant or, in more complex domains, in barter cycles and chains with many other participants before exchanging their endowed goods. My thesis research addresses the design, analysis, and real-world fielding of dynamic matching markets and barter exchanges.

Specifically, I study competing dimensions found in both matching markets and barter exchange, such as uncertainty over the existence of possible trades, trade-offs between efficiency and fairness, and inherent market dynamism. For each individual
dimension, I provide new theoretical insights as to the effect on market efficiency and match composition of clearing markets under models that explicitly consider those dimensions. I support each theoretical construct with new optimization models and techniques that focus on scalability and practical applicability. In the cases of uncertain trades and dynamic matching, where participants and potential trades arrive and depart over time, my algorithms perform substantially better than the status quo deterministic myopic matching algorithms used in practice, and also scale to larger instance sizes than prior methods. In the fairness case, I quantify the loss in system efficiency under a variety of equitable matching rules.

I address each dimension in “FutureMatch,” a framework for learning to match in a general dynamic model. It takes as input a high-level objective decided on by experts, then automatically (i) learns based on data how to make this objective concrete and (ii) learns the “means” to accomplish its goal—a task that humans handle poorly. FutureMatch now provides sensitivity analysis for matching policies at the UNOS nationwide kidney exchange.

My research shows that taking a holistic approach to balancing efficiency and fairness can often practically circumvent negative theoretical results. The balance is struck computationally via extensive optimization of realistic stochastic models of markets. Yet, theory lends necessary intuition to modeling decisions and validity to optimization techniques. Moving forward, I will continue to produce new theoretical results and optimization algorithms in support of market clearing frameworks that adhere to reality, with an eye toward fielding these new mechanisms.

Representative Papers.


and coordination mechanisms.

Research Summary. My main area of research is Algorithmic Game Theory and Algorithmic Mechanism Design. These fields lie in the intersection of Theoretical Computer Science and Economics and study the effects of strategic behavior of the participants in the performance of a system or an algorithm.

The main topic of my PhD thesis is approximately maximizing social welfare in general social choice and resource allocation problems without money, when the participants have unrestricted cardinal preferences over the set of outcomes. The goal is, for the different problems that I consider, to come up with truthful mechanisms with good approximation ratios or mechanisms with good Price of Anarchy guarantees. In the general social choice setting with unrestricted preferences, truthful deterministic mechanisms are severely limited by strong impossibility results. On the other hand, randomized mechanisms are possible and they do provide better approximation ratio guarantees. Comparing mechanisms and trying to come up with the best one is a topic of my thesis. A similar approach is adopted for the setting of one-sided matchings without money, where I study well-known mechanisms and prove their asymptotic optimality in terms of social welfare.

I have also been working on markets, studying both incentive properties and approximate fair solutions, for the fundamental Fisher market model, as well as markets with indivisibilities and subsets of Arrow-Debreu markets with quasilinear preferences. The subject of this work is to prove existence of pure Nash equilibria for these market models or come up with approximately clearing conditions that produce envy-free outcomes.

Another topic that I am interested in is fair division in general, in terms of incentives and economic properties as well as computational issues. I am interested in studying different well-known fair division protocols (such as the Adjusted Winner procedure) from a strategic point of view; proving equilibrium existence, Pareto-efficiency and social welfare guarantees in equilibrium. On the computational side, I have been working on fair division problems such as consensus halving or markets with indivisible items, trying to come up with hardness proofs and approximation algorithms for different goals.

I am also working on other subfields of algorithmic mechanism design, such as facility location problems, kidney exchange markets and structured assignment settings.

Representative Papers.

[1] Truthful Approximations to Range Voting (WINE 2014) with P.B. Miltersen

SAM GANZFRIEND

Thesis. Computing Strong Game-Theoretic Strategies and Exploiting Suboptimal Opponents in Large Games

Advisor. Tuomas Sandholm, Carnegie Mellon University

Brief Biography. Sam received a PhD in computer science from Carnegie Mellon University in 2015 for his dissertation Computing Strong Game-Theoretic Strategies and Exploiting Suboptimal Opponents in Large Games and holds an A.B. in math from Harvard. His research interests include artificial intelligence, game theory, multiagent systems, multiagent learning, large-scale optimization, large-scale data analysis and analytics, and knowledge representation. He created two-player no-limit Texas hold em agent Tartanian7 that won the 2014 Annual Computer Poker Competition and Claudico that competed in the inaugural 2015 Brains vs. Artificial Intelligence competition against the strongest human specialists in the world for that poker variant: the humans won the latter by a margin that was statistically significant at the 90% level but not at the 95% level, and many exciting lessons were learned. He organized the AAAI Workshop on Computer Poker and Imperfect Information in 2014 and 2015.

Research Summary. Important problems in nearly all disciplines and on nearly all application domains involve multiple agents behaving strategically; for example, deploying officers to protect ports, determining optimal thresholds to protect against phishing attacks, and finding robust policies for diabetes management. Such problems are modeled under the framework of game theory. In many important games there is information that is private to only some agents and not available to other agents – for instance, in auctions each bidder may know his own valuation and only know the distribution from which other agents’ valuations are drawn.

My research designs new approaches for strategic agents acting in large imperfect-information games. It includes novel algorithms, theoretical analysis, and large-scale implementation.

There are several major challenges that must be confronted when designing successful agents for large multiagent strategic environments. First, standard solution concepts such as Nash equilibrium lack theoretical justification in certain classes (e.g., games with more than two players). Second, computing these concepts is difficult in certain classes from a complexity-theoretic perspective. Third, computing these concepts is difficult in practice for many important games even for cases when they are well-motivated and polynomial-time algorithms exist (e.g., two-player zero-sum (competitive) games), due to enormous state spaces. And fourth, for all game classes, it is not clear if the goal should even be to compute a Nash equilibrium; one could achieve significantly higher payoff by learning to exploit opponents’ mistakes. However, such exploitation must be done in a way that does not open oneself up to being exploited in turn by strong deceptive opponents.

While the approaches are domain independent, most of them have been motivated by and applied to the domain of poker. Poker has emerged as a major AI challenge problem. Poker is not simply a toy game; it is tremendously popular for humans, and online poker is a multi-billion dollar industry. For the past ten years, there has been a competition between the strongest computer poker agents held annually at the top AI conference. The version of two-player no-limit Texas hold ’em played has approximately $10^{165}$ states in its game tree. Several of the techniques I developed were utilized to create agents that won the 2014 competition and that competed against the strongest human specialists in 2015.
Representative Papers.

[1] Endgame Solving in Large Imperfect-Information Games (AAMAS 2015) with T. Sandholm

XI (ALICE) GAO

Thesis. Eliciting and Aggregating Truthful and Noisy Information
Advisor. Yiling Chen, Harvard University

Brief Biography. Xi (Alice) Gao is currently a postdoctoral research fellow in Computer Science at University of British Columbia, where she holds a prestigious Canadian NSERC Postdoctoral Fellowship. Alice’s research tackles problems at the intersection of artificial intelligence, game theory, and crowdsourcing, using a mix of theoretical and experimental methods. Alice obtained her PhD in Computer Science from Harvard University in 2014. Her PhD dissertation received the 2014 Victor Lesser Distinguished Dissertation Runner-up Award and was also selected for Honorable Mention for the 2015 SIGecom Doctoral Dissertation Award. Her PhD research was supported by a Canadian NSERC Postgraduate Scholarship for Doctoral Students and she was named a 2014 Siebel Scholar. Previously, she earned her Bachelor’s degree in Computer Science and Mathematics from University of British Columbia.

Research Summary. My research is in algorithmic game theory and broadly at the intersection of artificial intelligence, game theory, and crowdsourcing. I am driven by the desire to understand the strategic interactions of self-interested participants in complex systems and I aim to better design these systems to achieve desirable outcomes. In pursuing these goals, I draw insights from various disciplines such as artificial intelligence, game theory, statistics, etc. Moreover, I enjoy tackling problems using a mix of theoretical analyses and experimental studies.

My dissertation research focuses on developing and analyzing methods for eliciting and aggregating dispersed information. I have addressed a number of problems including eliciting truthful estimates of uncertain events using prediction markets, eliciting truthful evaluations of products and services using peer prediction methods, and ranking multiple alternatives by adaptively eliciting and aggregating noisy information. Currently, I am investigating ways of eliminating collusion in peer prediction mechanisms by using limited, costly access to ground truth provided by trusted evaluators.

Representative Papers.

with T. Pfeiffer, A. Mao, Y. Chen, and D.G. Rand

NIKOLAI GRAVIN

Thesis. Incentive Compatible Design of Reverse Auctions
Advisor. Dmitrii Pasechnik, University of Oxford

Brief Biography. Nick finished graduate school at Saint-Petersburg department of Steklov Mathematical Institute in Russia in 2010. At the same time he was a PhD student at the mathematical department of Nanyang Technological University in Singapore, which he finished in 2012. His research interests are twofold. In Mathematics he has been working in graph theory, convex and discrete geometry. In Theoretical Computer Science he is particularly interested in Algorithmic Mechanism Design and Equilibria computations. Nick is a recipient of a prestigious Microsoft Research Fellowship awarded to the top students in Asia.

Research Summary. My research lies in the areas of algorithmic mechanism design and game theory, with connections to on-line algorithms and learning theory. It involves the design and analysis of approximation algorithms for a variety of optimization problems. In my work, I often ask the following questions: Which metrics should be used to quantify performance and efficiency of an algorithm or mechanism in an economic setting?

Digital goods auctions. My work on digital goods auctions illustrates the importance of these questions. In this setting a monopolistic seller seeks to maximize profit from the sale of a single good available in unlimited supply. Digital goods with negligible costs for duplication and distribution such as pay-per-view television and downloadable audio files make a perfect example. [GHW 01] initiated the worst-case analysis of this problem in the mechanism design framework. This and many subsequent work study pricing mechanisms in the form of single-round, sealed-bid truthful auctions for selling digital good. Our characterization of the extremal distributions for the class of all monotone benchmarks provided a missing tool to derive tight worst-case results for a big family of meaningful benchmarks.

Simple Mechanisms for Complex Markets. This line of my work focuses on Bayesian framework allowing to circumvent undesirable computational hardness and performance gaps of the worst-case analysis. To study the large-scale, computer-aided combinatorial markets which are becoming a reality, with examples of FCC spectrum auction and internet-powered marketplaces like Ebay the CS community has generated a subfield of work on developing efficient algorithms and incentive compatible mechanism for combinatorial allocation problems. In the model of combinatorial auction, there is a large set $M$ of $m$ objects for sale, and $n$ potential buyers. Each buyer has a private value function $v_i : 2^M \rightarrow \mathbb{R}_{\geq 0}$ mapping sets of objects to their associated values. The goal of the market designer is to decide how to allocate the objects among the buyers to maximize the social efficiency. In [3], [2] we study two simple and practical mechanisms in a Bayesian setting: item-bidding simultaneous auctions that achieve nearly optimal social welfare at any Bayesian Nash equilibrium; posted price mechanism yielding the first Bayesian polytime constant-approximation truthful mechanism for submodular buyers.
Representative Papers.


NIMA HAGHPANAH
Thesis. Optimal Multi-parameter Auction Design
Advisor. Jason Hartline, Northwestern University

Brief Biography. Nima Haghpanah is a postdoctoral associate at MIT CSAIL and Sloan School of Management, working with Costis Daskalakis and Itai Ashlagi. He obtained his PhD in 2014 from Northwestern University advised by Jason Hartline. His dissertation was on optimal multi-parameter auctions, and his research interests are mechanism design, pricing, algorithms, and optimization. Nima was awarded the Simon’s Award for Graduate Students in Theoretical Computer Science 2013-2015, and Yahoo! Key Scientific Challenges in 2012.

Research Summary. Mathematical models are often designed to understand simple rules of nature. The design of real world systems, however, requires models encompassing high levels of detail, making them more complex and harder to analyze. I study these complex design problems, focusing on the design of mechanisms and markets, using tools and concepts from microeconomics analysis, algorithms, and optimization.

Bayesian mechanism design studies the design of systems to optimize revenue when the information is privately held by strategic agents. Myerson’s (1981) seminal result studied this problem when there is a single item to sell. The result is celebrated because it identified optimal mechanisms that are practical and commonly used, and also proposed a universal analysis method based on virtual valuations. A fundamental assumption in Myerson’s analysis was single-dimensionality: there is only one way to serve an agent, and the agent’s utility is value minus payment. In my PhD thesis I studied a class of problems with wide variety of applications, in which there are multiple ways to serve an agent (e.g., the item may have different qualities or delivery times), and agents have general non-linear or multi-parameter preferences.

First, I extended the method of virtual values to analyze auctions in multi-parameter settings. I showed that virtual values that prove the optimality of a given auction exist, if the space of agent’s possible preferences can be integrated in a consistent manner. Formalizing this intuition gives a system of differential equations, the solution to which would imply the optimality of the give mechanism. I applied this framework to recover conditions implying optimality of several natural mechanisms. Second, I showed that under general conditions, revenue optimization is governed by an intuitive and well-known principle: maximizing marginal revenue. Third, I showed that even without analytical understanding, optimal multi-agent mechanisms can be efficiently optimized.
In another line of research, I study mechanisms and pricing strategies in presence of network or allocation externalities. I showed that in single-parameter settings, the standard approach of reducing the problem to a worst-case optimization of virtual values can not be used to solve the problem. Instead, I reduced the problem to a novel average-case problem and used it to derive constant factor approximations to these problems.

**Representative Papers.**

   with J. Hartline
2. The Simple Economics of Approximately Optimal Auctions (FOCS 2013)  
   with S. Alaei, H. Fu, and J. Hartline
3. Optimal Auctions with Positive Network Externalities (EC 2011)  
   with N. Immorlica, V. Mirrokni, and K. Munagala

DARRRELL HOY

*Thesis.* Strategic Computation via Non-Truthful Mechanism Design

*Advisor.* Jason Hartline, Northwestern University

*Brief Biography.* Darrell is a Ph.D. candidate in the Theory and Economics group of the EECS department at Northwestern University, and has been in the Boston area visiting the Harvard EconCS group since 2014. Darrell received his undergraduate degree from Dartmouth College, where he greatly enjoyed a liberal arts education, racing for the cycling team as well as beginning to experiment with research. Between Dartmouth and Northwestern, Darrell worked in finance for Bridgewater Associates and launched a website to help cyclists find good roads to ride, sweetopenroads.com, and can be found searching for good roads and trails wherever they may be.

*Research Summary.* Sometimes the output of a computation affects the input. The way Google runs their page-ranking algorithm affects how site designers build websites. When Yelp asks for reviews, restaurants can change how they serve and incentivize customers to leave reviews. In such settings, a computation is more complicated than a collection of circuits: it involves all of the agents making strategic decisions in accordance with their own incentives rather than their instructions. I call these strategic computations, and I want to understand them. I’m working to push non-revelation mechanism design to be more useful as a general model of these strategic computations. I am particularly focused on developing analytical tools that are robust to a) the details of equilibrium, and b) robust to changes in the decision making frameworks of users, for instance, bidders who are risk-averse.

On the first front, my recent work with Sam Taggart and Jason Hartline [1] refined the smooth games and mechanisms approach for the objective of revenue in auctions. In a first-price auction with a few light regularity assumptions, we found implementing a reserve price is sufficient to eliminate the impact of agents with misaligned incentives, even in asymmetric settings where we do not have an analytical characterization of equilibrium.

Our current theories of mechanism design rely strongly on precise assumptions of the decision-making behavior of the agents: that they are risk-neutral and always
choose their optimal action. I am working in this light to make our understanding of mechanisms more robust to the exact risk-attitudes and exact decision-making behavior of strategic agents. In work with Hu Fu and Jason Hartline [2], we showed that the first-price auction is approximately optimal when bidders exhibit a specific type of risk-attitude as well as when they are risk averse. I am very excited to continue pushing in these directions to broader notions of risk-aversion and other behavioral complications.

By pushing our understanding of non-revelation mechanism design to be more robust to the details of equilibrium and decision making behavior, I plan to push forward our understanding of how to think about and design general computations with strategic agents.

Representative Papers.

[1] Price of Anarchy for Auction Revenue (EC 2014)
   with J. Hartline and S. Taggart
   with H. Fu and J. Hartline
   with K. Jain and C. Wilkens

MOHAMMAD REZA KHANI

Thesis. Revenue Efficient Mechanisms for Online Advertising
Advisor. Mohammad T. Hajiaghayi, University of Maryland

Brief Biography. Reza is a fourth year Ph.D. student in Department of Computer Science at University of Maryland working under supervision of Mohammad T. HajiAghayi. He got his M.Sc. degree from University of Alberta working with Mohammad R. Salavatipour. He did his undergraduate studies in computer engineering at Amirkabir University of Technology.

Research Summary. Generalized Second Price (GSP) auction (the current mechanism of choice in online advertising) has appealing properties when ads are simple (text based and identical in size). But GSP does not generalize to richer ad settings, whereas truthful mechanisms, such as VCG do. A straight switch from GSP to VCG either requires all bidders instantly bid truthfully or incurs significant revenue loss. We propose a transitional mechanism which encourages advertisers to update their bids to their valuations, while mitigating revenue loss. In settings where both GSP ads and truthful ads exist, it is easier to propose a payment function than an allocation function. We give a general framework for these settings to characterize payment functions which guarantee incentive compatibility of truthful ads, by requiring that the payment functions satisfy two properties.

Next, we study revenue monotonicity (revenue should go up as the number of bidders increases) of truthful mechanisms in online advertising. This natural property comes at the expense of social welfare - one can show that it is not possible to get truthfulness, revenue monotonicity, and optimal social welfare simultaneously. In light of this, we introduce the notion of Price of Revenue Monotonicity (PoRM) to capture the loss in social welfare of a revenue monotone mechanism. We design truthful and revenue monotone mechanisms for important online advertising...
auctions with small PoRM and prove a matching lower bound.

Finally, we study how to measure revenue of mechanisms in the prior free settings. One of the major drawbacks of the celebrated VCG auction is its low (or zero) revenue even when the agents have high values for the goods and a competitive outcome would have generated a significant revenue. A competitive outcome is one for which it is impossible for the seller and a subset of buyers to ‘block’ the auction by defecting and negotiating an outcome with higher payoffs for themselves. This corresponds to the well-known concept of core in cooperative game theory. We define a notion of core-competitive auctions. We say that an incentive-compatible auction is $\alpha$-core-competitive if its revenue is at least $1/\alpha$ fraction of the minimum revenue of a core-outcome. We study designing core-competitive mechanisms for a famous online advertising scenario.

Representative Papers.


[2] Mechanism Design for Mixed Participants (to be submitted) with Y. Bachrach, S. Ceppi, I.A. Kash, and P. Key


ELI MEIROM

Thesis. Games and Dynamics in Large Communication Networks

Advisor. Ariel Orda and Shie Mannor, Technion

Brief Biography. Eli Meirom received a B.Sc. degree in Math and Physics from the Technion, Israel (summa cum laude), and an M.A in Physics (magna cum Laude). He is currently pursuing a Ph.D degree in Electrical Engineering at the Technion, Israel. Previously, he held research positions in IBM Research and St. Jude Medical. He published papers in various fields, including quantum information, solid state physics, machine learning, game theory and computer networks. He was awarded the Applied Materials and Mel Berlin fellowships. His current research interests are in the intersection of machine learning and social networks, e.g., social network analysis and multi-agent dynamics on graphs.

Research Summary. The interplay between the network topology, and the network performance is critical in all networks. Additionally, a network must operate in secure and reliable fashion, in order to perform its function. We ask: How does the behavior, and performance requirements of the users, affect the network structure? What is the effect of reliability requirements on the network topology? In computer networks, can we design generic anti-malware measures that are based only on network structure and properties, rather than malware attributes?

Most of the studies in Network Formation Games (NFG) assume identical players, whereas the Internet is composed of many types of ASs, such as minor ISPs, CDNs, tier-1 ASs etc. We constructed a heterogeneous network formation games, and analyzed both is static and dynamic properties. Furthermore, game theoretic analysis is rarely confronted with real-world data. We took a step further, and considered real inter-AS topology data. Our model and its analysis resulted in some
novel predictions regarding the evolution of the inter-AS topology.

In many NFG the resulting networks are very fragile. In a later work, we established a model that explicitly includes the agents’ reliability requirements. We provide dynamical analysis of topological quantities, and explain the prevalence of some network motifs, i.e., sub-graphs that appear frequently in the Internet.

Attacks, from denial of service to state-driven cyberwarfare threaten to compromise our modern infrastructure. Anti-virus software can find the signature of known worm or virus. But how do we intercept malware spread in actionable time-scales, before we even know what is spreading?

When people contract a virus they might miss a work day, though of course many other factors could produce such behavior. Malware, similarly, produces slight deviations in system behavior, for example, a spike in network activity. Can we use indications of abnormality that are so weak that on their own they are statistically indistinguishable from noise, to make an accurate global diagnosis about a spreading contagion? We have shown that, given a map of nodes that experience suspicious behavior, we were able to identify a malware outbreak. We addressed this problem from a dynamic perspective as well, and showed that monitoring the dynamics of these weak signatures enables early detection, very shortly after the initial infiltration occurred.

Representative Papers.

[1] Network Formation Games with Heterogeneous Players and the Internet Structure (EC 2014) with S. Mannor and A. Orda

ILAN NEHAMA

Thesis. Computational issues in Judgement Aggregation

Advisor. Noam Nisan, Hebrew University of Jerusalem

Brief Biography. Ilan received his B.A. in Math and B.Sc. in Computer Science at the Technion (Summa Cum Laude), and M.A. in Computer Science with specialization in Rationality at The Hebrew University under the supervision of Prof. Gil Kalai; Thesis: Implementing Social Choice Correspondences using k-Strong Nash Equilibrium (Summa Cum Laude, GPA: 97.77, 2/194). Ilan is a Ph.D. candidate at The Hebrew University (Benin School of Computer Science & Federmann Center for the Study of Rationality) and is expected to finish on 2016. During his graduate studies, he served as a lecturer in a Programming course, as well as a TA in several MA courses both in the Computer Science department - Mathematical Tools in CS, and the Economics department - Microeconomics A & Microeconomics B: Game Theory and Information Economics. Ilan’s works are mainly in theoretical Game Theory, Social Choice, and Judgement Aggregation, and on computational aspects and the usage of methods from Computer Science in these fields.
Research Summary. I am interested in a Computer Science approach to questions in Social Choice and specifically to Judgement Aggregation. Judgement Aggregation (JA) investigates which procedures a group could or should use to form collective judgements on a given set of propositions or issues, based on the judgements of the group members. Judgement Aggregation is the subject of a growing body of works in economics, computer science, political science, philosophy, law, and other related disciplines. I find this field highly applicable to agent systems, voting protocols in a network and other frameworks in which one needs to aggregate a lot of opinions in a systematic way without letting the voters deliberate or without assuming a deliberation process.

I’m interested in shedding light on phenomena in Judgement Aggregation using approximation and perturbation viewpoints. That is, studying the way phenomena studied in the literature change when perturbing the classic strict properties. E.g., requiring an aggregation rule to satisfy a property with high probability (but not for sure), generalizing players’ rationality constraints to being close to rational (bounded rationality). Dealing with probabilistic properties raises the question of choosing the ‘right’ underlying distribution or distributions family. It is clear that real-life distributions are not uniform, and indeed most of the current works that analyze rules using simulations, check non-uniform distributions. Nevertheless, there are very few analytical works dealing with such distributions. As part of the above perturbation paradigm, I study other natural distributions (e.g., Polya-Eggenberger models), and aim to extend works that study uniform distributions to analysis under other distributions families.

For example, I studied the perturbations of the ‘Doctrinal Paradox’ scenarios. In these scenarios, one looks for ‘consistent’ and ‘independent’ aggregation mechanism for a given agenda (=set of permissible opinions). In ‘Approximately Classic Judgement Aggregation’ I presented the relaxation where these two desired properties hold only with high probability, and showed that under uniform distribution, for conjunction and xor agendas, there is no non-trivial mechanisms that satisfy these perturbed constraints. In subsequent works, I show similar results for non-silencing agendas, which are most of the truth-functional agendas, and for non-uniform distributions, although still having independent representation.

Representative Papers.

   with E. Dokow, M. Feldman, and R. Meir

SVETLANA OBRAZTSOVA

Thesis. Essays on the Complexity of Voting Manipulation

Advisor. Edith Elkind, University of Oxford

Brief Biography. Currently a postdoctoral fellow at Israeli Centre of Research Excellence (I-CORE) in Algorithms. My first postdoctoral appointment was with the CoreLab, NTUA, Greece. Prior to that, a dual, 4-year PhD program of the Steklov
Institute of Mathematics (St.Petersburg, Russia) and NTU (Singapore). My PhDs are in Mathematics and were received in June 2011 (from the Steklov Institute) and in October 2012 (from NTU). My work has been accepted and acknowledged by the research community. The collaboration with Prof. Elkind was nominated for the Best Paper Award at The 10th AAMAS Conference (4 nominees from 575 submissions). Same paper was among the three works selected to represent AAMAS-2011 at IJCAI-2011 (the Best Papers from Sister Conferences Track). Another joint work has received the Pragnesh Jay Modi Best Student Paper Award at AAMAS-2012, and also served as a representative work at AAAI-2012. My work on graph theory has won Google Europe Anita Borg Memorial Scholarship in 2008.

**Research Summary.** My research interests fall into the realm of Computational Economics or, more accurately, the Computational Social Choice.

Initially, during my PhD studies, I'd concentrated on the voting manipulation complexity, publishing several joint papers with Edith Elkind on the influence of the tie-breaking rule on the said complexity, and the complexity of optimal manipulations.

However, accepting manipulation attempts as a given, I've quickly moved to the study of resulting Nash Equilibria (NEs), and general stable states. First, I've contributed to the study of biased voters, as means to reduce the set of NEs. E.g., I've investigated the effects of the truth and lazy biases. Second, I took interest in iterative voting, which lead to several results on its stability conditions and stable state characteristics. Taking the lead on the iterative voting processes, I've also researched Iterative Candidacy Games.

These results have proved of a great interest to the community in general, and a well grouped set of them was organised into the “Voting and Candidacy Games” Tutorial at AAMAS 2015.

There are several research questions that I continue to address, both building on my previous success and establishing new research directions. The first natural direction is to further exploit the idea of rewarding truthfulness, extending it to other voting rules. It is also possible to additionally enrich the truth-biased voting model by introducing the concept of “lie degree”, based on the distance between the submitted vote and the truthful preference. Thus making the current model a binary sub-case. A variety of distance measures can be used here, which suggests a rich ground for new ideas and publications.

Another research direction in my immediate plans is to further develop the concept of candidates’ game. In spite of recent contributions, including my own, it remains one of the least studied and, yet, most intriguing research prospects in Social Choice. Current results are only partial and specialised, and are in dire need for generalisation across voting rules and mutual voter-candidate choice dynamics.

Finally, a budding research direction is the link between voting manipulations and preference elicitation procedures, as expressed by the dynamics of these these processes. Inevitably with great success, the first set of results was presented with my co-authors to limited audiences, building up to their wider acceptance at IJCAI-2015.

**Representative Papers.**

with E. Elkind and N. Hazon


JOEL OREN

Thesis. Multi-Winner Social Choice: Algorithmic and Game-Theoretic Approaches

Advisor. Allan Borodin and Craig Boutilier, University of Toronto

Brief Biography. Joel Oren is a Ph.D student at the Department of Computer Science, University of Toronto, Canada. He is supervised by Allan Borodin and Craig Boutilier, and will be graduating in August, 2015. He received his M.Sc in Computer Science, at the University of Toronto under supervision of Allan Borodin. He received his B.Sc in Computer Science with honors from Ben Gurion University, Israel, where he was supervised by Avraham Melkman. He received the Ontario Graduate Scholarship award in 2010, 2011, and 2013. He has also received the Raymond Reiter award by the department of Computer Science, University of Toronto, for the year 2015.

Research Summary. I am interested in the study of influence diffusion in social networks. In particular, I focus on the game theoretic study of games and mechanisms for competing diffusion processes. Moreover, I take an active part in research on the design of efficient and parallelizable algorithms for large scale social networks, for addressing problems such as cascade detection, and fitting of influence diffusion model parameters. My goal is to reason about the spread processes that are captured by real-world datasets.

During my PhD studies, my research had two recurring themes. The first major theme of research is the study of stochastic processes in social networks. I performed research in approximation algorithms and mechanism design for competitive influence diffusion. I am presently engaged in applying current algorithmic paradigms for parallel computing (e.g., MapReduce) for data-mining tasks related to social networks, such as influence estimation and maximization, structural analysis, etc. With my collaborators, I have taken part in the design of highly-parallelizable algorithms for estimating influence diffusion processes in massive social networks, with provable performance guarantees.

The second area that I have contributed to deals with applying probabilistic models to decision-making problems at the core of computational social choice. With my collaborators, I have studied slate optimization and efficient preference elicitation, while crucially building on belief models of agent preferences. We have revisited existing problems in these regimes through a probabilistic lens, and offered a rigorous study, which offered great insight, that goes beyond existing worst-case models. Our work includes an analysis of the capabilities of top-\( k \) voting for preference elicitation, and online algorithms for slate optimization.

Representative Papers.

[1] Strategyproof Mechanisms for Competitive Influence in Networks (WWW 2013) with A. Borodin, M. Braverman and B. Lucier
S. Dughmi, V. Gkatzelis, and J. Hartline

[2] Influence at Scale: Distributed Computation of Complex Contagion in Networks (KDD 2015) with B. Lucier and Y. Singer


EMMANOUIL POUNTOURAKIS

Thesis. Simple Mechanisms in Static and Dynamic Settings

Advisor. Nicole Immorlica, Microsoft Research

Brief Biography. Emmanouil Pountourakis is currently a fifth year PhD student in the Department of Electrical Engineering and Computer Science at Northwestern University, advised by Nicole Immorlica. Since 2014 he has been a long term visitor at Microsoft Research, New England. He holds an undergraduate and masters degree in Computer Science from the University of Athens. During Summer 2012 he completed a research internship at CWI, Amsterdam. He was a student visitor in the Institute of Advanced Studies at Hebrew University for the Special Semester in Algorithmic Game Theory in 2011. Emmanouil Pountourakis has a broad interest in algorithmic mechanism design. His current research focuses on revenue maximization in static and dynamic environments. In the past he has worked on a variety of topics including cost-sharing, matching, and mechanism design without money.

Research Summary. Most of my research lies in algorithmic mechanism design and lately focuses on revenue maximization in static and dynamic settings. I am particularly interested in the study of anonymous pricing. For several years this widely studied mechanism was known to have a revenue approximation ratio within [2,4] for selling a single item. My recent work gives the first improvement of this gap to [2.23, e]. Furthermore, I am currently studying anonymous pricing in the dynamic environment of repeated sales of a single item. This setting gives rise to various strategic behaviors with negative implications for the revenue. Surprisingly, anonymous pricing suffers less than its discriminatory counterpart and may provide higher revenue. My on-going work further investigates this phenomenon. Also, I am interested in the interaction of revenue maximization and different behavioral models. My recent work studies optimal contract design with a present-biased agent. When making a decision, the present-biased agent overestimates her present utility by a multiplicative factor. The contract designer exploits this behavior to maximize his revenue. My work introduces regulations and studies optimal contracts under them in the hopes of reducing the exploitative power of the mechanism.

Earlier problems I’ve worked on include cost-sharing, matching, and mechanism design without money. My contributions are briefly outlined below. A central challenge in cost sharing is the design of group-strategyproof mechanisms, that is mechanisms that are resilient to group manipulation. My work gives a complete characterization of group-strategyproof cost sharing mechanisms and studies their performance in terms of budget balance, that is what fraction of the cost can be covered using payments assigned to the agents. In another work I generalize the stable-marriage problem. Agents have limited information about their peers illustrated by a social graph. This relaxes the stability constraint and gives rise to the optimization problem of finding a maximum size stable matching. My work
gives an approximation algorithm with matching lower bound. Finally, I studied the problem of task allocation without payments using the assumption of binding reports: the agent incurs a cost which is the maximum of the cost they reported and their actual cost. My work studies the problem of choosing a base of a matroid, e.g. spanning tree, and designs optimal truthful approximate mechanisms under this assumption.

Representative Papers.


BAHARAK RASTEGARI

Thesis. Stability in Markets with Power Asymmetry

Advisors. Kevin Leyton-Brown and Anne Condon, University of British Columbia

Brief Biography. Baharak Rastegari is a research associate at the School of Computing Science, University of Glasgow (UofG). She is currently working on an EPSRC project titled “Efficient Algorithms for Mechanism Design Without Monetary Transfer”, which is a joint project between the universities of Glasgow, Liverpool, and Oxford. Her main areas of interest are Game Theory and Bioinformatics. For the past five years she has been focused on solving problems concerning matching markets under preferences; before that, she worked on revenue properties of combinatorial auctions and the prediction of RNA secondary structures. She received her Ph.D. from the University of British Columbia (UBC), Canada, in 2013. She holds an M.Sc. in Computer Science (UBC 2004) and a B.Sc. in Computer Engineering (Sharif University of Technology, Iran, 2002). She enjoys teaching and has given several lectures in various courses, including Foundations of Multiagent Systems (UBC) and Algorithmics (UofG).

Research Summary. I enjoy solving mathematical problems and in particular designing efficient algorithms for various problems, or proving that none exists. My particular focus has been on two-sided matching markets, where agents have preferences over one another; these preferences might be partially known, and might not necessarily be strict. The goal is to compute matchings of the agents to one another that are optimal with respect to the given preferences. Optimality can refer to classical concepts such as stability (which ensures that no two agents have an incentive to form an arrangement outside of the matching) or Pareto optimality (which guarantees that no coalition of agents can improve without harming someone else). Applications arise in entry-level labor markets such as the allocation of junior doctors to hospitals.

In a paper in EC 2013 we considered two-sided matching markets with incomplete information — that is, agents’ own preference profiles are only partially known and true preferences can be learned via so-called interviews. The goal was to identify a centralized interview policy, i.e. an algorithm that adaptively schedules
interviews in order to produce a matching that is stable with respect to agents’ true preferences, and that is furthermore optimal for one given side of the market. We showed that an interview-minimizing policy can be computed in exponential time and we gave evidence showing that it is likely that no polynomial-time computable policy exists unless P=NP. Additionally, we provided a polynomial time algorithm that identifies an interview-minimizing policy for a restricted setting inspired by real world applications.

In a paper in EC 2014 we studied truthful mechanisms for finding large Pareto optimal matchings in two-sided matching markets with one-sided preferences (the so-called “house allocation problem”). We provided a natural and explicit extension of the classical Random Serial Dictatorship Mechanism to the case where preferences may include ties. Consequently we obtained a universally truthful randomized mechanism for finding a Pareto optimal matching and showed that it achieves an approximation ratio of \(\frac{e}{e-1}\).

In earlier work, appearing at AAAI 2007 and SODA 2009, we studied combinatorial auctions involving the so-called revenue monotonicity property (which guarantees that a seller’s revenue weakly increases as the number of bidders grows), investigating the existence of truthful mechanisms for this restriction.

Representative Papers.


NISARG SHAH

Advisor. Ariel D. Procaccia, Carnegie Mellon University

Brief Biography. Nisarg Shah is a Ph.D. candidate in the Computer Science Department at Carnegie Mellon University, advised by Ariel Procaccia. His broad research agenda in algorithmic economics includes topics such as computational social choice, fair division, game theory (both cooperative and noncooperative), and prediction markets. He focuses on designing theoretically grounded methods that have practical implications. Shah is the winner of the 2013-2014 Hima and Jive Graduate Fellowship and the 2014-2015 Facebook Fellowship.

Research Summary. I am interested in the general problem of how to use inputs from multiple agents for computing a social outcome; examples include political elections, crowdsourcing, and multi-agent resource allocation. My thesis research investigates real-world social computing settings in which monetary exchange is prohibited, and uses theoretical insights to design well-founded solutions.

For example, in computational fair division, my latest project [1], in collaboration with a large California school district, deals with the design and implementation of a method for fairly allocating classrooms to charter schools. On the theoretical level, we show that our approach is provably fair and provides worst-case optimization.
guarantees. And from the practical viewpoint, a scalable implementation of our mechanism requires a number of innovations; its deployment is an ongoing, intricate project. I have also done significant work on the fair allocation of computational resources in clusters (see, e.g., [2]). In particular, we were among the first to study dynamic fair division in a setting where agents arrive over time, thereby pushing the conceptual limits of fair division theory itself.

As another example, my work in computational social choice focuses on settings where an objective ground truth exists, and the input votes provide noisy estimates of this ground truth. In a sequence of papers (see, e.g., [3]), we design social choice methods that provide accuracy guarantees with respect to wide families of possible noise distributions, or even with respect to worst-case noise. My latest project handles correlated noise that arises from an underlying social network. In an ongoing collaboration with Facebook, we aim to design and deploy more efficient rules when the social network structure is known.

Going beyond these specific examples, I am excited about the broader potential of algorithmic economics to make a real-world impact. This potential is evidenced by widely deployed game-theoretic algorithms for protecting critical infrastructure sites (an area that I have contributed to); and by the popularity of the fair division website Spliddit.org, which I am helping to develop. As the field continues to mature, and its theoretical foundations become firmer, I am certain that more opportunities will arise for applying algorithmic economics for societal good.

Representative Papers.

[1] Leximin Allocations in the Real World (EC 2015) with A.D. Procaccia and D. Kurokawa

OR SHEFFET

Thesis. Beyond Worst-Case Analysis in Privacy and Clustering: Exploiting Explicit and Implicit Assumptions

Advisor. Avrim Blum, Carnegie Mellon University

Brief Biography. I’m a postdoctoral fellow at the Center for Research on Computation and Society at the School for Engineering and Applied Sciences at Harvard University, under the supervision of Prof. Salil Vadhan. Before joining Harvard, I was a Research Fellow at the Theoretical Foundations of Big Data program at the Simons Institute for the Theory of Computing in UC Berkeley. I completed my PhD in computer science from Carnegie Mellon University, advised by Prof. Avrim Blum. I got my M.Sc in computer science from the Weizmann Institute of Science, where I was advised by Prof. Oded Goldreich. I have a B.Sc in CS and Math from the Hebrew University in Jerusalem, Israel, where I worked with Prof. Nati Linial as part of the Amirim honors program.

Research Summary. My interests lie in many fields within computer science that touch on, and benefit from, rigorous mathematical theory. Projects I have worked
on span areas such as algorithm design, machine learning and clustering, ranking and voting, algorithmic game theory and social network analysis. Complementary to this range of interests, my focus in recent years has been on the notion of differential privacy — a powerful, rigorous mathematical guarantee of privacy.

The main goal of my work is to design new differentially private data analysis techniques in the above-mentioned fields. In the coming years I will further pursue the study of the back-and-forth connections between differential privacy and these fields, as well as aim to establish new connections with other fields of big data analysis. One particular direction I am actively pursuing nowadays is the ability to do statistical inference with differential privacy. Apparently, existing techniques in statistics — such as sampling from a posterior or regularizations — preserve privacy for the right choice of parameters.

The fact that privacy concerns are rooted in economic incentives is well-known. (To illustrate, think of the next two questions: Will you let me read your emails? Will you let me read your emails for a million dollars?) It is therefore natural to study the motivation for privacy from a game theoretic perspective, of selfish utility-maximizing agents. A recent work of mine, which I am currently continuing, studies the behavior of rational agents under concrete privacy concerns, where we show that some privacy concerns lead to agents behaving at equilibrium in a way that is differentially private, while in a different setting agents’ behavior are diametrically different.

I am currently a member of Harvard’s Privacy Tools project, aimed at implementing differentially private techniques in order to release information about real datasets. As part of my involvement with the project I have interacted with researchers from very different fields, like statisticians, social scientists and even lawyers. Though they approach data and think of data-driven tools in a very different way than in CS, I found this collaboration to be very rewarding and I plan on continuing such collaborations in the future. This is one of the goals I have set for myself: to promote the use of differentially private tools and to assist in the diffusion of differential privacy from CS to other scientific disciplines.

Representative Papers.

[1] Privacy Games (WINE 2014) with Y. Chen and S. Vadhan

VASILIS SYRGKANIS

Thesis. Efficiency of Mechanisms in Complex Markets

Advisor. Eva Tardos, Cornell University

Brief Biography. Vasilis Syrgkanis is a Postdoc researcher at Microsoft Research, NYC. He received his PhD in 2014, from the Computer Science Department of Cornell University under the supervision of Prof. Eva Tardos. His research interests include algorithms, game theory, auction theory, mechanism design, crowdsourcing,
econometrics, online learning theory and computational complexity. He is the recipient of the Simons Graduate Fellowship in Theoretical Computer Science 2012-2014 and his research has received the best paper award at the ACM Conference on Economics and Computation. During his PhD he spent three summers as a research intern at Microsoft Research. Prior to his PhD he completed his undergraduate in Electrical Engineering and Computer Science at the National Technical University of Athens.

Research Summary. My research addresses the design and analysis of complex electronic marketplaces. It lies at the intersection of computer science and economics and more specifically in the areas of algorithms, game theory, mechanism design, econometrics and online learning theory, addressing optimization problems in the presence of incentives. I am interested in developing theoretical tools for analyzing and designing online markets, focusing on their distinct characteristics and their large scale nature. Some of the key topics I have worked on are:

Analysis and Design of Distributed Mechanisms. How efficient is a market composed of simple, distributed mechanisms for allocating resources and how should we design these local mechanisms in a way that global market efficiency is guaranteed? Most of mechanism design has focused on the design of centralized mechanisms that run in isolation. In “Composable and Efficient Mechanisms”, we tackle the problem of designing distributed mechanisms and give an essential local property that each mechanism should satisfy for the market to achieve global efficiency, even under learning behavior and incomplete information. We show that this property is satisfied by several simple mechanisms, many of which are currently used in practice. Our work unifies a large set of results in the recent literature of characterizing the efficiency of simple and distributed mechanisms, including my work on the efficiency of sequential auctions, and has been subsequently applied and generalized in several settings. In “Bayesian Games and the Smoothness Framework” I also provide a more general approach for quantifying the efficiency in any incomplete information game.

Algorithmic Game Theory and Data Science. How can approaches from algorithmic game theory impact traditional econometrics and how can we use data to inform our theorems? I have explored two directions in this area. In “Econometrics for Learning Agents”, we use an online learning theory approach to model strategic behavior in repeated game theoretic environments and based on that propose an econometric theory for inferring private parameters of participants. In “Robust Data-Driven Efficiency Guarantees in Auctions”, we propose an approach for incorporating observed data of strategic behavior to infer efficiency guarantees in games that are better than the worst-case theoretical guarantees.

Representative Papers.

[1] Composable and Efficient Mechanisms (STOC 2013) with E. Tardos

BO TANG

Thesis. On Optimization Problems in Auction Design

Advisor. Paul Goldberg, University of Oxford

Brief Biography. Bo is PhD student in Economics and Computation Group at the Department of Computer Science, University of Liverpool, supervised by Prof. Xiaotie Deng, Prof. Paul Goldberg and Dr. Giorgos Christodoulou. Before coming to Liverpool, he got a bachelor degree from Shanghai Jiao Tong University in China. He was a Research Intern at Microsoft Research Asia and a Research Assistant at Nanyang Technological University and Columbia University.

Research Summary. Bo investigated the effect of agents' manipulation in economic market in the following three projects: Price-taking vs Strategic manipulation, PoA of Simple Auctions Auctions and Auction Design with a Revenue Target.

Price-taking vs Strategic manipulation: The interplay of demand and price has been modeled as market equilibrium in economics. It have been shown that this classical model is not robust under strategic playing. Bo studied how much utility the buyer can gain by manipulation. For a general class of valuations, he proved this improvement is bounded by his utility as a price-taker and diminishes when the market grows larger and also provided several sufficient conditions for that price-taking approaches best strategic behavior. As a corollary, when the buyers are even allowed to form coalitions, the pricing-taking behavior is also a good approximation to the best response.

Simultaneous Simple Auctions Auctions: These simple auctions were applied by eBay to sell miscellaneous items via running single-item bidding auctions on each item simultaneously. Nevertheless, these succinct auctions disregard the interdependence between items like substitution and complementation. Thus, in such auctions the resulting allocation might be inefficient, that is, it doesn’t allocate items to the buyers who want them most. Bo examined this efficiency loss in such item-bidding auctions and provided a tight lower bound for a class of valuations by constructing an inefficient Nash equilibrium. This result closes the gap of the efficiency bound for simultaneous simple auctions which has been studied in a series of literature from ICALP 2008 to STOC 2013.

Auction Design with a Revenue Target: The common objective for an auctioneer is to maximize the expected revenue raised from this auction. Actually, the objective can be generalized as a particular function of revenue when the seller is of special types like risk-averse, risk-seeking and goal-oriented. For instance, an auctioneer in debt would like to maximize the probability to earn a target revenue and pay off his debt. In contrast to the results for expected revenue, Bo showed the NP-hardness of computing the optimal auction with these objective functions even in single-parameter settings. On the positive side, polynomial-time algorithms can be developed to find the optimal mechanism for special cases based on a novel characterization of optimal auctions with a revenue target.

Representative Papers.

[2] Pricing Ad Slots with Consecutive Multi-unit Demand (SAGT 2013)
   with P. Goldberg, X. Deng, Y. Sun and J. Zhang

The Simulated Greedy Algorithm for Several Submodular Matroid Secretary Problems (STACS 2013) with T. Ma and Y. Wang

PANOS TOULIS
Thesis. Causal Inference under Network or Strategic Interference
Advisor. David C. Parkes (co-advised by Edoardo M. Airoldi and Donald B. Rubin), Harvard University

Brief Biography. I obtained my B.Sc. in Electrical Engineering from the Aristotle University (Greece) in 2005. Between 2006-2009 I worked on applications of intelligent agent systems, and between 2008-2009 I worked in the UAE on the creation of the first Arabic-speaking humanoid robot. In 2009, I moved to the U.S. and obtained my M.Sc. in CS at Harvard, and in 2011 I joined the Statistics Ph.D. program. At Harvard, I work on projects at the intersection of game theory, causal inference, and large data analysis through stochastic gradient descent. I received the 2015 Arthur P. Dempster prize for my work in implicit stochastic gradient descent, and the 2013 Thomas R. Ten Have award for my work in causal inference with interference. In 2012, I helped the Obama For America analytics team to do experimental design in voter mobilization on Facebook. My work has been supported by the 2015 LinkedIn EGC award, the 2012 Google US/Canada Ph.D. Fellowship in Statistics, and the Hellenic Harvard Foundation.

Research Summary. In social and economic contexts, there is an abundance of algorithms on how to mobilize voters over social media, or set prices in online ad auctions, or do viral marketing for a new product. However, there is a shortage of methods to empirically evaluate them. In the context of statistical experimentation, the algorithms are considered to be treatments applied on certain units of analysis, e.g., voters, auctions, or customers. A fundamental problem in the evaluation of such treatments is interference. My research has focused on causal inference of treatment effects under three different forms of interference.

In social network interference, units affect each other through a pre-existing social network, e.g., friends affecting each other in their voting behavior. A key challenge is to use better randomizations of treatment on networks, and to use statistical models to disentangle the interference spillover effect from the primary effect of the treatment itself.

In strategic interference, units affect each other through their strategic actions; for example, in an online ad auction advertisers adjust their bids in response to new prices, thus affecting the competition. A key challenge is then to adjust causal inference to estimate long-term effects, i.e., effects that would be observed if we waited long enough until a new equilibrium was reached in response to new prices.

In experimental strategic interference, a new problem arises when the treatments to be evaluated are themselves self-interested agents, each having a strategic choice of what version of treatment to apply. In an experimental evaluation, the experimenter wants to know how an agent would perform if it adopted its natural behavior, defined as the choice of treatment version that the agent would make if there was no competition. However, agents can game the experiment by adopting different behaviors than their natural behavior, e.g., by applying more risky treatment versions. The goal is therefore to design an incentive-compatible experiment
where agents will choose to adopt their natural behavior.

Parallel to causal inference, I have also been interested in incentive problems of mechanisms operating on random graphs in the context of kidney exchanges, and in estimation problems with large data sets using implicit stochastic approximations.

Representative Papers.


DANIEL URIELI

Thesis. Learning Agents for Sustainable Energy
Advisor. Peter Stone, University of Texas at Austin

Brief Biography. Daniel Urieli is a PhD candidate (graduating in 2015) in The Department of Computer Science at The University of Texas at Austin. Daniel works with Professor Peter Stone on designing autonomous learning agents for sustainable energy problems. As a part of his research, Daniel designed a state-of-the-art, smart-grid energy trading agent that won several research competitions in 2013 and 2015. Before that, Daniel designed a learning agent for smart HVAC thermostat control, which is a part of a pending U.S. patent application by UT Austin. Previously, Daniel was a main contributor to the UT Austin Villa team, which won first place at the international RoboCup competitions in 2011 and 2012, in the 3D simulation league. Before joining UT Austin, Daniel completed a dual major B.Sc. in mathematics and computer science, and an M.Sc. (summa-cum-laude) in computer science at Tel Aviv University, and developed software for microprocessor power-delivery optimization at Intel.

Research Summary. The vision of a smart electricity grid is central to the efforts of moving society to a sustainable energy consumption. The main goals of the smart grid include (1) integration of intermittent, renewable energy sources, (2) reducing the peak electricity demand, and (3) automated energy efficiency. A main milestone for achieving these goals is “customer participation in power markets through demand-side-management”. Demand-side management refers to adapting customer demand to supply conditions. Our research advances towards this milestone by designing state-of-the-art autonomous learning agents for energy trading and for energy efficiency.

In the context of goals (1)-(2) we designed TacTex, an autonomous energy trading agent that won several Power Trading Agent Competitions (Power TAC). The goal of Power TAC is to test novel energy market structures in simulation. This is important due to the high cost of failure in the real-world (like California-2001). In Power TAC, autonomous brokers compete for making profits in a realistic simulation of future smart-grid energy markets. Such brokers must be able to continually (1) learn (2) predict (3) plan (4) adapt in uncertain conditions. Our research gives
insights regarding (1) computational techniques that are required for designing a successful broker (2) the overall impact of such autonomous brokers on the economy. We formalized the energy trading problem as two interdependent (intractable) utility-maximization problems. TacTex approximates their solutions by combining online reinforcement learning with efficient model-based optimization. Using TacTex, we investigated a widely-proposed method for demand-side management called Time-Of-Use tariffs (TOU), achieved state-of-the-art performance, and pointed out challenges and impacts of using TOU in competitive markets.

In the context of goal (3), we developed a thermostat-controlling agent that learns and adapts using advanced reinforcement learning. Our agent saves 7%-15% of the yearly energy consumption of a heat-pump HVAC system while maintaining occupants’ comfort unchanged compared with the widely-used strategy, as observed in simulated experiments using a realistic simulator developed by the U.S. Department of Energy. Since HVAC systems are among the largest energy consumers, such savings can have a significant impact the total electricity demand. Our agent is a part of a pending U.S. patent application by The University of Texas at Austin.

Representative Papers.


ANGELINA VIDALI

Thesis. Game-theoretic Analysis of Networks: Designing Mechanisms for Scheduling

Advisor. Elias Koutsoupias, University of Oxford

Brief Biography. Angelina Vidali is a Postdoctoral Researcher at Pierre and Marie Curie University-LIP6. She received her PhD from the Department of Informatics of the University of Athens (Greece), advised by Elias Koutsoupias. She also held Postdoctoral Researcher Positions at the Max Planck Institute for Informatics (Germany), at the University of Vienna (Austria) and at Duke University (USA). At Duke she organized an interdisciplinary seminar series (Departments of Economics, Computer Science and Fuqua School of Business) sponsored by Yahoo. Her research and studies have been supported by grants from the Vienna Science and Technology Fund, the Alexander von Humboldt foundation, the Alexandros Onassis foundation, the University of Athens, the Greek State Scholarship Foundation and the Greek Secretariat for Research and Technology.

Research Summary. My research lies in the intersection of computer science and economics; a timely, new, exciting research area with unexplored, well-motivated research directions, facing more and more challenges as electronic markets, cloud computing and crowdsourcing gain in market share and as markets and processes get reshaped by social networks. Computer science with its methodology and novel
approaches addresses new questions and sheds new light on fundamental problems in economics. The internet enables us to run auctions, crowdsourcing contests and to compute tasks using cloud computing. The bidders/players/machines are not physically present but connected through the internet and linked through a social network structure, making these auctions easily accessible to a broader public and a part of our everyday life. MSN, Google, Yahoo and eBay need to design auctions for new settings such as sponsored search auctions, display ads, digital goods and pricing of cloud computing services, that will maximize their revenue but also guarantee customer satisfaction. As new markets emerge we need to build realistic new models and analyze them, while classic results from economics improve our intuition and provide us a solid background. This emerging new area has a lot more to contribute to computer science and economics in the coming years.

Representative Papers.


ELAINE WAH

Thesis. Computational Models of Algorithmic Trading in Financial Markets
Advisor. Michael Wellman, University of Michigan

Brief Biography. Elaine Wah is currently a PhD candidate in Computer Science & Engineering at the University of Michigan. Her research interests lie at the intersection of finance and artificial intelligence, specifically in applying computational methods to study algorithmic trading in financial markets. Her dissertation work employs agent-based modeling and simulation to capture current market structure and to investigate the impact of algorithmic trading on market participants. She is a recipient of an NSF IGERT Fellowship and a Rackham Predoctoral Fellowship, and she received the Pragnesh Jay Modi Best Student Paper Award at AAMAS 2015. She has interned previously in the Division of Economic and Risk Analysis at the U.S. Securities and Exchange Commission, and she is spending summer 2015 as a Research Intern at Microsoft Research NYC. Prior to Michigan, she completed a BS in Electrical Engineering at the University of Illinois at Urbana-Champaign and an MS in Computer Science at UCLA.

Research Summary. Algorithmic trading, the use of quantitative algorithms to automate the submission of orders, is responsible for the majority of trading activity in today’s financial markets. To better understand the societal implications of such trading, I construct computational agent-based models comprised of investors and algorithmic traders. I examine two overlapping types of algorithmic traders: high-frequency traders (of recent Flash Boys fame) who exploit speed advantages for profit, and market makers who facilitate trade and supply liquidity by simultaneously maintaining offers to buy and sell. I employ simulation and empirical game-theoretic analysis to study trader behavior in equilibrium, that is, when all
traders best respond to their environment and other agents’ strategies. I focus on the impact of algorithmic trading on allocative efficiency, or overall gains from trade.

I also investigate the potential for a frequent call market, in which orders are matched to trade at discrete periodic intervals rather than continuously, to mitigate the latency advantages of high-frequency traders. Frequent call markets have been proposed as a market design solution to the latency arms race perpetuated by high-frequency traders in continuous markets, but the path to widespread adoption of these call markets is unclear. I demonstrate that switching to a frequent call market eliminates the advantage of speed and promotes efficiency, and I formulate a game of strategic market choice to characterize the market conditions under which fast and slow traders choose to trade in a frequent call market versus a continuous double auction.

Representative Papers.


MATT WEINBERG

Thesis. Algorithms for Strategic Agents

Advisor. Costis Daskalakis, MIT

Brief Biography. Matt is currently a postdoc in the Computer Science department at Princeton University, hosted by Mark Braverman. From 2010-2014 he was a PhD student with Costis Daskalakis in Computer Science at MIT. Prior to that, Matt completed his B.A. in Math at Cornell University, where he worked with Bobby Kleinberg. During his time at MIT, Matt spent a summer interning with Microsoft Research New England, mentored two high school students in AGT research through the MIT PRIMES program, and supervised an undergraduate research project (UROP). He also spent the summers from 2009-2011 doing math and crypto research with the Department of Defense and the Institute for Defense Analyses. Before college, Matt grew up in Baltimore, MD. Outside of work, he spends the majority of his time training, teaching, and competing in taekwondo.

Research Summary. My research focuses largely on Algorithmic Mechanism Design, and has also made contributions to optimal stopping theory and convex optimization. At a high level, I like to study fundamental problems in Algorithmic Mechanism Design and distill from them, to the extent possible, purely algorithmic questions. Resolving such questions then develops new tools for these fundamental problems of study in a way that also contributes to more classical areas of Theoretical Computer Science.

One example of my work in this direction culminated in my thesis, and addresses the following question: In traditional algorithm design, some input is given and some output is desired. How much (computationally) harder is it to solve the
same problem when the input is held instead by strategic agents with their own preferences over potential outputs? This broad question captures, for instance, the problem of optimal mechanism design from a computational perspective. My thesis provides a generic reduction from solving any optimization problem on strategic input to solving a perturbed version of that same problem when the input is directly given. In other words, we have shown how to answer questions in mechanism design by solving purely algorithmic problems.

My work also addresses mechanism design from other angles, such as understanding the quality of simple versus optimal auctions. Surprisingly, even in settings where the optimal auction is prohibitively complex or computationally intractable, we are able to show that very simple auctions can still perform quite well. My research in this area also develops new prophet inequalities and other online algorithms. Recently, I’ve also become interested in tackling from a mechanism design perspective more applied problems where strategic interaction is involved, such as peer grading in MOOCs.

Representative Papers.

[1] Understanding Incentives: Mechanism Design Becomes Algorithm Design (FOCS 2013) with Y. Cai and C. Daskalakis

JAMES R. WRIGHT

Thesis. Behavioral Game Theory: Predictive Models and Mechanisms

Advisor. Kevin Leyton-Brown, University of British Columbia

Brief Biography. James Wright is a Ph.D. candidate in computer science at the University of British Columbia, advised by Kevin Leyton-Brown. He holds an M.Sc. from the University of British Columbia (2010) and a B.Sc. from Simon Fraser University (2000). He studies problems at the intersection of behavioral game theory and computer science, with a focus on applying both machine learning techniques and models derived from experimental and behavioral economics to the prediction of human behavior in strategic settings. He also studies the implications of behavioral game theoretic models on multiagent systems and mechanisms. James’s expected graduation date is June 2016.

Research Summary. A wealth of experimental evidence demonstrates that human behavior in strategic situations is often poorly predicted by classical economic models. Behavioral game theory studies deviations of human behavior from the standard assumptions, and provides many models of these deviations. These models typically focus on explaining a single anomaly. Although understanding individual anomalies is valuable, the resulting models are not always well-suited to predicting how people will behave in generic settings, which limits their application to questions of interest in algorithmic game theory, such as “What is the optimal mechanism for implementing a particular objective?”.
I am interested applying machine learning techniques to construct behavioral game theoretic models that have high predictive accuracy, and in applying these models to problems in algorithmic game theory. As an example of the first direction, I previously analyzed and evaluated behavioral models in simultaneous-move games, eventually identifying a specific class of models (iterative models) as the state of the art. I then proposed and evaluated an extension that improves the prediction performance of any iterative model by better incorporating the behavior of nonstrategic agents.

Despite growing interest in behavioral game theory over the past decade, many important questions about its application to areas such as mechanism design remain open. For example, foundational analytic techniques such as the revelation principle may not be straightforwardly applicable under some classes of behavioral model. One direction of my current research aims to determine to which classes of behavioral model do such principles apply, and how to handle the cases where they don’t apply.

**Representative Papers.**

1. Beyond Equilibrium: Predicting Human Behavior in Normal-Form Games (AAAI 2010) with K. Leyton-Brown
2. Behavioral Game-Theoretic Models: A Bayesian Framework for Parameter Analysis (AAMAS 2012) with K. Leyton-Brown
3. Level-0 Meta-Models for Predicting Human Behavior in Games (EC 2014) with K. Leyton-Brown

**JIE ZHANG**

*Thesis.* Incentive Ratio and Market Equilibrium

*Advisors.* Xiaotie Deng, City University of Hong Kong

*Brief Biography.* From 2008.08 to 2011.07 Jie was a PhD student in City University of Hong Kong, advised by Xiaotie Deng. During the last six months of the PhD he was in Harvard University as a visiting student, hosted by Yiling Chen. From 2011.10 to 2014.03 he was a postdoc at Aarhus University, working under Peter Bro Miltersen. After that he moved to University of Oxford for another postdoc (research associate in the UK), working with Elias Koutsoupias.

*Research Summary.* My research mainly focuses on Algorithmic Game Theory. It analyzes strategic behaviors of rational agents, and designs efficient algorithms and mechanisms in multi-agent environments under incentive constraints, as well as equilibrium analysis.

Identifying, understanding and modeling agents’ incentives and strategic behavior in cooperative and competitive environments is considered to be one of the most important subjects in the study of Internet markets. The analysis of agents’ behavior aids mechanism designers to better achieve their objectives, such as information aggregation, revenue maximization, social welfare optimization, and so on. My objective is to design efficient algorithms and mechanisms to align the incentives of the agents with that of society, by employing algorithmic game theory methodology.

I have worked mostly, but not only, on the following topics: game theoretical analysis of market equilibrium; Prediction markets; mechanism design; fixed-point
models and complexity of computing Nash equilibrium; fair division in resource allocation.

Representative Papers.

[2] How Profitable are Strategic Behaviors in a Market? (ESA 2011) with N. Chen and X. Deng

YAIR ZICK

Thesis. Arbitration, Fairness and Stability: Revenue Division in Collaborative Settings

Advisor. Edith Elkind, University of Oxford

Brief Biography. Yair Zick is a postdoctoral research fellow in the computer science department of Carnegie Mellon University, hosted by Anupam Datta and Ariel D. Procaccia. His research interests span cooperative game theory, computational social choice and their applications to domains such as security, privacy, machine learning and education. He completed his PhD at Nanyang Technological University under the supervision of Edith Elkind where he was funded by the Singapore A*STAR scholarship. As a graduate student, he has coauthored nine publications (seven of which as a main author), all appearing in top AI conferences. His first paper, “Arbitrators in Overlapping Coalition Formation Games” received the AAMAS 2011 Pragnesh Jay Modi best student paper award; his dissertation “Arbitration, Fairness and Stability: Revenue Division in Collaborative Settings”, has received the 2014 IFAAMAS Victor Lesser distinguished dissertation award.

Research Summary. As a graduate student, I mostly worked on cooperative game theory, and coauthored several papers on cooperative games and computational social choice. My thesis mainly focused on overlapping coalition formation (OCF). In OCF games, each player possesses some divisible resource (say, time or processing power), and may contribute a fractional amount of it to joint tasks with other players. These fractional coalitions generate revenue, which must be divided among participants. An outcome (a division into coalitions plus a division of revenue) of an OCF game is stable if no subset of agents can deviate—reallocate resources and revenue such that all of its members are strictly better off. The key observation here is that when agents deviate, they may still be invested in projects involving non-deviators. For example, if an agent receives payoffs from several projects but would like to withdraw only from one of them, the profitability of deviation strongly depends on how non-deviators react.

I studied OCF games with arbitration functions. These functions describe the way non-deviators react to deviation; their lenience governs the stability of OCF games. The structure of arbitration functions has far-reaching implications on stability, as well as the computational problem of finding such outcomes.

Upon graduation, I joined Carnegie Mellon University as a postdoctoral research fellow. During my time here, I expanded my research interests to include machine learning and education.
learning, privacy, security, and fair division. I am currently involved in several exciting projects. We are exploring an interesting link between game theory and causality in machine learning environments. In addition, we apply PAC learning techniques to cooperative games and to fair allocation of indivisible goods. Our results in an ongoing rent division project have been implemented on the Spliddit.org website, with more results on the way! In a foray to the field of AI and education, we implement machine learning models in order to elicit various student metrics from course data.

I am passionate about applying game theoretic notions to other fields, and I always enjoy learning about new fields and techniques. Furthermore, I am keenly interested in empirical analysis of game theoretic solution concepts: what outcomes are considered fair by people? Do their notions of fairness coincide with our formal definitions?

Representative Papers.

## Index

**agent-based simulation**  
Elaine Wah, 32

**algorithmic trading**  
Elaine Wah, 32

**algorithms**  
Baharak Rastegari, 23  
Nima Haghpanah, 14

**artificial intelligence**  
Sam Ganzfried, 10

**auction theory**  
Mohammad Reza Khani, 16  
Nick Arnosti, 3

**autonomous energy trading**  
Daniel Urieli, 30

**Bayesian equilibrium**  
Nikolai Gravin, 13

**behavioral models**  
James R. Wright, 34

**behavioural experiments**  
Xi (Alice) Gao, 12

**big data algorithms**  
Joel Oren, 21

**cascades**  
Joel Oren, 21

**causal inference**  
Panos Toulis, 29

**combinatorial auctions**  
Baharak Rastegari, 23  
Nikolai Gravin, 13

**combinatorial optimization**  
Mohammad Reza Khani, 16

**computational judgement aggregation**  
Ilan Nehama, 18

**cost sharing**  
Angelina Vidali, 31  
Emmanouil Pountourakis, 22

**decision theory**  
Ilan Nehama, 18

**differential privacy**  
Or Sheffet, 25

**dynamic posted price schemes**  
Ilan Cohen, 7

**econometrics**  
Vasilis Syrgkanis, 26

**epidemic detection**  
Eli Meirom, 17

**equilibrium computation**  
Yun Kuen Cheung, 5

**experimental design**  
Panos Toulis, 29

**fair division**  
Aris Filos-Ratsikas, 9  
Jie Zhang, 35  
Nisarg Shah, 24

**game theory**  
Baharak Rastegari, 23  
James R. Wright, 34  
Nisarg Shah, 24  
Or Sheffet, 25  
Sam Ganzfried, 10  
Vasilis Syrgkanis, 26  
Yair Zick, 36

**imperfect information**  
Sam Ganzfried, 10

**incentives**  
Panos Toulis, 29

**information elicitation**  
Xi (Alice) Gao, 12

**interference**  
Panos Toulis, 29

**kidney exchange**  
John P. Dickerson, 8

**learning**  
Yair Zick, 36

**learning agents**  
Daniel Urieli, 30

**machine learning**  
James R. Wright, 34
market design
  Elaine Wah, 32
  Jie Zhang, 35
  Nick Arnosti, 3
  Yun Kuen Cheung, 5

matching
  Aris Filos-Ratsikas, 9
  Baharak Rastegari, 23
  Jie Zhang, 35
  John P. Dickerson, 8
  Nick Arnosti, 3

mechanism design
  Angelina Vidali, 31
  Aris Filos-Ratsikas, 9
  Baharak Rastegari, 23
  Bo Tang, 28
  Darrell Hoy, 15
  Emmanouil Pountourakis, 22
  Ilan Cohen, 7
  Jie Zhang, 35
  Matt Weinberg, 33
  Nima Haghipanah, 14
  Or Sheffet, 25
  Vasilis Syrgkanis, 26

multi-agent systems
  Nisarg Shah, 24

Nash equilibrium
  Bo Tang, 28

network formation
  Eli Meirom, 17

non-truthful auctions
  Darrell Hoy, 15

online algorithms
  Matt Weinberg, 33

online learning
  Vasilis Syrgkanis, 26

optimal auctions.
  Matt Weinberg, 33

overlapping coalition formation
  Yair Zick, 36

peer prediction
  Xi (Alice) Gao, 12

preference aggregation
  Markus Brill, 4

prior-free setting
  Mohammad Reza Khani, 16

privacy
  Yair Zick, 36

ranking
  Or Sheffet, 25

revenue management
  Angelina Vidali, 31
  Bo Tang, 28
  Emmanouil Pountourakis, 22
  Mohammad Reza Khani, 16
  Nima Haghipanah, 14

risk-aversion
  Darrell Hoy, 15

smart grid
  Daniel Urieli, 30

social choice
  Emmanouil Pountourakis, 22
  Ilan Nehama, 18
  Nisarg Shah, 24
  Svetlana Obraztsova, 19

social networks
  Eli Meirom, 17
  Joel Oren, 21

stochastic optimization
  John P. Dickerson, 8

strategyproofness
  Markus Brill, 4

tatonnement
  Yun Kuen Cheung, 5

tournament solutions
  Markus Brill, 4

voting
  Aris Filos-Ratsikas, 9
  Svetlana Obraztsova, 19
Multi-Item Auctions Defying Intuition?

CONSTANTINOS DASKALAKIS¹
Massachusetts Institute of Technology

The best way to sell \( n \) items to a buyer who values each of them independently and uniformly randomly in \([c, c+1]\) is to bundle them together, as long as \( c \) is large enough. Still, for any \( c \), the grand bundling mechanism is never optimal for large enough \( n \), despite the sharp concentration of the buyer’s total value for the items as \( n \) grows. Optimal multi-item mechanisms are rife with unintuitive properties, making multi-item generalizations of Myerson’s celebrated mechanism a daunting task. We survey recent work on the structure and computational complexity of revenue-optimal multi-item mechanisms, providing structural as well as algorithmic generalizations of Myerson’s result to multi-item settings.

Categories and Subject Descriptors: F.0 [Theory of Computation]: General
General Terms: Algorithms, Economics, Theory
Additional Key Words and Phrases: Auctions, Multidimensional Mechanism Design

1. INTRODUCTION

Optimal mechanism design is a problem with important applications and deep mathematical structure. In its basic formulation, studied in this survey, a seller has \( n \) items to sell to \( m \) interested buyers. Each buyer knows his own values for the items, but the seller and the other buyers only know a distribution from which these values are assumed to be drawn. The goal is to design a sales procedure, called a mechanism, that optimizes the expected revenue of the seller.

The basic version of the problem and its myriad extensions have familiar applications. Here are a few quick ones: When auction-houses sell items, this is the problem that they face. This is also the problem that governments face when auctioning a valuable public resource such as wireless spectrum. Finally, the problem arises every millisecond as auctions are used in sponsored search and the allocation of banner advertisements.

When it comes to selling a single item, optimal mechanism design is really well understood. Building on Myerson’s celebrated work [Myerson 1981], it has been studied intensely for decades in both Economics and Computer Science. This research has revealed surprisingly elegant structure in the optimal mechanism, as well as robustness to the details of the distributions, and has had a deep impact in the broader field of mechanism design.

While all this progress has been taking place on the single-item front, the multi-item version of the problem has remained poorly understood. Despite substantial research effort, it is not even known how to optimally sell two items to one buyer. On the contrary, multi-item auctions appear to have very rich structure, often

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exhibiting unintuitive properties.

In this survey, we present recent progress on the structure and computation of optimal multi-item auctions. Our survey will focus on the philosophy and intuition behind the results, but we will also give substantial technical detail. Our intention is not to present a complete account of results on multi-item mechanisms, but to motivate what we view as a fresh perspective on the problem.

Structure. In Section 2, we take a brief tour of the wondrous land of single-item problems, where we discuss the surprising simplicity of optimal mechanisms. Then, in Section 3, we present multi-item examples showing various ways in which the simplicity of optimal single-item mechanisms fails to generalize. A few of these examples are particularly striking as they illustrate quite unintuitive properties of optimal multi-item mechanisms. The structure of these mechanisms appears so rich that we go one step back, in Section 4, discussing approaches that may be able to accommodate this richness. Then, in Section 5, we present a duality based approach to the multi-item problem, showing how it can be used to characterize single-bidder mechanisms. We also show how this framework can be used to demystify the examples of Section 3, which all pertain to a single bidder. In Section 6, we turn to computation, presenting computationally efficient algorithms for the multi-item multi-bidder problem. As a byproduct, these algorithmic results offer a crisp characterization of the structure of optimal multi-item multi-bidder mechanisms. Finally, in Section 7, we wrap up with a short summary and future directions.

2. THE WONDRous MYERSON-LAND

Consider the task of selling an item to a single buyer with the goal of maximizing the seller’s revenue. The following is a well-known fact.

Fact 1 [Myerson 1981; Riley and Zeckhauser 1983]. The optimal way to sell one item to a buyer whose value for the item is drawn from some known distribution $F$ is a take-it-or-leave-it offer of the item at some price in $\arg \max \{x \cdot (1 - F(x))\}$.

Example 1. The optimal way to sell one item to a buyer whose value for the item is uniformly distributed in $[0, 1]$ is to price the item at 0.5. The expected revenue is 0.25.

While it is perhaps intuitive that this should be true, it still surprising that, among all possible communication protocols that the seller and buyer could engage in, the optimal one would require minimal communication and involve no randomization at all.

In this light, it is even more surprising that the optimal auction would maintain its simplicity when multiple buyers are involved.

Fact 2 [Myerson 1981]. When one item is sold to $m$ buyers whose values for the item are i.i.d. from a known, regular distribution $F$, the optimal mechanism is a second price auction with reservation price $\arg \max \{x \cdot (1 - F(x))\}$.

$^2$A distribution $F$ with density $f$ is called regular iff $x - \frac{1 - F(x)}{f(x)}$ is monotone increasing.
There are several reasons why this fact is surprising, besides the simplicity of the optimal mechanism:

(1) The mechanism is deterministic, namely the auctioneer does not need access to a random generator to implement the allocation and pricing.

(2) The mechanism requires only one round of communication, namely the bidders submit bids to the auctioneer who then decides the outcome without any further exchanges with them, except to announce the outcome.

(3) The mechanism is dominant strategy truthful (DST), but it is optimal among the larger class of Bayesian Incentive Compatible (BIC) mechanisms.3

In fact, these properties carry over to settings where bidder values are not necessarily i.i.d. and their distributions are not necessarily regular.

**Fact 3 [Myerson 1981].** When one item is sold to m buyers whose values for the item are independently drawn from known distributions, the optimal mechanism is a virtual welfare maximizer, namely:

(1) **Bidders are asked to report bids for the item:** b₁,...,bₘ.

(2) **Bids are transformed into what are called “ironed virtual bids,”** h₁(b₁),...,hₘ(bₘ), where each hᵢ(·) depends on the corresponding bidder’s value distribution (but not on the other bidders’ distributions and not even m).4

(3) **The item is allocated to the bidder with the highest ironed virtual bid, with some lexicographic tie-breaking.**

(4) **The winner of the item is charged his threshold bid, namely the smallest bid he could place and still win the item.**

Facts 1-3 are both surprising and powerful, providing simple, yet sharp and versatile machinery for revenue optimization in single-item and, more broadly, single-dimensional environments.5 Importantly, they have provided solid foundation for a tremendous literature that has brought tools from approximation algorithms and probability theory into mechanism design. Building on the shoulders of Myerson, this literature strives to understand how to make the theory robust, further improving the simplicity of mechanisms and reducing their dependence on the details of the bidders’ distributions, both at a quantifiable loss in revenue. See, e.g., [Hartline 2013; Chawla and Sivan 2014; Roughgarden 2015] for recent surveys of this work.

---

3A mechanism is called Dominant Strategy Truthful iff it is in the best interest of every bidder to truthfully report their value to the mechanism, regardless of what the other bidders report. Bayesian Incentive Compatible mechanisms are a broader class, but we postpone their definition, as it is slightly technical and not important for our discussion right now. See Definition 6.

4The precise functional form of the hᵢ’s is not important for this survey.

5In a single-dimensional environment, the seller can provide service to several buyers, subject to constraints on which buyers can receive service simultaneously. Each buyer has a value for receiving service, which is distributed according to some distribution known to the seller and the other buyers. Single-dimensional environments clearly generalize single-item environments, where only one buyer can be served the item.
3. AUCTIONS DEFYING INTUITION

Despite remarkable progress on the single-item front over the past few decades, revenue optimization in multi-item settings has remained poorly understood. We do not even have a sharp characterization of optimal two-item mechanisms, even when there is a single buyer. On the contrary, multi-item mechanisms exhibit such rich structure that it is difficult to imagine what a generalization of Myerson’s results could look like. Or, better said, the generalizations that we can imagine can be shot down via simple examples.

To illustrate the richness of multi-item mechanisms, let us consider some simple multi-item settings and their corresponding optimal mechanisms. All our examples will involve a seller with \( n \) items and a single additive buyer. Such a buyer is characterized by a private vector \((v_1, \ldots, v_n)\) of values for the items and derives utility \( \sum_{i \in S} v_i - p \), whenever he pays \( p \) to get the items in set \( S \subseteq [n] \). If \( S \) and \( p \) are random his utility is \( \mathbb{E}[\sum_{i \in S} v_i - p] \).

### 3.1 Bundling

When it comes to multiple items, a natural question to ask is whether we can use Myerson’s technology to design optimal mechanisms, and indeed what exactly it is that we should be selling. The following simple example illustrates that we may need to bundle items, even when there is a priori no interaction between them.

**Example 2.** Suppose \( n = 2 \) and the buyer’s values are i.i.d., uniform in \([1, 2]\).

Since the buyer is additive, and his values for the items are independent, getting one of the two items will not affect his marginal value for getting the other item as well. Since there is no interaction between the item values, it is natural to expect that the optimal mechanism should sell the two items separately. By Fact 1, this would mean pricing each item at 1 and letting the buyer decide which of them to buy. Simple calculations show that the expected revenue of this mechanism is 2.

Interestingly, there is a flaw in this logic. While it is true that item-values do not “interact with each other,” we may still want to capitalize on the fact that the buyer’s average item-value is better concentrated than his value for a specific item. Indeed, it is better to only offer the bundle \([1, 2]\) of both the items at price 3. Given that \( \Pr[\sum_{i} v_i \geq 3] = 3/4 \), the expected revenue of the seller is now \( 9/4 > 2 \). It can be shown that this is the optimal mechanism [Daskalakis et al. 2014].

Example 2 illustrates the following.

**Fact 4.** Optimal multi-item mechanisms may require bundling, even when there is a single additive buyer with independent values for the items.

Our intuition for the effectiveness of bundling in Example 2 appealed to the concentration of the buyer’s surplus (i.e. total value for both items). There was still a flaw in our logic, however, and this time a less tangible one: Why is the surplus the right benchmark to compare against? The following result, discussed in more detail in Section 5.8, illustrates a setting where the structure of the optimal mechanism is different in two asymptotic regimes.

**Theorem 1 [Daskalakis et al. 2015].** Consider selling \( n \) items to a buyer whose values for the items are i.i.d. uniform in \([c, c+1]\). The following are true:
(1) For all $n$, there exists $c_0$ such that, for all $c > c_0$, the optimal mechanism only offers the bundle of all the items together at some price.

(2) For all $c$, there exists $n_0$ such that, for all $n > n_0$, the optimal mechanism does not only offer the bundle of all items together at some price.

Part 2 of the above theorem is especially counter-intuitive. As $n \to \infty$, the buyer’s average value for the items, $\sum \frac{v_i}{n}$, becomes more and more concentrated around its mean, $c + 0.5$. It is clear that the seller cannot hope to extract higher revenue than the buyer’s total expected value, $n(c + 0.5)$, and offering the grand bundle for $n(c + 0.5 - \epsilon)$, for the tiniest discount $\epsilon > 0$, would make the buyer accept to purchase it with probability arbitrarily close to 1 as $n \to \infty$. Still, for no $n$ does this intuition materialize, and it never becomes optimal to only sell the grand bundle.

3.2 Randomization

Recall that optimal single-item mechanisms do not require randomization. Our next example illustrates that this is not the case in multi-item settings.

**Example 3.** Suppose $n = 2$, and $v_1$ is distributed uniformly in $\{1, 2\}$ while $v_2$ is independently distributed uniformly in $\{1, 3\}$. In this example, an optimal deterministic mechanism prices item 1 at 1 and item 2 at 3. Its expected revenue is 2.5.

Still, we can do better. We can offer the buyer two options: The first is to pay 4 and get both items. The second is to pay 2.5 to get a “lottery ticket” that allocates item 1 with probability 1 and item 2 with probability 1/2. The expected revenue of this mechanism is 2.625, which can be shown to be optimal [Daskalakis et al. 2014].

So randomization is necessary for revenue maximization. It turns out its effect on the revenue may actually be quite dramatic.

**Fact 5.** Optimal multi-item mechanisms may require randomization. The gap between the revenue of the optimal randomized and the optimal deterministic mechanism can be arbitrary large, even when there are two items and a single buyer [Briest et al. 2010; Hart and Nisan 2013].

3.3 Menu Size Complexity

In our previous examples, we described the optimal mechanism as a menu of options for the buyer to choose from. If the optimal mechanism were guaranteed to be deterministic, describing it as a menu would require a bounded number of options, as there is a finite number of possible bundles that the mechanism may offer. Given Fact 5, however, it becomes unclear how to specify the optimal mechanism. The following example illustrates that representing it as an explicit menu may be infeasible.

**Example 4** [Daskalakis et al. 2013]. Suppose $n = 2$, and $v_1$ is distributed according to the Beta distribution with parameters $(3, 3)$ while $v_2$ is independently distributed according to the Beta distribution with parameters $(3, 4)$. Then the

---

6The Beta distribution with parameters $(\alpha, \beta)$ is distributed in $[0, 1]$ according to the density function $f(x) \propto x^{\alpha-1}(1-x)^{\beta-1}$.
optimal mechanism needs to offer uncountably many lotteries.

The alarming feature of Example 4 is that it becomes unclear whether the allocation and price rule of the optimal mechanism can be effectively described via a small number of parameters, even if the buyer’s distribution can be.\(^7\) In Section 5.6.2, we will show that the optimal mechanism for Example 4 can actually be described via a small number of parameters. However, this may not be true in general.

3.4 Non-Monotonicity

It seems intuitive that a seller with more valuable items should expect a higher revenue from selling them. One way to quantify this intuition is the following.

Consider two distributions \(F\) and \(G\) such that \(F\) first-order stochastically dominates \(G\), denoted \(F \succeq_1 G\). This means that, for all \(x \in \mathbb{R}\), \(F(x) \leq G(x)\), i.e. \(F\) and \(G\) can be coupled so that \(F\) always samples a value larger than the value sampled by \(G\). It easily follows from Fact 1 that a seller selling an item to some buyer whose value for the item is distributed according to \(F\) makes higher revenue than if the buyer’s value were distributed according to \(G\).

Surprisingly this fails to hold in multi-item settings!

**Fact 6 [Hart and Reny 2012].** There exist distributions \(F\) and \(G\) such that \(F \succeq_1 G\) but the optimal revenue from selling two items to a buyer whose values are i.i.d. from \(F\) is smaller than if they were i.i.d. from \(G\).

4. CROSSROADS

It is clear from our examples in the previous section that, in the close neighborhood of Myerson’s setting, optimal mechanisms exhibit rich structure and may defy our intuition. It is not even clear if they have a finite effective representation. In view of these complications, there are several directions we may want to pursue:

1. Forget about trying to understand optimal mechanisms and pursue approximations directly. Even though optimal mechanisms are complex, there may still be simple mechanisms that can be shown to guarantee some good fraction of the optimal revenue.
2. Forget about characterizing the structure of optimal mechanisms and study instead whether they can be computed efficiently.
3. Develop new machinery to characterize the structure of optimal mechanisms. Despite their apparent complexity and fragility, there may still be a different lens through which they exhibit more structure.

It is a priori dubious whether the approximation approach can lead anywhere. Without understanding the optimal mechanism, how can we possibly establish the approximate optimality of some other mechanism? It is quite surprising then that this approach has actually been quite fruitful:

\(^7\)Of course, in a somewhat perverse way, the buyer’s distribution itself “indexes” the optimal mechanism for that distribution. But this does not count as it is unclear if there is an efficient procedure that can implement the mechanism given this description. We will touch upon this point a bit later, when we discuss the computation of optimal mechanisms in Section 6.
In multi-item settings with additive buyers, [Hart and Nisan 2012; Li and Yao 2013; Babaioff et al. 2014; Yao 2015; Rubinstein and Weinberg 2015] design approximate mechanisms by carefully decomposing the support of the buyers’ distributions into regions and, due to lack of a better benchmark, using expected welfare as an obvious upper bound to the optimal revenue, competing against this stronger benchmark in some of the regions. Here is a very interesting and clean result that they obtain for the setting of the previous section.

Theorem 2 [Babaioff et al. 2014]. When $n$ items are sold to a buyer whose values for the items are independent, a mechanism that either only prices individual items or only prices the bundle of all the items obtains at least $1/6$-th of the optimal revenue.

In multi-item settings with unit-demand buyers, [Chawla et al. 2007; Chawla et al. 2010] upper bound the optimal revenue by the optimal revenue in a related single-dimensional setting, and define sequential posted price mechanisms for the multi-item setting competing against this stronger upper bound.

These approximation results provide simple ways through which a constant fraction of the optimal revenue can be attained, bypassing the difficulties coming from our lack of understanding of optimal multi-item mechanisms. Moreover, they help us understand the tradeoffs between simplicity, optimality and generality of mechanisms. For example, Theorem 2 tells us that, if we are willing to sacrifice generality in the model and $5/6$-ths of the revenue, a very simple mechanism will work for us. Still such approximation results only apply to restricted settings, e.g. they typically assume independence of the distributions across items, and it is unclear how to extend them to broader settings: correlation among items, more complex buyer valuations, and more complex allocation constraints.

Ultimately, it is our belief that a cohesive theory of optimal multi-item mechanisms cannot be obtained through disparate approximation results applying to different multi-item environments. And even where these approximation results do apply they may still not provide fine-tuned insight into the salient features of the setting responsible for revenue. As a simple example, the non-monotonicity of revenue (Fact 6) is not foreseeable from Theorem 2 alone. On the contrary, the theorem hints towards the incorrect conclusion.

In this light, over the past several years we have pursued the challenge of characterizing optimal multi-item mechanisms from both an algorithmic and a structural perspective. In the next two sections, we give a flavor of our progress on these fronts. In both sections, we discuss our philosophy as well as give an overview of results and techniques. We note that our goal is not the coverage of all results in the literature, but mostly the philosophy behind them. So we will focus on our philosophy and a biased sample of primarily our own results.

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8These are buyers who only want to purchase one item.
5. THE STRUCTURE OF MULTI-ITEM MECHANISMS

5.1 Philosophy

In Sections 3.1–3.4 we gave several multi-item examples, making claims about their optimal mechanisms. Taking a step back, how did we go about proving these claims? More broadly, how is the optimality of some object, such as a mechanism, established? Two principled approaches for doing this are the following. The first entails formulating the problem at hand as a convex minimization/concave maximization problem, and then showing that the object whose optimality we want to establish satisfies first-order conditions. The second is to develop a dual formulation and identify a solution to the dual that matches the value attained by the object whose optimality is to be established in the primal.

In single-dimensional environments, Facts 1-3 can be shown using the first approach. Through a chain of deductions, [Myerson 1981] expresses the expected revenue of a mechanism as an expected (virtual) welfare quantity, which can be optimized in a point-wise manner.

In multi-item environments, it is not clear how to generalize Myerson’s approach, except under significant restrictions on the value distributions; see [Rochet and Stole 2003; Manelli and Vincent 2007]. This motivates the second approach, which we have pursued in the context of a single, additive buyer in [Daskalakis et al. 2013; 2015]. We give a flavor of our approach with examples in the following sections.

5.2 Setting

In this section, we restrict our attention to a seller with $n$ items and an additive buyer whose values for these items are jointly distributed according to some distribution $F$. For simplicity, we assume that $F$ is supported on some box $X = \prod_i [x_i, x_i^+] \subseteq \mathbb{R}_+^n$ and is continuously differentiable with bounded partial derivatives. We call $X$ the typeset of the buyer, and its elements the buyer’s possible types. If a buyer of type $x$ is allocated a subset $S$ of the items and pays price $t$ for it, his utility is $\sum_{i \in S} x_i - t$. Our buyers are also risk-neutral, so if $S$ and $t$ are random, then their utility is $\mathbb{E}_{S,t} [\sum_{i \in S} x_i - t]$.

The seller only knows $F$, but the buyer knows his realized type, and the seller’s goal is to design a mechanism to optimize her expected revenue. By the revelation principle the optimal mechanism can be described in the form of a menu whose entries are lotteries. Each lottery specifies a vector of probabilities $p_1, \ldots, p_n$ and a price $t$. If purchased, it will allocate each item $i$ independently with probability $p_i$. Given a menu of lotteries, the buyer will choose the lottery that optimizes his utility, given his type. If all lotteries give him negative utility, then he will not purchase any of them.

5.3 Convex Optimization Formulation

To develop our duality framework we start with expressing our mechanism design problem as a convex optimization problem. Here we have a few options.

(1) Represent the mechanism as a menu of lotteries for the buyer to choose from. There are a few reasons why we dislike this approach. First, the menu of the optimal mechanism may be uncountable in size, as Example 4 illustrates. So we would have to represent it as a continuous set. Second, given some representation
of the menu, it is cumbersome to express the expected revenue resulting from this menu, as such an expression would have to incorporate the buyer’s optimization over lotteries in the menu. Finally, each lottery in the menu is multi-parametric, comprising \( n \) allocation probabilities as well as a price.

(2) Represent the mechanism as a menu, but also keep track of which lottery in the menu each possible type of buyer will purchase. In this case, a mechanism can be represented as a pair of functions: (i) the allocation function \( \mathcal{P} : X \rightarrow [0, 1]^n \) specifying the allocation probabilities of the lottery that each type will purchase, if any; and (ii) the price function \( \mathcal{T} : X \rightarrow \mathbb{R} \) specifying the price that each type will pay for the purchased lottery, if any.

Now, the representation of the mechanism is simpler, and we can easily express its expected revenue as follows:

\[
\int_X \mathcal{T}(x) dF(x).
\]

Of course, we need to add some consistency constraints to make sure that our modeling is faithful:

\[
\forall x, x' \in X : x \cdot \mathcal{P}(x) - \mathcal{T}(x) \geq x \cdot \mathcal{P}(x') - \mathcal{T}(x');
\]

\[
\forall x \in X : x \cdot \mathcal{P}(x) - \mathcal{T}(x) \geq 0.
\]

Constraint (2) expresses that no type prefers a different lottery to the one we maintain for this type, while Constraint (3) expresses that each type will actually buy this lottery.

Finding the pair of functions \((\mathcal{P}, \mathcal{T})\) optimizing (1) subject to the constraints (2) and (3) was the approach taken by [Myerson 1981] and is quite standard. In the multi-item setting, however, this representation is still cumbersome to work with as, besides having an \( n \)-variate input, the compound function \((\mathcal{P}, \mathcal{T})\) also has an \((n + 1)\)-dimensional output.

(3) Given the complexity of the standard representation, we decide to optimize over mechanisms indirectly. Rather than optimizing revenue in terms of the mechanism’s allocation and price functions, we want to explore whether we can optimize revenue in terms of the buyer’s utility from participating in the mechanism.

Indeed, facing some mechanism, a buyer of type \( x \) will decide to buy some lottery \((p_x, t_x)\), thereby enjoying utility \( p_x \cdot x - t_x \) from his decision.\(^9\) So, every mechanism induces a function \( u : X \rightarrow \mathbb{R} \), where \( u(x) \) expresses the utility of the buyer when his realized type is \( x \) and he buys his favorite lottery in the mechanism, if any. Our goal is to optimize over mechanisms indirectly by optimizing over \( u \)’s, which raises two questions:

(1) Given a function \( u : X \rightarrow \mathbb{R} \), can we recognize whether there is a mechanism inducing this function \( u \) as the utility of the buyer?

(2) If \( u \) is induced by a mechanism, is there enough information in \( u \) to uniquely specify the expected revenue of whatever mechanism induces \( u \), and can we get our hands on a mechanism that induces \( u \)?

\(^9\)Let us assume that the lottery \((0, 0)\) is always in our menu to account for the possibility that the buyer may derive negative utility from all lotteries in the menu and hence decide to buy none.
A priori it is unclear whether \( u \) is informative enough about the mechanism(s) that induce(s) it. In fact, it appears that it is losing information about these mechanisms. Nevertheless, the answer to each of the two questions above is actually “yes,” due to the following theorem by Rochet.

**Theorem 3 [Rochet 1987].** Function \( u : X \to \mathbb{R} \) is induced by a mechanism iff \( u \) is 1-Lipschitz continuous with respect to the \( \ell_1 \)-norm, non-decreasing, convex and non-negative. Moreover, if these conditions are met then \( \nabla u(x) \) exists almost everywhere in \( X \), and wherever it exists:

- \( \nabla u(x) \) are the allocation probabilities of the lottery purchased by type \( x \), and
- \( \nabla u(x) \cdot x - u(x) \) is the price of the lottery purchased by type \( x \) in any mechanism inducing \( u \).

The theorem follows by combining Constraints (2) and (3). For a concise derivation of the theorem, please refer to Lecture 21 of [Daskalakis 2015]. In order to ground it to our experience, let us plot the utility induced by the optimal mechanism in Example 1. The utility is shown in Figure 1. We can verify that it satisfies the conditions of the theorem, and its derivatives contain information about the allocation function of the mechanism.

![Figure 1](image-url)  
Fig. 1. The utility induced by a take-it-or-leave it offer of an item at 0.5.

With Theorem 3, we can formulate our mechanism design problem as follows:

\[
\sup \int_X (\nabla u(x) \cdot x - u(x))dF(x) \\
s.t. \ |u(x) - u(y)| \leq |x - y|_1, \forall x, y \in X \\
u : \text{non-decreasing} \\
u : \text{convex} \\
u(x) \geq 0, \forall x \in X.
\]

Note that our formulation aims at optimizing the expected price paid by the buyer, which is given by \( \nabla u(x) \cdot x - u(x) \) when his type is \( x \), subject to the constraints...
on the function $u$. Since $\nabla u(x)$ may only be undefined at a measure zero subset of $X$ (given that $F$ has no atoms), we can take “$\nabla u(x)$” to mean anything when it is undefined, and this will not affect our revenue. For concreteness, from now on, whenever we write $\nabla u(x)$, we will mean any subgradient of $u$ at $x$ (which will exist as $u$ is convex).

The advantage of our new formulation is that the variable $u$ is a scalar function of the buyer’s type. The downside is that the objective function is cumbersome and the constraints, especially convexity, are difficult to handle. We can eliminate the first issue with a little massaging of the objective, using the divergence theorem. Our objective can equivalently be written as follows, where $f$ denotes the density of $F$ and $\hat{\eta}(x)$ denotes the outer unit normal vector at point $x \in \partial X$ of the boundary of $X$—see Lecture 21 of [Daskalakis 2015] for a concise derivation.

\[
\int_X (\nabla u(x) \cdot x - u(x))dF(x) \equiv 
\int_{\partial X} u(x)f(x)(x \cdot \hat{\eta}(x))dx - \int_X u(x)(\nabla f(x) \cdot x + (n+1)f(x))dx. 
\]  
(4)

Our massaged objective is linear in our variable $u$. Indeed, it can be viewed as the “expectation” of $u$ with respect to the signed measure $\mu$ with the following density:

\[
f(x)(x \cdot \hat{\eta}(x))1_{x \in \partial X} - (\nabla f(x) \cdot x + (n+1)f(x)).
\]

Equipped with the above definition, we can express our mechanism design problem as the following convex optimization problem.

\[
(P): \sup \int_X u(x)d\mu(x) \\
\text{s.t.} \quad |u(x) - u(y)| \leq |x - y|, \forall x, y \in X \quad \text{(6)} \\
u : \text{ non-decreasing} \quad \text{(7)} \\
u : \text{ convex} \quad \text{(8)} \\
u(x) \geq 0, \forall x \in X. \quad \text{(9)}
\]

Balancing $\mu$. For $u(x) = 1$, integral 4 becomes $-1$. Hence, so does integral 5, and therefore $\int_X du = -1$. So $\mu(X) = -1$. It is convenient to have $\mu$ balanced, namely satisfy $\mu(X) = 0$. So we add an atom of +1 at point $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ to make this happen. Accordingly, from now on, $\mu$ is in fact the measure with the following density:

\[
1_{x = \bar{x}} + f(x)(x \cdot \hat{\eta}(x))1_{x \in \partial X} - (\nabla f(x) \cdot x + (n+1)f(x)).
\]

To be formal here, we should not define $\mu$ through its density. It is more accurate to define $\mu$ using Riesz’s representation theorem, as the unique Radon measure such that the bounded linear functional (5) of bounded continuous functions $u$ equals $\int_X ud\mu$. Accordingly, all measures in our duality framework will be Radon measures. Having said that, we invite the reader to forget about Radon measures and Riesz’s theorem for the remainder of this survey. Formally, whenever we talk about measures we mean Radon measures, and $\mu$ is as defined by Riesz’s representation theorem.
5.4 Interpretation of Convex Formulation

Figure 2 shows $\mu$ for the single-item setting of Example 1, while Figure 3 shows $\mu$ for the setting of Example 4. As measure $\mu$ is now two-dimensional we have color-coded it. The dashed curve separates $X_+ \setminus \{\vec{0}\}$ from $X_-$, where $X_+$ and $X_-$ are subsets of $X$ where $\mu$ is positive and negative respectively. Notice the atom of +1 measure at point $\vec{0}$ in both figures.

Formulation $(P)$ is asking us to maximize $\int_X u d\mu$. So we would like to make $u$ as large as possible where $\mu$ is positive and make it as small as possible where $\mu$ is negative. However, Constraints 6–9 impose interesting tradeoffs on how we should implement this optimally, as $u$ should be continuous, not too steep, and convex-nondecreasing.

For instance, the optimal $u$ against the measure of Figure 2 turns out to be the one shown in Figure 1. How do we know this? It is not completely obvious, but we know it should be true from Fact 1/Example 1.

In the multi-item setting, what we need is new machinery that will allow us to identify optimal such tradeoffs. As we have already discussed, however, we know of no direct way to find these tradeoffs by studying formulation $(P)$ directly. Our approach is instead to develop a dual optimization problem that will hopefully provide insight about the optimal solution to $(P)$.

5.5 Duality: Building Intuition

It is a priori not clear whether $(P)$ has a strong dual problem, i.e. a minimization problem whose optimal value matches that of the optimal value of $(P)$. The reason is that it is an infinite-dimensional program, and we are not aware of infinite-dimensional programming tools that can handle our constraints [Luenberger 1968; Anderson and Nash 1987].

We will show that a strong dual actually does exist in Section 5.7. Prior to that let us build some intuition though, grounding it to our experience in the finite-dimensional world. In this section, we will make an analogy to mim-cost perfect matching duality that will lead us to formulate a weak dual of $(P)$, upper-bounding but not necessarily matching the value of $(P)$. But first let us...
massage \((P)\) to the following equivalent formulation:

\[
(P) : \quad \sup_X \int_X u(x) d\mu(x) \\
\text{s.t.} \quad u(x) - u(y) \leq |x - y|, \forall x, y \in X \\
u : \text{ convex} \\
u(0) = 0
\]

where we denote by \(|x - y| = \sum_i \max(0, x_i - y_i)\). Notice that Constraint (11) already implies that \(u\) must be non-decreasing as well as 1-Lipschitz with respect to the \(\ell_1\)-norm. Combined with (13), it also implies non-negativity. So, if anything, our new constraints might have restricted the set of functions that we are optimizing over. But, it is also easy to see that Constraints (6) and (7) imply Constraint (11). Moreover, Constraint (13) could have been added to the original formulation without changing the optimal value, given that \(\mu(X) = 0\). So the two formulations are actually equivalent.

Next, let us write \(\mu\) as the difference \(\mu_+ - \mu_-\) of two non-negative measures \(\mu_+\) and \(\mu_-\), so that

\[
\int_X u d\mu = \int_X u d\mu_+ - \int_X u d\mu_-
\]

for all measurable \(u\). Given that \(\mu(X) = 0\), it follows that \(\mu_+(X) = \mu_-(X)\). Moreover, let \(X_+\) and \(X_-\) be a partition of \(X\) so that \(\mu_+\) is supported on \(X_+\) and \(\mu_-\) is supported on \(X_-\). With this notation, let us consider the following relaxation of \((P)\):

\[
(P') : \quad \sup_X \int_X u d\mu_+ - \int_X u d\mu_- \\
\text{s.t.} \quad u(x) - u(y) \leq |x - y|, \forall x \in X_+, y \in X_- \\
u(0) = 0
\]

where we have dropped the convexity constraint, and only maintain Constraint 11 for \(x \in X_+\) and \(y \in X_-\). It is clear that any feasible solution to \((P)\) is also a feasible solution to \((P')\) and, therefore, the optimum of \((P')\) upper bounds the optimum of \((P)\).

We are thus ready to employ our finite-dimensional intuition. Suppose temporarily that sets \(X_+\) and \(X_-\) were finite, and measures \(\mu_+, \mu_-\) were uniform over \(X_+\) and \(X_-\) respectively. In this case \((P')\) becomes a problem of assigning potential values to the nodes of the complete bipartite graph \((X_+, X_-, X_+ \times X_-)\) with the goal of optimizing the total potential (gaining the potential of nodes in \(X_+\) and losing the potential of nodes in \(X_-\)) subject to the constraint that for every \(x \in X_+\) and \(y \in X_-\) the difference in potential between \(x\) and \(y\) cannot exceed \(|x - y|\). It is well-known that the dual of this problem is a min-cost perfect matching problem on the same graph, where the weights are as in Figure 4.
Going back to the infinite-dimensional problem, a perfect matching should correspond to a coupling of the measures $\mu_+$ and $\mu_-$, defined formally as follows.

**Definition 1.** A coupling between two non-negative measures $\mu_1$ and $\mu_2$ defined over some $S \subseteq X$ and satisfying $\mu_1(S) = \mu_2(S)$ is a non-negative measure $\gamma$ over $S \times S$ such that, for all measurable subsets $S' \subseteq S$, it holds that:

$$
\int_{S' \times S} d\gamma = \mu_1(S')\mu_2(S) \quad \text{and} \quad \int_{S \times S'} d\gamma = \mu_1(S)\mu_2(S').
$$

We denote by $\Gamma(\mu_1, \mu_2)$ the set of all couplings between $\mu_1$ and $\mu_2$.

Given the above definition and our finite-dimensional intuition, it makes sense to propose the following as a dual to $(P')$:

$$
(D') : \inf \int_{X \times X} |(x - y)_+|d\gamma(x, y)
\text{ s.t. } \gamma \in \Gamma(\mu_+, \mu_-)
$$

It is indeed quite straightforward to establish that the optimum of $(D')$ upper-bounds the optimum of $(P')$.

**Lemma 1 [Daskalakis et al. 2013].** $(D')$ is a weak dual of $(P')$.

**Proof (Lemma 1):** For any feasible solution $u$ of $(P')$ and $\gamma$ of $(D')$ we have the following:

$$
\int_X ud\mu = \int_X ud(\mu_+ - \mu_-) = \int_{X \times X} (u(x) - u(y))d\gamma(x, y)
\leq \int_{X \times X} |(x - y)_+|d\gamma(x, y),
$$

Fig. 4. Min-Cost Perfect Matching.
where the second equality follows from the feasibility of $\gamma$ in $(D')$ and the inequality follows from the feasibility of $u$ in $(P')$. □

5.6 Applications of Weak Duality

So far, we have formulated revenue optimization as a convex optimization problem $(P)$, defined a relaxation $(P')$ of $(P)$ and identified a weak dual $(D')$ of $(P')$ and therefore of $(P)$. Namely, here is where we stand:

$$\text{OPT}(P) \leq \text{OPT}(P') \leq \text{OPT}(D').$$

Given that $(D')$ is only a weak dual of $(P)$, it is unclear how to use it to certify optimality of mechanisms. Indeed, it could be that $u$ is an optimal solution to $(P)$, but there is no solution $\gamma$ of $(D')$ that matches it in value of the objective. So what is the point of $(D')$?

The mismatch between the values of $(P)$ and $(D')$ motivates the development of a strong dual of $(P)$ in the next section. Nevertheless, our experience shows that $(D')$ often gives a simple heuristic that can be used to certify optimality of mechanisms. In the remainder of this section, we illustrate how $(D')$ can be applied for this purpose.

Suppose that $u$ is the utility function of some mechanism that we want to show is optimal for some distribution $F$. So we want to show that $u$ is an optimal solution to $(P)$, where $\mu$ is the signed measure derived from $F$ according to (10). Even though $(D')$ is only a weak dual of $(P)$ we could still be optimistic, trying to find a feasible solution $\gamma$ of $(D')$ that achieves the same objective value as that achieved by $u$ in $(P)$.

What does $\gamma$ need to satisfy for this to happen? Inspecting the proof of Lemma 1 (which also applies to feasible solutions of $(P)$ and $(D')$) we get the following sufficient condition:

$$\gamma(x, y)\text{-almost everywhere: } u(x) - u(y) = |x - y|_+. \quad 11$$

This condition would force the only inequality in the proof to be tight. Given that the gradient of $u$, wherever defined, corresponds to allocation probabilities, the following condition is also sufficient:

$$\gamma(x, y)\text{-almost everywhere:}$$

\[
\begin{array}{ll}
\nabla u \text{ exists on all points of the segment } (x, y) \quad \land \quad \forall i : \\
& x_i > y_i \implies \text{receive item } i \text{ with probability 1} \\
& x_i < y_i \implies \text{receive item } i \text{ with probability 0}
\end{array}
\]

Equipped with sufficient condition (16), we exhibit how it can be used to show optimality of mechanisms. We start with a simple example in Section 5.6.1, pro-

\[11\text{What this means is that the set of points } (x, y) \in X \times X \text{ where the equality fails to hold has measure 0 under } \gamma.\]
ceeding to a more complex example in Section 5.6.2. It will be instructive to think of our couplings $\gamma$ as routing infinitesimal amounts of flow between pairs of points in $X$, and Condition (16) as restricting the directions of these flows.

5.6.1 Certifying Optimality of Mechanisms Using Duality

**Example 5.** Suppose 2 items are sold to a buyer whose values for the items are i.i.d., uniform $[0,1]$.

**Claim 1 [Manelli and Vincent 2006].** The optimal mechanism in Example 5 is to price each individual item at $2/3$, and the bundle of both items at $4-\sqrt{2}/3$.

**Proof (Claim 1):** First, let us compute the measure $\mu$ induced by the uniform distribution over $X = [0,1]^2$ according to (10). The resulting measure comprises a total of $-3$ surface measure distributed uniformly over $[0,1]^2$, a total of $+2$ single-dimensional measure distributed uniformly over the top and the right edge of $[0,1]^2$, and an atom of $+1$ at the origin.

In Figure 5, we partition $[0,1]^2$ into four regions $R_0, R_1, R_2, R_3$ corresponding to the subsets of types that will purchase each lottery in the menu, or no lottery at all. (The tie-breaking at the boundaries between regions is unimportant as it corresponds to a measure 0 set of types.) We also depict $\mu$ and write down the utility function restricted to each region.

**Fig. 5.** Measure $\mu$ for two i.i.d. uniform $[0,1]$ items, and the partition of the typeset induced by the mechanism that prices each individual item at $2/3$ and the bundle of both items at $4-\sqrt{2}/3$. We also depict the utility of the buyer depending on which region his type falls into.
In particular, region $R_0$ corresponds to the types that do not want to purchase anything. In this region, there is $-1$ total surface measure uniformly distributed, as well as an atom of $+1$ total measure at the origin. Region $R_1$ corresponds to the types that will purchase bundle $\{1, 2\}$ at price $\frac{4-\sqrt{2}}{3}$. There is a total of $-\frac{2+\sqrt{2}}{3}$ surface measure uniformly distributed here, as well as a total of $+\frac{2+\sqrt{2}}{3}$ single-dimensional measure spread uniformly on the top and right edges of this region. Finally, regions $R_2$ and $R_3$ correspond to types that will purchase items 1 and 2 respectively at price $\frac{2}{3}$. Each of these have a total of $-\frac{2-\sqrt{2}}{3}$ surface measure uniformly distributed, as well as a $+\frac{2-\sqrt{2}}{3}$ single-dimensional measure uniformly distributed on their right and, respectively, top edges.

Our goal is to find a coupling $\gamma$ between $\mu_+$ and $\mu_-$ satisfying Condition (16). Given that our sufficient condition is sensitive about the existence of $\nabla u$, we should consider couplings of $\mu_+$ and $\mu_-$ in each region separately. This is conceivable as $\mu$ is balanced in each region, namely $\mu_+(R_i) = \mu_-(R_i)$, for all $i = 0, 1, 2, 3$. In fact, our path is cut out for us given the functional form of $u$ in each region and the form of Condition (16). In particular, we are seeking a coupling between $\mu_+$ and $\mu_-$ so that

—In region $R_0$ we are only allowed to transport measure in north-east directions, as both items are allocated with probability 0.
—In region $R_1$ we are only allowed to transport measure in south-west directions, as both items are allocated with probability 1.
—In region $R_2$ we are only allowed to transport measure in north-west directions, as item 1 is allocated with probability 1 and item 2 is allocated with probability 0.
—In region $R_3$ we are only allowed to transport measure in south-east directions, as item 2 is allocated with probability 1 and item 1 is allocated with probability 0.

In fact, we will be more optimistic, restricting our transports to be westward and southward in regions $R_2$ and $R_3$ respectively. All in all, we are seeking a coupling of $\mu_+$ and $\mu_-$ that pushes measure in the directions shown in Figure 6 in each region.

Now it is easy to verify that the way our regions and measure $\mu$ are set up, it is possible to couple $\mu_+$ and $\mu_-$ where all transports take place according to the figure. So Condition (16) is satisfied, and the resulting coupling certifies the optimality of $u$. □

We refer the reader to [Giannakopoulos and Koutsoupias 2014] for an application of the afore-described approach to $n = 3, \ldots, 6$ i.i.d. uniform $[0, 1]$ items. While written in a slightly different language, their proof establishes the existence of solutions to $(D')$ matching the value achieved by the optimal mechanism in $(P)$. It still remains an interesting open problem to determine the optimal mechanism for $n > 6$. We conjecture that using $(D')$ remains sufficient, but it becomes analytically challenging to define the transports in high dimensions.

5.6.2 Reverse-Engineering Optimal Mechanisms Using Duality. In Section 5.6.1, we started with a conjectured optimal mechanism. Given the mechanism, we partitioned the typeset into regions, depending on what lottery each type will purchase. We then used Condition 16 to guide us with what directions we
should use to transport measure in our coupling, in each region separately, as determined by the gradient of the utility function induced by the conjectured optimal mechanism.

In fact, we can reverse-engineer these steps when we do not have a conjecture about the optimal mechanism. Let us go back to Example 4, where we had two independent Beta items with parameters $(3, 3)$ and $(3, 4)$. Also, recall Figure 3 where we show the measure $\mu$ induced by the product of these distributions, according to (10).

How might the optimal mechanism for this example look like? It is reasonable to try to find a mechanism with the following properties:

—For sufficiently small values of $v_1$ and sufficiently large values of $v_2$ the mechanism offers item 2 with probability 1 and item 1 with probability strictly smaller than 1. Let us denote $A$ the unknown subset of the typeset where this may happen.

—For sufficiently small values of $v_2$ and sufficiently large values of $v_1$ the mechanism offers item 1 with probability 1 and item 2 with probability strictly smaller than 1. Let us denote $B$ the unknown subset of the typeset where this may happen.

Now let us revisit Condition (16). If we were to use this condition to show optimality of our yet-unknown mechanism, we would certainly be allowed to:

(1) transport measure southward in region $A$; and
(2) transport measure westward in region $B$. 
We may also be able to push measure eastward in region $\mathcal{A}$, if item 1 is allocated with probability 0 in this region. Similarly, we may be able to push measure northward in region $\mathcal{B}$, if item 2 is allocated with probability 0 in this region. As we want to be versatile, let us ignore these extra possibilities, insisting on southward transports in region $\mathcal{A}$ and westward transports in region $\mathcal{B}$.

With these restrictions on our transports let revisit measure $\mu$, trying to close in on subsets of $X$ that regions $\mathcal{A}$ and $\mathcal{B}$ may occupy. In Figure 7, we have drawn a monotone strictly concave curve $S_{\text{top}}$ that traces, for each $x_1$, the top-most point $(x_1, x_2)$ such that, restricted to the vertical segment between points $(x_1, x_2)$ and $(x_1, 1)$, $\mu_+ \succeq_1 \mu_-$, where $\succeq_1$ denotes first-order stochastic dominance between measures defined as follows.

**Definition 2.** If $\mu_1, \mu_2$ are two non-negative measures defined on some $S \subseteq X$ such that $\mu_1(S) = \mu_2(S)$, we say that $\mu_1$ first-order stochastically dominates $\mu_2$, denoted $\mu_1 \succeq_1 \mu_2$, iff there exists a coupling $\gamma \in \Gamma(\mu_1, \mu_2)$ between $\mu_1$ and $\mu_2$ such that, almost everywhere with respect to $\gamma(x, y)$, $x$ is coordinate-wise larger than or equal to $y$. Equivalently, for all non-decreasing measurable functions $u$, $\int_S u d\mu_1 \geq \int_S u d\mu_2$.

Given that $X_+ \setminus \{0\}$ is an increasing set, point $S_{\text{top}}(x_1)$ must belong to $X_-$, if it exists, and it can be identified by finding the largest $x_2$ such that the total measure on the segment between points $(x_1, x_2)$ and $(x_1, 1)$ under $\mu$ is 0. Notice that for some $x_1$’s there fails to be a segment with these properties, so $S_{\text{top}}$ is undefined for those $x_1$’s. Similarly, the monotone strictly concave curve $S_{\text{right}}$ traces, for each $x_2$, the rightmost point $(x_1, x_2)$ such that, restricted on the horizontal segment between points $(x_1, x_2)$ and $(1, x_2)$, $\mu_+ \succeq_1 \mu_-$. Again, this curve is not defined for all $x_2$, and it lies within $X_-$. 

Fig. 7. Reverse-engineering the optimal mechanism from $\mu$. 

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To accommodate Constraints (1) and (2) on our transports of measure in the (still unidentified) regions $A$ and $B$, it suffices to pick $A$ to be any region defined by $S_{\text{top}}$, the left and top boundaries of $[0, 1]^2$ and a vertical segment $v_A = (v, v')$ for some $v \in S_{\text{top}}$ and $v'$ on the top edge of $[0, 1]^2$, as in the figure. Similarly, we can pick $B$ to be any region defined by $S_{\text{right}}$, the bottom and right boundaries of $[0, 1]^2$ and a horizontal segment $h_B = (h, h')$, for some $h \in S_{\text{right}}$ and $h'$ on the right edge of $[0, 1]^2$. For any region $A$ as above, by the definition of $S_{\text{top}}$, we can couple $\mu_+$ restricted in $A$ with $\mu_-$ restricted in $A$ while respecting Constraint (1), as we can do this separately for every “vertical slice” of $A$. Similarly for any $B$ as above, we can couple $\mu_+$ restricted in $B$ with $\mu_-$ restricted in $B$ while respecting Constraint (2).

The next question is how to pick segments $v_A$ and $h_B$, and what to do with the rest, $X \setminus A \cup B$, of the typeset. A natural approach is to assume that the remainder of the typeset is partitioned into two regions, $W$ and $Z$, of types that will purchase the bundle $\{1, 2\}$ of both items at some price $p_{\{1, 2\}}$ and of types that will not purchase anything. Clearly, the utility in $W$ will be of the form $u(x) = x_1 + x_2 - p_{\{1, 2\}}$ while the utility in $Z$ will be $u(x) = 0$. Hence, the boundary between these regions must be a $45^\circ$ segment.

Given this, a natural way to finish would be to try to identify a point $v \in S_{\text{top}}$ and $h \in S_{\text{right}}$ such that:

(i) The straight segment, $(vh)$, between points $v$ and $h$ has a $45^\circ$ angle; more precisely, we want that $(h_1 - v_1, h_2 - v_2) \propto (1, -1)$.

(ii) The region, $Z$, under the curve $S$ defined by the initial part of $S_{\text{top}}$ (between points $S_{\text{top}}(0)$ and $v$), the straight segment $(vh)$, and the initial part of $S_{\text{right}}$ (between points $h$ and $S_{\text{right}}(0)$) has total measure $\mu(Z) = 0$ and is convex; i.e., $Z$ is convex.

(iii) The region, $W$, enclosed by the straight segments $v_A$, $(v'h')$, $S_{\text{right}}$ and $(vh)$, satisfies $\mu_+|_W \geq_1 \mu_-|_W$, where $\mu_+|_W$ and $\mu_-|_W$ denote respectively the restrictions of measures $\mu_+$ and $\mu_-$ in region $W$.

Note that, if we can identify points $v$ and $h$ satisfying Requirements (i)–(iii), we are done. Indeed, let us define the function $u : x \mapsto \ell_1(x, Z)$, mapping each type $x$ to its $\ell_1$ distance from set $Z$. Clearly:

—From (ii), $Z$ is convex, hence $u(x)$ is also convex. Given that $Z$ is a decreasing set, $u$ is non-decreasing. It is also clearly non-negative and 1-Lipschitz. So $u$ is feasible for (P). If we could also find a coupling $\gamma$ between $\mu_+$ and $\mu_-$ that satisfies Condition (16), this would establish that $u$ is optimal for (P).

We proceed to do this next separately for $\mu_+$ and $\mu_-$ restricted to each region.

—Region $Z$: $u(x) = 0$, for all $x \in Z$, conforming to our intention that $Z$ is the set of types that do not purchase anything. Given that $\nabla u(x) = 0$, for all $x \in Z$, Condition (16) implies that we are allowed to transport measure in north-east directions in this region. Given that $Z$ lies entirely within $X_+ \cup \{0\}$, $\mu_+$ only resides at $(0, 0)$. Moreover, $\mu(Z) = 0$. Hence, it is possible to couple $\mu_+$ and $\mu_-$ in $Z$ with only north-east transports.

—Region $W$: $u(x) = x_1 + x_2 - p^*$, for all $x \in W$, where $p^*$ is the intercept of segment $(vh)$ if it were extended to hit the $x_1$ axis. This conforms to our intention that the types in $W$ purchase the grand bundle. Given that $\nabla u(x) = 1$, for all
$x \in W$, Condition (16) implies that we can transport measure in south-west directions in this region. Given that $\mu_+|_W \succeq \mu_-|_W$, as per Requirement (iii), we can indeed couple $\mu_+$ and $\mu_-$ in this region with only transports in south-west directions, by Definition 2.

---

**Region A:** Given that $Z$ is convex (and therefore the lower boundary of region $A$ is less steep than $45^\circ$ as per the location of this boundary with respect to the segment $(v h)$), for all $x \in A$, $u(x)$ is the vertical distance between $x$ and $S_{\text{top}}$. So $\frac{\partial u}{\partial x} = 1$, which conforms to our intention that item 2 should be allocated with probability 1 in this region. Moreover, this means by Condition (16) that we are allowed to transport measure southward in our coupling between $\mu_+$ and $\mu_-$ in this region. And, by the definition of $S_{\text{top}}$, restricted to this region $\mu_+$ and $\mu_-$ can be coupled with only southward transports.

---

**Region B:** Similarly, given that $Z$ is convex (and therefore the left boundary of region $B$ is steeper than $45^\circ$), $u(x)$ is the horizontal distance between $x$ and $S_{\text{right}}$ for all $x \in B$. Via similar arguments as those employed for region $A$, $\mu_+$ and $\mu_-$ can be coupled in this region with only westward transports, respecting Condition (16).

By the above discussion, if we can identify points $v$ and $h$ satisfying Requirements (i)–(iii), this means that we have identified our optimal mechanism. It turns out that, for our specific distributions, Requirements (i)–(iii) can be satisfied and we can analytically compute the points $v$ and $h$, as shown in Figure 7. So we have managed to reverse-engineer the optimal mechanism for our setting by exploiting Condition (16). The mechanism can be described indirectly by specifying $Z$. In terms of $Z$, the utility function of the optimal mechanism is $u : x \mapsto \ell_1(x, Z)$. From $u$ we can also find the lotteries offered by the optimal mechanism: The types in $W$ receive both items with probability 1. Each type $x \in A$ receives item 2 with probability 1 and item 1 with probability that equals minus the slope of $S_{\text{top}}$ at point $S_{\text{top}}(x_1)$. Similarly, each type $x \in B$ receives item 1 with probability 1 and item 2 with probability that equals minus the inverse slope of $S_{\text{right}}$ at point $S_{\text{right}}(x_2)$.

Some remarks are in order before we conclude this section:

---

First, as a byproduct of our derivation above we have proven our claim in Example 4 that the optimal mechanism offers an uncountably large menu of lotteries. Indeed, recall that $S_{\text{top}}$ is strictly concave. Hence, the allocation probability of item 1 differs in every vertical strip within this region. Thus, a continuum of lotteries are offered to the types in $A$. The same is true for the types in $B$.

---

On the other hand, as promised in Section 3.3, there does exist a succinct description of the optimal mechanism. All we need to maintain is an analytic description of the boundary of region $Z$.

---

We emphasize again that the approach followed in this section to reverse-engineer the optimal mechanism is not guaranteed to succeed. Indeed, it is based on a weak dual ($D'$) of our optimal mechanism design formulation ($P$). Based on this weak dual, it identifies a complementary slackness condition, (16), which ties solutions to ($P$) and ($D'$) in a particular way. It then makes guesses about the
optimal mechanism, and tries to follow through with these guesses using Condition (16). Despite the fact that it is not guaranteed to succeed, the approach is quite successful in identifying optimal mechanisms. In [Tzamos 2015], Christos Tzamos provides an applet where the heuristic approach of this section is applied to reverse-engineer the optimal mechanism for user-specified Beta distributions.

—Given that the heuristic method proposed in this section may not succeed, it remains important to develop a technique that is guaranteed to succeed. This is what we do in the next section.

5.7 A Strong Duality Theorem

In previous sections, we identified a weak dual \((D')\) of our formulation, \((P)\), of optimal mechanism design as a convex optimization problem. Based on the weak dual, in Section 5.6.1, we gave an example where we were able to certify the optimality of a mechanism. In Section 5.6.2, we also showed how we can use the weak dual to reverse-engineer the optimal mechanism. Still, \((D')\) is a weak dual, so there are settings where both certifying and reverse-engineering the optimal mechanism will fail [Daskalakis et al. 2015].

We would thus like to obtain a strong dual of formulation \((P)\), whose optimum is guaranteed to match that of \((P)\). The advantage of a strong dual would be that, for every mechanism design problem \((n, F)\), the optimal mechanism \(u\) for \((P)\) would have a certificate of optimality in the form of a solution to the dual. Thus, we would know what type of certificate to look for, and we would also be able to reverse-engineer optimal mechanisms by exploiting the structure of those certificates.

The challenge we encounter though is that a priori it is not clear if a tight dual to \((P)\) should exist. Indeed, \((P)\) is an infinite-dimensional optimization problem and contains constraints on the variable \(u\), such as convexity, which are non-standard. In particular, we are not aware of a duality framework, based on infinite-dimensional linear programming, that can accommodate such constraints directly. We prove our own extension of Monge-Kantorovich duality to accommodate the constraints of \((P)\), establishing the following.

**Theorem 4** [Daskalakis et al. 2015]. Formulation \((P)\) has a strong dual formulation \((D)\), taking the form of an optimal transport problem, as follows:

\[
(P): \sup_{u \text{ satisfies } (11), (12) \text{ and } (13)} \int_X u \, d\mu = \inf_{\gamma \in \Gamma(\mu_+, \mu_-)} \int_{X \times X} |(x - y)_+| \, d\gamma : (D)
\]

Moreover, both \(\sup\) and \(\inf\) are attainable.

We next explain the statement of Theorem 4, comparing \((D)\) to the weak-dual \((D')\) of Section 5.5. We then discuss the point of deriving a strong dual, before turning to some important applications of our strong duality theorem.

5.7.1 Discussion of Theorem 4. First, to clarify the notation used in Theorem 4, \(\geq_{\text{cvx}}\) is the standard notion of convex dominance between measures, defined as follows.
Definition 3. For two measures $\mu_1$ and $\mu_2$ over $X$ we say that $\mu_1$ convexly dominates $\mu_2$, denoted $\mu_1 \succeq_{\text{cvx}} \mu_2$, iff, for all measurable, non-decreasing and convex functions $u$, $\int_X u \, d\mu_1 \geq \int_X u \, d\mu_2$.

Compared to the first-order stochastic dominance of Definition 2, convex dominance is a weaker requirement as it only requires that $\int_X u \, d\mu_1 \geq \int_X u \, d\mu_2$ for convex, non-decreasing $u$’s, as opposed to all non-decreasing $u$’s.

As non-decreasing, convex functions model utility functions of risk-seeking individuals, when $\mu_1, \mu_2$ are probability measures and $\mu_1 \succeq_{\text{cvx}} \mu_2$, this means that any risk-seeking individual prefers a prize distributed according to $\mu_1$ to a prize distributed according to $\mu_2$. More generally, when $\mu_1 \succeq_{\text{cvx}} \mu_2$, this essentially means that $\mu_1$ can be obtained from $\mu_2$ through a sequence of operations allowed to:

—send (positive) measure to coordinate-wise larger points—this makes the integral $\int u \, d\mu_1$ larger than $\int u \, d\mu_2$ since $u$ is non-decreasing; or
—spread (positive) measure so that the mean is preserved—this makes the integral $\int u \, d\mu_1$ larger than $\int u \, d\mu_2$ since $u$ is convex.

Now that our understanding of convex dominance is grounded, let us compare our strong dual $(D)$ to the weak-dual $(D')$ of Section 5.5. The two problems are very similar. They are both min-cost transportation problems, seeking a coupling $\gamma(x, y)$ between two non-negative measures under which the total cost under the same cost function $|x - y|_1$ is minimized. The difference between the two problems is that $(D')$ seeks a coupling between $\mu_+$ and $\mu_-$, the positive and negative parts of measure $\mu$ derived from $F$ according to (10). $(D)$ is also allowed to pre-process $\mu$ without incurring any cost before seeking a coupling between the positive and negative parts of the processed measure. In particular, $(D)$ is allowed to choose any measure $\mu' \succeq_{\text{cvx}} \mu$ and couple the positive and negative parts of that measure $\mu'$. This way it may reduce the transportation cost, which Theorem 4 says is guaranteed to match the optimum of $(P)$.

5.7.2 The Point of Theorem 4. So, what is the point of obtaining a strong dual of our mechanism design problem? We have already alluded to the benefits of strong duality above. By analogy, these should also be clear to anyone familiar with the uses of strong linear programming duality in combinatorial optimization. Let us expand a bit. In comparison to our weak dual $(D')$, our strong dual $(D)$ is more powerful as it allows us to certify the optimality of any mechanism. In particular, for any mechanism design problem defined by some $n$ and $F$, we are guaranteed to find a measure $\mu' \succeq_{\text{cvx}} \mu$ and a coupling $\gamma$ between $\mu'_+ \mu'_-$ whose transportation cost equals the optimal revenue. In particular, we can identify conditions tying a solution $u$ to $(P)$ with a solution $(\mu', \gamma)$ to $(D)$ that, whenever satisfied, establish the joint optimality of both $u$ and $(\mu', \gamma)$. Additionally, using these conditions as a guiding principle, we can reverse-engineer optimal mechanisms in settings where we do not have a conjecture about what the optimal mechanism is, as we did in Section 5.6.2 using weak duality. Except now this approach is guaranteed to work. We refer the interested reader to [Daskalakis et al. 2015] for these conditions, as well as examples where they are used to certify optimality of mechanisms. The approach is similar to Sections 5.6.1 and 5.6.2, so we do not expand more here.
We prefer to turn instead to exciting applications of the strong duality theorem to obtain characterization results.

5.8 Characterizing Optimal Mechanisms

Using our strong duality theorem (Theorem 4) to certify optimality of mechanisms may be cumbersome. A pertinent question is this: Can we somehow use the theorem behind the scenes to develop simple characterization results of mechanism optimality? In this section, we obtain such a characterization: given a proposed mechanism for some setting of \( n \) and \( F \), we identify a collection of necessary and sufficient conditions on \( \mu \) for the mechanism to be optimal. In particular, these conditions do not involve the dual problem at all. They are just conditions on the measure \( \mu \) derived from \( n \) and \( F \) using (10). We proceed to give a flavor of our characterization, and some applications. We start with a characterization of grand-bundling optimality, proceeding to general mechanisms afterwards.

5.8.1 Characterization of Grand-Bundling Optimality.

Let us complement Definition 3 with the following definition.

**Definition 4.** For two measures \( \mu_1 \) and \( \mu_2 \) over \( X \) we say that \( \mu_1 \) second-order stochastically dominates \( \mu_2 \), denoted \( \mu_1 \succeq_2 \mu_2 \), iff for all measurable, non-decreasing and concave functions \( u \), \( \int_X u d\mu_1 \geq \int_X u d\mu_2 \).

As concave and non-decreasing functions model utility functions of risk-averse individuals, if \( \mu_1, \mu_2 \) are probability measures and \( \mu_1 \succeq_2 \mu_2 \), this means that any risk-averse individual prefers a prize distributed according to \( \mu_1 \) to a prize distributed according to \( \mu_2 \). More generally, \( \mu_1 \succeq_2 \mu_2 \) essentially means that \( \mu_1 \) can be obtained from \( \mu_2 \) via a sequence of operations that shift positive measure to coordinate-wise smaller points and do mean-preserving merges of positive measure.

With the above definition, we can use our duality theorem behind the scenes to obtain the following characterization of grand-bundling optimality.

**Theorem 5 [Daskalakis et al. 2015].** For all \( n \) and \( F \), the mechanism that only offers the bundle of all items at some price \( p \) is optimal if and only if measure \( \mu \) defined by (10) satisfies \( \mu|_W \succeq_2 0 \succeq_{\text{cvx}} \mu|_Z \), where \( W \) is the subset of types that can afford the grand bundle at price \( p \), \( Z \) the subset of types who cannot, and \( \mu|_W \), \( \mu|_Z \) are respectively the restrictions of \( \mu \) in subsets \( W \) and \( Z \).

Sufficient conditions for grand-bundling optimality have been an active line of inquiry—see e.g. [Manelli and Vincent 2006; Pavlov 2011]. Theorem 5 provides a single condition, in the form of two stochastic dominance relations, that is necessary and sufficient.

As a corollary of this theorem, we can show Theorem 1 of Section 3.1, pertaining to the counter-intuitive behavior of the optimal mechanism for \( n \) i.i.d., uniform \([c, c+1]\) items:

—The first part of this theorem is an extension of Pavlov’s result for \( n = 2 \) [Pavlov 2011]. Its proof is by a geometric construction showing that the sufficient condition will be met for all \( n \)'s as long as \( c \) is large enough.

—The second, and more counter-intuitive, part of the theorem is shown by arguing that all prices \( p \) will fail to satisfy \( 0 \succeq_{\text{cvx}} \mu|_Z \). Essentially, what happens in this
Multi-Item Auctions Defying Intuition?

setting is that all the positive measure is surface measure on the facets \( x_1 = c + 1 \), and there is an atom of +1 at the origin \( \vec{c} \). All other facets and the interior of the cube \([c, c + 1]^n\) have negative measure. Given this, if \( n \) is large enough, measure \( \mu|_Z \) exhibits a phase transition in terms of the price \( p \): if \( p \) is large enough that \( Z \) has a positive-measure intersection with facet \( x_1 = c + 1 \), function \( u(x) = 1_{\{x_1 = c + 1\}} \) is an explicit witness that constraint \( 0 \succeq_{cvx} \mu|_Z \) is violated, as \( \int u d\mu|_Z > 0 \). On the other hand, if \( p \) is small enough that \( Z \) has measure 0 intersection with this facet, there turns out not to be enough negative mass in region \( Z \) to balance the positive atom at \( \vec{c} \), as required for \( 0 \succeq_{cvx} \mu|_Z \) to hold.

5.8.2 Characterization of General Mechanisms. Our characterization of grand-bundling optimality from the previous section naturally extends to arbitrary mechanisms. Let us briefly discuss this generalization, referring the reader to [Daskalakis et al. 2015] for more details.

Consider a mechanism \( \mathcal{M} \) for some setting \( n \) and \( F \). \( \mathcal{M} \) induces a partition of the typeset \( X \) into subsets of types that will decide to purchase different lotteries in the menu offered by \( \mathcal{M} \). Assuming that \( \mathcal{M} \) offers a finite number of lotteries, this partition may look like Figure 8, where each cell \( c \) corresponds to a subset of types that will purchase the same lottery \((p^c, t^c)\), where \( p^c \) is a vector of allocation probabilities and \( t^c \) a price.

![Fig. 8. A partition of the typeset induced by some finite menu of lotteries.](image)

If the mechanism is a grand-bundling mechanism, then there are only two regions and Theorem 5 defines a pair of stochastic dominance conditions that are necessary and sufficient for its optimality. Naturally, our generalization to general mechanism will require one condition per cell \( c \) of the partition. To describe them, let us define vector \( \vec{v}^c \) in terms of \( p^c \) as follows:

\[
\forall i : v_i^c = \begin{cases} 
1, & \text{if } p_i^c = 0 \\
-1, & \text{if } p_i^c = 1 \\
0, & \text{otherwise}
\end{cases}
\]

Moreover, for a vector \( \vec{v} \in \{-1, 1, 0\}^n \), let us define a stochastic dominance relation with respect to \( \vec{v} \) as follows:
Definition 5. For a vector $\vec{v} \in \{-1,1,0\}^n$ and two measures $\mu_1$ and $\mu_2$ over $X$, we say that $\mu_1$ convexly dominates $\mu_2$ with respect to $\vec{v}$, denoted $\mu_1 \succeq_{\text{cvx}} \vec{v} \mu_2$, iff, for all measurable and convex functions $u$ that are non-decreasing in all coordinates $i$ such that $v_i = 1$ and non-increasing in all coordinates $i$ such that $v_i = -1$: 

$$\int_X ud\mu_1 \geq \int_X ud\mu_2.$$ 

Clearly, $\mu_1 \succeq_{\text{cvx}(\vec{1})} \mu_2 \iff \mu_1 \succeq_{\text{cvx}} \mu_2 \iff \mu_2 \succeq_{\vec{v}} \mu_1$. So Theorem 5 can be restated as specifying the following necessary and sufficient conditions for grand-bundling optimality: $0 \succeq_{\text{cvx}(\vec{1})} \mu|_Z$ and $0 \succeq_{\text{cvx}(-\vec{1})} \mu|_W$, where $Z$ and $W$ are the subsets of types that purchase nothing and the grand bundle respectively.

Our general characterization is the following:

**Characterization of General Mechanisms**

$$(\mathcal{M} \text{ is optimal}) \iff \left( \text{for every cell } c \text{ in the partition of } X \text{ induced by } \mathcal{M}: 0 \succeq_{\text{cvx}(\vec{v}^c)} \mu|_c, \text{ where } \mu|_c \text{ is the restriction of } \mu, \text{ defined in (10), to cell } c \right).$$

The interested reader is referred to [Daskalakis et al. 2015] for more details.

5.9 Concluding Remarks

In Sections 5.1–5.8 we took up the challenge of understanding the structure of multi-item mechanisms, trying to demystify their counter-intuitive behavior identified in Section 3.

Our contribution was a duality framework based on which we can certify the optimality of mechanisms in every single-buyer setting. In particular, we showed that the optimal mechanism design problem has a tight dual problem, taking the form of an optimal transportation problem. Given a proposed mechanism, we can test if it is optimal by identifying solutions to the dual achieving the same value, as we did in Section 5.6.1, except that more generally we can also pre-process $\mu$ with cost-free convex shuffling operations before finding our transports, as allowed by the strong dual ($D$). Given our strong duality, dual certificates are guaranteed to exist. Moreover, if we do not have a conjectured optimal mechanism for some setting of interest, we can exploit the dual to reverse-engineer it, as we did in Section 5.6.2. Finally, we developed “duality-theory oblivious” tools, characterizing the optimality of mechanisms in terms of the buyer’s distribution. These tools use the power of the duality framework behind the scenes, presenting clean necessary and sufficient conditions for a mechanism to be optimal.

All in all, we believe that our framework provides a new perspective on multi-item mechanisms, presenting a tool whose applicability is universal, given our strong duality. Our work opens interesting lines for future investigation, and we are particularly interested in:

1. extending the duality framework to accommodate multiple buyers; and
2. developing technical machinery to facilitate testing stochastic dominance relations.
While we recognize that a lot remains to be done, we believe that the afore-described framework represents the beginning of a principled approach towards a structural understanding of multi-item multi-buyer mechanisms.

6. THE COMPUTATIONAL COMPLEXITY OF MULTI-ITEM MECHANISMS

6.1 Philosophy

The duality based framework of the previous section targeted closed-form characterizations of optimal mechanisms. A related goal is to study the computational complexity of optimal mechanisms. In particular, we are interested in whether optimal mechanisms can be computed and implemented computationally efficiently. There are several reasons why this question is important:

— First, if optimal mechanisms were computationally intractable, then why should practitioners care about them, especially in settings with a large number of bidders or items? Computational intractability would justify using approximate mechanisms in practical applications.

— Moreover, as we have seen, getting closed-form descriptions of optimal mechanisms is a challenging task. Despite intense work in the literature, including that of the previous section, we are still far from characterizing optimal multi-bidder mechanisms. When closed-form descriptions are unknown, being able to compute optimal mechanisms is an interesting middle ground.

— Additionally, computing optimal mechanisms would be a great tool for researchers who want to gain familiarity with the structure of these mechanisms and/or test hypotheses about their structure or performance of approximate solutions.

— Finally, one might expect that studying optimal mechanism design from an algorithmic point of view may reveal structure that might be hard to observe using non-algorithmic tools.

For all the above reasons, we find it important to study the computational complexity of optimal mechanisms. Over the past few years, our and other groups have made tremendous progress in the complexity of optimal mechanism design [Cai and Daskalakis 2011; Cai et al. 2012a; Alaei et al. 2012; Cai et al. 2012b; Daskalakis et al. 2012; Cai and Huang 2013; Cai et al. 2013a; 2013b; Alaei et al. 2013; Bhalgat et al. 2013; Daskalakis et al. 2014; Chen et al. 2014; Daskalakis and Weinberg 2015; Daskalakis et al. 2015]. See also [Hartline 2013; Chawla and Sivan 2014; Cai et al. 2015] for recent surveys, covering some of this work. We proceed to give a flavor of what is known, restricting our attention to multi-item multi-bidder settings with additive bidders. As discussed below, all our results in this section extend to much broader settings.

6.2 Setting

In this section, we restrict our attention to a seller with $n$ items and $m$ additive bidders interested in those items. The type of each bidder is a vector $t_i$ of values for the items, which are jointly distributed according to some distribution $F_i$. In particular, $t_{ij}$ will denote the value of bidder $i$ for item $j$. We assume that the distribution $F_i$ is known to the seller and all the other bidders, but only bidder $i$
knows his realized type. We also assume that bidder types are independent and use \( \vec{t} \) to denote the types of all bidders, also called the type profile. Finally, given a type vector \( \vec{t} \) and \( i \), we denote, as is customary, by \( \vec{t}_{-i} \) the vector containing the types of all bidders except the type of bidder \( i \). Accordingly, we use \( (t_i; \vec{t}_{-i}) \) to denote \( \vec{t} \).

The goal of the seller is to design a mechanism that optimizes his expected revenue, when the expectation is computed with respect to the types of the bidders, the randomness in the mechanism, if any, and the randomness in the strategies of the bidders, if any. The mechanism is allowed to be any protocol that interacts with the bidders and has the bidders interact with each other in some fashion. Whatever the protocol is, it is supposed to output an allocation \( x \in \{0, 1\}^{mn} \) of items to bidders, where \( x_{ij} \) indicates whether item \( j \) is given to bidder \( i \). As we assume that there is exactly one copy of each item, any allocation ever output by the mechanism must satisfy that

\[
\sum_i x_{ij} \leq 1, \text{ for all items } j.
\]

We call \( \mathcal{F} \) the subset of \( \{0, 1\}^{mn} \) satisfying the above constraints. The mechanism can also charge prices, as long as the bidders accept to pay those prices.

While it is hard to optimize over protocols, it follows from the revelation principle that it suffices to optimize over a simpler class of mechanisms called “direct mechanisms.” These mechanisms are described by an allocation function \( X : \vec{t} \mapsto \Delta(\mathcal{F}) \), mapping type profiles to distributions over feasible allocations, and a price function \( P : t \mapsto \Delta(\mathbb{R}^m) \) mapping type profiles to distributions over price vectors, and are implemented as follows:

—The bidders are asked to report their types to the mechanism.
—If \( \vec{t} \) are the reported types, the mechanism samples \( x \sim X(\vec{t}) \) and \( p \sim P(\vec{t}) \), implements allocation \( x \) and charges prices according to \( p \).

For convenience, we will use \( \mathcal{X}(\vec{t}) \) to denote a random variable distributed according to \( X(\vec{t}) \), and similarly \( \mathcal{P}(\vec{t}) \) to denote a random variable distributed according to \( P(\vec{t}) \).

While, in principle, the bidders need not be truthful about their types, due to the revelation principle we can also assume without loss of generality that it will be in their best interest to do so. We may also assume that it is not hurtful to them to participate in the mechanism. In particular, we may assume that the mechanism is Bayesian Incentive Compatible and satisfies Individual Rationality according to the following definition.

**Definition 6.** We say that a direct mechanism \((X, P)\) is Bayesian Incentive Compatible (BIC) iff for all bidders \( i \) and types \( t_i \) and \( t_i' \) in the support of \( F_i \):

\[
\mathbb{E}_{\vec{t}_{-i}}[t_i \cdot \mathcal{X}(\vec{t}) - \mathcal{P}(\vec{t})] \geq \mathbb{E}_{\vec{t}_{-i}}[t_i \cdot \mathcal{X}(t_i'; \vec{t}_{-i}) - \mathcal{P}(t_i', \vec{t}_{-i})].
\] (17)

We say that a direct mechanism \((X, P)\) satisfies Individual Rationality or is IR iff for all bidders \( i \) and types \( t_i \) in the support of \( F_i \):

\[
\mathbb{E}_{\vec{t}_{-i}}[t_i \cdot \mathcal{X}(\vec{t}) - \mathcal{P}(\vec{t})] \geq 0.
\] (18)
expresses that the expected utility of bidder \(i\) cannot be improved by mis-reporting his type, while (18) expresses that the expected utility of bidder \(i\) is non-negative if he is truthful. Both expectations are with respect to the types of the other bidders as well as the randomness in the allocation and price functions, if any.

Under the assumption that bidders will report their types truthfully to a BIC, IR mechanism, its revenue can be expressed as follows:

\[
\mathbb{E}_T \left[ \sum_i \mathcal{P}_i(t) \right].
\]  

(19)

With all the context provided above, the seller is given distributions \(F_1, \ldots, F_m\) over bidder types and seeks to compute a BIC, IR mechanism of optimal revenue.

6.3 Computationally Efficient in What Exactly?

When it comes to studying multi-item mechanisms from a computational perspective what we need to answer first is what the input is, and how its description complexity is measured. There are several ways one can go about this, including the following.

— One approach is to assume that the bidders’ distributions come from a parametric family of distributions, and specify the parameters of each bidder’s distribution. This approach would allow us to accommodate both discrete and continuous distributions. In this case, the description complexity of the distributions is the description complexity of all parameters required to describe them.

— Another approach is to assume that the distributions are discrete and provide them explicitly, by listing the elements in their support and the probability that each element is sampled. The explicit description is reasonable for distributions with a small and discrete support. Here the description complexity of the distributions is the description complexity of all the elements in their support and their associated probabilities.

— Finally, one can assume to have sample access to the distribution \(F_i\) of each bidder. Here each distribution can be thought of as a subroutine that a seller can call to get an independent sample from \(F_i\). The description complexity of the distributions is harder to define in this model. One way to do this is to assume that the subroutines only output numbers of certain bit complexity, or truncate all numbers output by these subroutines to some accuracy. We do not want to dwell on this point too much though.

We will restrict our attention to the simplest model, assuming that all our distributions are explicit. The parametric and sample-access models are important to study as well, but we are not aware of efficiently computable mechanisms that can accommodate these models in reasonable generality without losing revenue. In fact, we should not expect to get general, exactly optimal algorithms in these settings given the following result:

**Theorem 6** [Daskalakis et al. 2014]. Consider the algorithmic problem of designing an optimal mechanism for selling \(n\) items to a single, additive buyer whose values for the items are independently distributed, according to distributions
that are supported on two rational numbers with rational probabilities. In particular, the distribution of each item $i$ is specified by a pair of rational numbers $\{a_i, b_i\}$ and a rational probability $p_i$.

Unless $\text{ZPP} \supseteq \text{P}^\#P$, any mechanism (of any type, direct or indirect) that can be computed in expected polynomial time cannot be both optimal and executable in expected polynomial time.

Notice that the mechanism design problem in the statement of Theorem 6 conforms to the parametric model for describing distributions. And the theorem provides a particularly simple setting where, unless we spent super-polynomial time, we will not be able to compute an efficiently executable mechanism that is also optimal. As the sample-access model is not easier than the parametric one, the same result applies to this model as well. Finally, we will only remark here that the assumption $\text{ZPP} \supseteq \text{P}^\#P$ is a standard complexity-theoretic assumption about the relations of complexity classes $\text{P}$, $\text{ZPP}$ and $\text{P}^\#P$. The interested reader is referred to standard complexity theory textbooks for more discussion.

While Theorem 6 is quite discouraging, especially given the simplicity of the mechanism design problem that is shown intractable, it still does not preclude efficiently computable mechanisms that are near-optimal. In particular, it is an interesting open problem to determine whether there exist efficiently computable mechanisms that achieve a $(1 - \epsilon)$-fraction of the optimal revenue, for any desired accuracy $\epsilon > 0$, as long as one is willing to invest time polynomial in $1/\epsilon$ and the parameters of the distribution for their computation. This problem is open even in the simple setting of Theorem 6.

6.4 Computing Optimal Mechanisms, and a Bonus

We turn to the explicit model of representing bidder distributions, and ask whether optimal mechanisms can be computed efficiently. There is still a wrinkle we need to overcome however. As we said a direct mechanism is a pair of functions $X : \hat{t} \mapsto \Delta(F)$ and $P : \hat{t} \mapsto \Delta(\mathbb{R}^m)$. So to describe these functions explicitly, we need to specify a distribution over allocations and a distribution over price vectors for every possible type profile. This is problematic though as revealed by a little calculation. Suppose that the support of every bidder’s type distribution has size $k$. Then to specify each distribution we need to give $k$ numbers and $k - 1$ probabilities. So to describe all these distributions only requires $O(mk)$ numbers. On the other hand, there are $k^m$ possible type profiles. So we cannot hope to compute $X$ and $P$ explicitly. Besides, even the outputs of these functions are distributions over high-dimensional spaces.

We thus need to be smart about how we represent mechanisms. We need to represent them implicitly, whilst still being able to compute on the implicit representation. In the additive setting that we consider here, it turns out that the so-called “reduced-form” is a good representation:

**Definition 7.** The reduced form $(\hat{x}, \hat{p})$ of a mechanism $(X, P)$ is a collection of single-variate functions $\hat{x}_i : T_i \to [0, 1]^n$, $i = 1, \ldots, m$, and $\hat{p}_i : T_i \to \mathbb{R}$, $i = 1, \ldots, m$, where $T_i$ is the support of $F_i$, which are related to $X$ and $P$ as follows. For all $i$ and types $t_i \in T_i$:
Multi-Item Auctions Defying Intuition?

\[ \hat{x}_{ij}(t_i) = \mathbb{E}_{\vec{t} - i}[X_{ij}(t_i; \vec{t} - i)]; \text{ in particular, the } j\text{-th coordinate of } \hat{x}_{i}(t_i) \text{ is the probability that the mechanism allocates item } j \text{ to bidder } i, \text{ conditioning on his reported type being } t_i \text{ and assuming that the other bidders report their types truthfully.} \]

\[ \hat{p}_i(t_i) = \mathbb{E}_{\vec{t} - i}[P_i(t_i; \vec{t} - i)]; \text{ in particular, } \hat{p}_i(t_i) \text{ is the expected price charged to bidder } i, \text{ conditioning on his reported type being } t_i \text{ and assuming that the other bidders report their types truthfully.} \]

The reduced form is called “reduced” because it is losing information about the mechanism. It can be viewed as projecting \((X, P)\), which is a high-dimensional object, to a lower-dimensional space. So maintaining mechanisms in their reduced forms, creates two computational challenges:

1. Given a reduced form \((\hat{x}, \hat{p})\), is it possible to verify computationally efficiently whether it is feasible, that is whether there exists an actual mechanism \((X, P)\) whose reduced form agrees with \((\hat{x}, \hat{p})\)?
2. Given a reduced form \((\hat{x}, \hat{p})\) that is feasible, is it possible to implement some mechanism with this reduced form computationally efficiently?

Border and Che et al. have provided a collection of linear constraints that are necessary and sufficient for reduced-form feasibility [Border 1991; 2007; Che et al. 2011]. See also [Hart and Reny 2014]. These constraints have a nice interpretation as max-flow/min-cut constraints in a related flow network. However, they cannot be used towards computationally efficient algorithms for the above problems, as they are exponentially many. We can improve these results as follows:

**Theorem 7** [Cai et al. 2012a; Alaei et al. 2012]. The answers to both questions above are “yes.” Namely, given a reduced form \((\hat{x}, \hat{p})\), we can verify in polynomial time whether it is feasible. Moreover, given a feasible reduced form \((\hat{x}, \hat{p})\), we can compute in polynomial time a pair of polynomial-time algorithms for sampling the allocation and price functions of a mechanism \((X, P)\) whose reduced form is \((\hat{x}, \hat{p})\).

Theorem 7 is very handy as it allows us to formulate polynomial-size linear programs for finding optimal mechanisms. In particular, it is not hard to see the following:

- The set of all possible reduced forms is convex, as they are projections of allocation and price functions, which themselves are a convex set as distributions over deterministic allocation and price functions.
- Given Theorem 7, there exists a polynomial-time algorithm that determines feasibility of reduced forms. This gives a computationally efficient separation oracle for the set of feasible reduced forms.
- The expected revenue of a mechanism can be expressed as a linear function of its reduced form.
- The BIC and IR constraints are also expressible as linear constraints in the reduced form.
- Finally, Theorem 7 implies that, given the reduced form of a mechanism, we can efficiently implement it.
Using the Ellipsoid algorithm, the above observations culminate to the following result:

**Theorem 8 [Cai et al. 2012a].** Consider a mechanism design setting with $m$ additive bidders and $n$ items. Given an explicit description of the bidders’ type distributions, we can compute and implement an optimal mechanism in polynomial-time.

Our new theorem provides an important counterpart to our structural results from Section 5, encompassing multi-bidder settings as well. In fact, the theorem can be generalized to much broader settings involving more complex allocation constraints [Cai et al. 2012b], budgets [Bhalgat et al. 2013; Daskalakis et al. 2015], non-additive bidders [Cai et al. 2013b], and even going beyond revenue and welfare to other interesting objectives, such as maximizing fairness or minimizing makespan [Daskalakis and Weinberg 2015]. Taken together our algorithmic results provide a very crisp understanding of the complexity of Bayesian mechanism design. In a recent article, we provide a concise overview of our work on this front, and refer the interested reader to this overview [Cai et al. 2015], as well as the surveys mentioned earlier [Hartline 2013; Chawla and Sivan 2014].

Let us conclude this section with a small treat. We promised earlier that the algorithmic perspective might actually reveal structure in the optimal mechanism that could be hard to see otherwise. While developing our algorithmic framework, we encountered remarkable structure in the optimal mechanism that is worth sharing here. Recall, from Fact 3 that Myerson’s optimal single-item mechanism is a virtual welfare maximizer. It turns out that this holds in arbitrary settings. Here is the structure of the optimal mechanism for the setting of Theorem 8.

**Theorem 9 [Cai et al. 2012b].** When $n$ items are sold to $m$ additive bidders, the optimal mechanism is a virtual welfare maximizer, namely:

1. The bidders are asked to report their types to the mechanism. Say that the reported types are $t_1, \ldots, t_m$.
2. The reported types are transformed into virtual types, $h_1(t_1), \ldots, h_m(t_m)$, where each $h_i : T_i \rightarrow \mathbb{R}^n$ maps an additive type $t_i$ to another additive type $h_i(t_i)$.
3. Each item is allocated to the bidder with the highest virtual value for this item, with some lexicographic tie-breaking.
4. Finally, prices are charged so that the mechanism is BIC.

So the optimal mechanism has the exact same form in multi-item settings as it has in single-item settings! The differences between Theorem 9 and Fact 3 are the following:

—In Myerson’s single-item setting, the virtual transformations $h_i$ are deterministic, while in the multi-item setting they are actually randomized. Of course, we knew this had to be the case as by Fact 5 randomization is necessary in multi-item settings.

—As emphasized in Fact 3, in Myerson’s setting, each $h_i$ only depends on bidder $i$’s distribution, but not on the other bidders’ distributions and not even on how many other bidders show up. In the multi-item setting, each $h_i$ may depend on
the distributions of all bidders, but importantly its argument is only bidder \(i\)'s type.

Despite these differences, it is remarkable that the same structure applies to both single-item and multi-item settings. Again the above structure generalizes well-beyond the additive setting—see [Cai et al. 2013b; 2015].

7. CONCLUSIONS

In the past few decades, we witnessed the tremendous impact of Myerson’s result [Myerson 1981] in mechanism design. Building on his result, we now understand virtually every aspect of revenue optimization in single-item settings. Moreover, we have seen numerous extensions, robustifying the result with respect to the details of the bidders’ distributions, and expanding its applicability to a myriad of single-dimensional settings, accommodating budgets, online arrivals and departures of bidders, complex allocation constraints, non-linear designer objectives, and many more. Algorithmic and approximation techniques have played an important role in exploring these extensions, and indeed it has been thanks to the sharpness and simplicity of Myerson’s result that this interplay between computation and mechanism design has been so fruitful. See [Hartline 2013; Chawla and Sivan 2014; Roughgarden 2015] for recent surveys of this literature.

Unfortunately, multi-item revenue optimization has not enjoyed the same fate due to our lack of understanding of multi-item mechanisms. Indeed, optimal multi-item mechanisms exhibit such rich structure that it is not clear whether there is a lens through which we can gain a crisp understanding of their properties. In this survey, we provided two approaches, based on duality theory and optimization, through which we obtained a fresh perspective on multi-item mechanism design. We have used these approaches to characterize the structure of multi-item mechanisms and showcased procedures, both analytical and algorithmic, via which the optimal mechanism can be identified.

We believe that the results presented in this survey have begun to resemble a cohesive theory of multi-item auctions, opening exciting directions for future investigation. To identify just a few:

— It is important to extend the duality-based framework of Section 5 to accommodate multiple bidders.

— What is the sensitivity of the structural and algorithmic results on the details of the bidder type distributions?

— In settings where the seller knows the bidder distributions, but the bidders do not, what is the Bayesian-optimal dominant strategy truthful mechanism?

Ultimately, we feel that the foundations have been laid for exciting developments in optimal multi-item mechanism design, expecting a lot more progress on this front in the next years.

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Multi-Item Auctions Defying Intuition?


Finding Any Nontrivial Coarse Correlated Equilibrium Is Hard

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One of the most appealing aspects of correlated equilibria and coarse correlated equilibria is that natural dynamics quickly arrive at approximations of such equilibria, even in games with many players. In addition, there exist polynomial-time algorithms that compute exact correlated and coarse correlated equilibria. However, in general these dynamics and algorithms do not provide a guarantee on the quality (say, in terms of social welfare) of the resulting equilibrium. In light of these results, a natural question is how good are the correlated and coarse correlated equilibria—in terms natural objectives such as social welfare or Pareto optimality—that can arise from any efficient algorithm or dynamics.

We address this question, and establish strong negative results. In particular, we show that in multiplayer games that have a succinct representation, it is NP-hard to compute any coarse correlated equilibrium (or approximate coarse correlated equilibrium) with welfare strictly better than the worst possible. The focus on succinct games ensures that the underlying complexity question is interesting; many multiplayer games of interest are in fact succinct. We show that analogous hardness results hold for correlated equilibria, and persist under the egalitarian objective or Pareto optimality.

To complement the hardness results, we develop an algorithmic framework that identifies settings in which we can efficiently compute an approximate correlated equilibrium with near-optimal welfare. We use this framework to develop an efficient algorithm for computing an approximate correlated equilibrium with near-optimal welfare in aggregative games.

Categories and Subject Descriptors: F.2.0 [Analysis of Algorithms and Problem Complexity]: General

General Terms: Theory, Algorithms, Economics

Additional Key Words and Phrases: Coarse correlated equilibrium, welfare maximization, complexity of equilibria, price of anarchy

1. INTRODUCTION

Questions related to the complexity of equilibria lie at the core of algorithmic game theory. While computation of Nash equilibria has in recent years been shown to be computationally hard even in games with two players [Chen et al. 2009], algorithmic results for correlated equilibria (CE) and coarse correlated equilibria (CCE) have been more positive. Even in games with many players, there exist a number of natural dynamics that quickly converge to these solution concepts; see, e.g., [Young 2004]. In particular, these dynamics induce efficient computation of approximate CE and CCE in multiplayer games. In fact, exact CE and CCE are efficiently computable in many classes of multiplayer games [Papadimitriou and Roughgarden 2008; Jiang and Leyton-Brown 2013].
Beyond computation of equilibria, another significant thread of research in algorithmic game theory has been the study of the quality of equilibria, often as measured by the social welfare of the equilibrium or its ratio to the social welfare of the socially optimal outcome (c.f. the extensive literature on the price of anarchy (PoA) [Nisan et al. 2007]). Given that we know it is possible to efficiently compute some CE and CCE, it is natural to ask how good an equilibrium we get as the output of such a procedure. For example, do existing efficient dynamics find the best such equilibria, or at least ones that approximately optimize the social welfare?

In their notable work, [Papadimitriou and Roughgarden 2008] show that determining a socially optimal CE is NP-hard in a number of succinct multiplayer games. This hardness result leaves open the question of computing near-optimal CE/CCE, i.e., whether there exist efficient algorithms that compute CE/CCE with welfare at least, say, \( \alpha \) times the optimal, for a nontrivial approximation ratio \( \alpha \leq 1 \). This question forms the basis of our work [Barman and Ligett 2015].

**Technical Aside (succinct games):** In general multiplayer games the size of the normal form representation, \( N \), is exponentially large in the number of players; one can compute a CE/CCE that optimizes a linear objective by solving a linear program of size polynomial in \( N \), and hence the computational complexity of equilibrium computation is not interesting for general games. However, most games of interest—such as graphical games, polymatrix games, congestion games, and anonymous games—admit a succinct representation (wherein the above-mentioned linear program can be exponentially large in the size of the representation), and hence it is such succinctly representable games that we (and previous works) study.

**Results:** In [Barman and Ligett 2015] we establish that, unless \( P = NP \), there does not exist any efficient algorithm that computes a CCE in succinct multiplayer games with welfare better than the worst possible CCE. We also extend the hardness result to approximate CCE. Therefore, while one can efficiently compute an approximate CCE in succinct multiplayer games, one cannot provide any nontrivial welfare guarantees for the resulting equilibrium (unless \( P = NP \)). Furthermore, these hardness results also hold specifically for potential games (generally considered to be a very tractable class of games), and persist even in settings where the gap between the best and worst equilibrium is large. It is relevant to note that these complexity barriers provide new motivation for studying the price of anarchy (the quality of the worst equilibrium) for CE and CCE, since generally that is the best thing we can hope to compute.

Our work also complements these hardness results by developing an algorithmic framework for computing an approximate CCE with welfare that is additively close to the optimal. This framework establishes a sufficient condition under which the above-mentioned complexity barriers can be circumvented. In particular, we show that if in a given game we can efficiently obtain an additive approximation for a modified-welfare maximization problem, then we can efficiently compute an approximate CE with high welfare. The modified welfare under consideration can be thought of as a Lagrangian corresponding to the equilibrium constraints. We instantiate this algorithmic framework to compute a high-welfare approximate CCE in aggregative games.
In the interest of space, the theorems presented below only address CCE; analogous results hold for CE. Also, details of the above mentioned positive results appear in [Barman and Ligett 2015].

2. PROBLEM DEFINITIONS AND RESULTS

We consider games with $n$ players and $m$ actions per player. Write $A_p$ to denote the set of actions available to the $p$th player and $A$ to denote the set of action profiles, $A := \prod_p A_p$. The (normalized) utility of player $p$ is denoted by $u_p : A \to [0, 1]$. We use $w(a)$ to denote the welfare of action profile $a \in A$, $w(a) = \sum_p u_p(a)$. Given a distribution $x$ over the set $A$, we use $u_p(x)$ and $w(x)$ to denote the expected utility of player $p$ and the expected welfare, respectively.

**Definition 2.1** $\varepsilon$-Coarse Correlated Equilibrium. A probability distribution $x$ over the action profiles $A$ is said to be an $\varepsilon$-coarse correlated equilibrium if for every player $p$ and every action $i \in A_p$ we have $\sum_{a \in A} [u_p(i, a_{-p}) - u_p(a)]x(a) \leq \varepsilon$.

The definition of a CCE is obtained by setting $\varepsilon = 0$ in the above inequality. Next we establish the hardness of the decision problem NT which is formally defined below. Note that acronym NT stands for nontrivial.

**Definition 2.2** NT. Let $\Gamma$ be an $n$-player $m$-action game with a succinct representation. NT is defined to be the problem of determining whether $\Gamma$ admits a CCE $x$ such that $w(x) > w(x')$. Here $x'$ denotes the worst CCE of $\Gamma$, in terms of social welfare $w$.

To prove the subsequent theorem we start with a succinct $n$-player $m$-action game $G$ from a class of games in which computing a welfare-maximizing action profile is NP-hard. We reduce the problem of determining an optimal (welfare maximizing) action profile, say $a^*$, in $G$ to solving NT in a modified succinct game $G'$, which is obtained by providing each player in $G$ with an additional action. Intuitively, the reduction works by ensuring that $a^*$ is an optimal CCE in $G'$ and any CCE with welfare better than the worst possible one can in fact be used to determine $a^*$; see [Barman and Ligett 2015] for a complete proof.

**Theorem 2.3.** NT is NP-hard in succinct multiplayer games.

The hardness of NT implies that, under standard complexity-theoretic assumptions, it is impossible to efficiently compute a CCE that achieves a nontrivial approximation guarantee in terms of social welfare.

We also establish the hardness of computing an approximate CCE that has high social welfare. Specifically, we consider the problem of computing a $\frac{1}{2n^3}$-CCE with welfare $(1 + \frac{1}{n})$ times better than the welfare of the worst CCE. Note that there exist regret-based dynamics (cf. [Young 2004]) that converge to the set of $\varepsilon$-CCE in time polynomial in $1/\varepsilon$. Therefore, in polynomial time we can compute a $\frac{1}{2n^3}$-CCE. But, as the following theorem shows, it is unlikely that we can efficiently find a $\frac{1}{2n^3}$-CCE with any nontrivial welfare guarantee. Below we use acronym ANT to refer to the problem of determining an approximate CCE with nontrivial welfare.

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1Multiple examples of such classes of games are given in [Papadimitriou and Roughgarden 2008].

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Definition 2.4 ANT. Let $\Gamma$ be an $n$-player $m$-action succinct game. ANT is defined to be the problem of determining whether there exists a $\frac{1}{\sqrt{n^2}}$-CCE $x$ in $\Gamma$ such that $w(x) \geq (1 + \frac{1}{n^2})w(x')$, where $x'$ denotes the worst CCE of $\Gamma$, in terms of social welfare $w$.

Theorem 2.5. In succinct multiplayer games, ANT is NP-hard under randomized reductions.

Hardness results analogous to Theorem 2.3 and 2.5 hold for CE as well. Note that a classical interpretation of a CE is in terms of a mediator who has access to the players’ payoff functions and who draws outcomes from a correlated equilibrium’s joint distribution over player actions and privately recommends the corresponding actions to each player. The equilibrium conditions ensure that no player can benefit in expectation by unilaterally deviating from the recommended actions. Therefore, the problem we study here is exactly the problem that a mediator faces if she wishes to maximize social welfare.

In addition, it is shown in [Barman and Ligett 2015] that the above mentioned hardness results also hold specifically for potential games. This follows from the fact that the reduction used in the proofs of Theorem 2.3 and Theorem 2.5 gives us a potential game.

3. OPEN PROBLEMS

Overall, this work establishes a notable dichotomy: while one can efficiently compute an approximate CE and CCE, one cannot provide any nontrivial welfare guarantees for the resulting equilibria, unless $P = NP$. A number of interesting questions follow.

A relevant extension is to show that such hardness results hold even in specific classes of succinct games such as polymatrix games and graphical games. In terms of positive results, it would be interesting to determine additional classes of games which (like aggregative games) admit efficient computation of high-welfare CE and CCE. Developing dynamics that quickly converge to high-welfare CE/CCE in particular classes of games also remains an interesting direction for future work.

REFERENCES


We present a protocol for eliciting dynamic beliefs from forecasters. At time $t = 0$, forecasters hold beliefs about a random variable of interest that will realize publicly at time $t = 1$. Between $t = 0$ and $t = 1$, forecasters observe private information that impacts their beliefs. We design a class of protocols that, at the outset, elicit forecasters’ beliefs about the random variable and elicit beliefs about any private information they expect to receive over time, and then elicit the private information that forecasters receive as they receive it. We show that any alternative elicitation mechanism can be approximated by protocols in our class. The information elicited can be used to solve optimally an arbitrary dynamic decision problem.

Categories and Subject Descriptors: J.4 [Social and Behavioral Sciences]: Economics

General Terms: Economics, Theory, Measurement, Design

Additional Key Words and Phrases: Scoring rules, Elicitability, Forecasting, Decision Theory

In this letter we introduce the idea and purpose of dynamic belief elicitation as explored in our working paper [Chambers and Lambert 2014]. In many economic situations, decision makers have less payoff-relevant information than do some other agents. To deal with these situations, the decision maker may delegate the decision to the better informed agents. Alternatively, the decision maker may extract beliefs from the agents and then use these beliefs to decide on the best course of action. Our paper is about the latter case; specifically, it addresses the problem of eliciting beliefs from the agents.

To illustrate, consider a simple stylized situation. The decision maker is an investor who contemplates buying shares in a risky venture. The venture, if successful, yields a payoff $B = 1000$ to the investor. If it fails, it pays nothing. The outcome of the venture will be known at time $t = 1$. The investor must decide at $t = 0$. If she buys the shares, she incurs a cost $C = 450$.

This simple setup is easy to analyze. If the investor is risk neutral without discounting, she is better off buying the shares when the expected benefit exceeds the cost, or $p \times B > C$, where $p$ is the probability that the project is successful. Note that to make the optimal decision, she only needs to assess the probability $p$. If she is unable to form such assessments, she can elicit probability beliefs from better informed forecasters using scoring rules and related methods.

In our context, a scoring rule is a function $s(\hat{p}, x) \in \mathbb{R}$ that takes as input a probability forecast $\hat{p}$ and a realization $x = 1$ if success and $x = 0$ if failure, and assigns a score to the forecaster. It is typically assumed that forecasters want...
to maximize their average score, for example when the scoring rule is used as a direct incentive scheme with scores mapping to payments or lottery tickets, or when it is used as an indirect incentive scheme that compares average scores of different forecasters over time. (Strictly) Proper scoring rules ensure that it is a strict best response for the forecaster to provide his true belief, meaning that $E[s(p, X)] > E[s(\hat{p}, X)]$ whenever $p = \Pr(X = 1)$ and $\hat{p} \neq p$, where $X$ is the random variable associated with the project outcome. For example, the Brier score $s(\hat{p}, x) = 1 - (\hat{p} - x)^2$ is proper [Brier 1950]. More generally, scoring rules elicit information about the uncertainty of random variables. Such information can be used to solve optimally a static decision problem, in which the decision maker must choose among a set of alternatives at one particular instant.

However, in many situations of practical interest, decisions can be made at different times. These decision problems are dynamic. In such situations, assessing the uncertainty of payoff-relevant random variables at every decision stage is not enough. To decide optimally, the decision maker must know how and when uncertainty unravels over time. To illustrate this point, let us revisit our investor story.

The investor can now decide to delay the decision to invest to time $t = \frac{1}{2}$. However, if she invests at that time, she loses the offer made to her at $t = 0$: her cost of participation increases from 450 to 600. By adding the option to invest at a later time, the optimal time-0 decision—invest or delay—depends not only on her current belief about chances of success, but also on how she expects her belief to change over time. Consider one instance where the investor believes at $t = 0$ that the project has 50% chances of success, and does not expect to receive any further information by $t = \frac{1}{2}$. With such beliefs, the investor is best off investing at $t = 0$, as she expects to make $1000 \times 0.5 - 450 = 50$ by investing right away (better than delaying the decision to invest or not investing at all). In another instance, suppose that the investor continues to believe in 50% chances of success at $t = 0$, but now expects to conduct further investigation by $t = \frac{1}{2}$. The outcome of the investigation is a signal at $t = \frac{1}{2}$, taking values good news or bad news, each occurring with equal probability. If good news was to arrive, the investor expects to revise her assessment to 80% chances of success, while in case of bad news, she would update her assessment to 20%. With such beliefs, if she delays decision and invest at $t = \frac{1}{2}$ but only upon observing good news, she makes on average $0.5 \times (1000 \times 0.8 - 600) = 100$, which is better than investing right away. So what does this simple analysis tell us? When dealing with dynamic decision problems, beliefs about future beliefs (or equivalently beliefs about future information) matter. This property is common to most dynamic decision problems—unless decisions are reversible, beliefs over future beliefs impact optimal decisions.

If the investor is not well informed, she will want to extract beliefs from well informed forecasters. Scoring rules and related methods can help her extract static beliefs, i.e., beliefs about the chances of success, at both decision stages. However they fail to extract a valuable piece of information: the forecaster’s belief at $t = 0$ that concerns his future belief at $t = \frac{1}{2}$. Without such information, the investor cannot make decisions optimally.

Our paper extends the theory of scoring rules to the time dimension, allowing to
extract such information. To fix ideas, consider a forecaster who is about to form a belief at time $t = \frac{1}{2}$ about success of the project ($X = 1$). Denote by $P$ this belief. Note that $P$ is a random variable, because the forecaster generally does not know at the outset what his belief will be later on. At $t = 0$, the forecaster forms a prior about his future belief, captured by a cumulative distribution $F$ about $P$.

We want to design a scoring function $s(\hat{F}, \hat{p}, x)$ taking as input a report of belief over beliefs, $\hat{F}$, made at time 0, a report of the posterior belief $\hat{p}$ at time $\frac{1}{2}$, and a realization $x$, so that to maximize his expected score, the forecaster must provide his true prior belief over beliefs, $F$, at time 0, and his true posterior belief over $X$ at time $\frac{1}{2}$. Our main results show that such scoring functions exist and show how to construct them; in many cases an analytic expression can be found. Here for example we can use the following scoring function:

$$s(\hat{F}, \hat{p}, x) = \int_{\hat{p}}^{1} \left(1 + \int_{0}^{\alpha} \hat{F} d\alpha\right)(\alpha - x) d\alpha - \frac{1}{2} \int_{0}^{1} \left(\int_{0}^{\alpha} \hat{F} d\alpha\right)^{2} d\alpha.$$ 

Knowing $F$ and the realization of $P$ turns out to be exactly what is needed to solve optimally any two-period decision problem where the only payoff-relevant variable is $X$. Solving a three-period decision problems requires beliefs over beliefs over the final posterior belief of $X$, and so forth.

Our framework includes this instance but is significantly more general. The forecaster may receive high-dimensional private information at multiple times (even continuously or at random times) before the realization of the public random variable of interest, which itself can be high-dimensional. We design a family of protocols that elicit the entire belief structure of the forecaster at the outset, and the updated structure whenever the forecaster observes new private information.

A common approach in the mechanism design and scoring rules literatures relies on convex analysis, making use of the fact that payoffs or scores are the subdifferentials of convex value functions [McCarthy 1956; Savage 1971; Rochet 1985; Gneiting and Raftery 2007]. In our framework, the high dimensionality of the object being communicated and the presence of private information makes this approach less practical. Instead, we apply an idea that originates from Allais and relies on the use of randomization devices [Allais 1953]. Allais applied his technique to elicit preferences of an individual over pairs of choices in the context of revealed preferences. In the context of probability elicitation, similar ideas have been used by [Becker et al. 1964] and [Matheson and Winkler 1976].

Each protocol in our setup is described by a pair $(D, \mu)$, where $D$ is a collection of “simple” dynamic decision problems and $\mu$ is a probability measure over $D$. Every such protocol works as follows: at the outset, the elicitor selects a decision problem $d^{*} \in D$ at random according to $\mu$. The forecaster knows $D$ and $\mu$, but does not know $d^{*}$, which the elicitor keeps secret. At time 0, the forecaster is asked to announce his belief structure, and then asked to send updates at subsequent times whenever he receives new information. The elicitor solves the decision problem $d^{*}$ optimally according to the information communicated by the forecaster over time. Upon realization of the public random variable, the elicitor collects the overall payoff from $d^{*}$ and transfers it to the forecaster (such a payoff need not be monetary, it can be interpreted as a score). For example, the scoring function $s(\hat{F}, \hat{p}, x)$ displayed above
corresponds to the expected payoff for a particular \((D, \mu)-\)protocol. In our more general setup, we use a class \(D\) whose elementary decision problems correspond to making choices when holding a hierarchy of option menus to exercise with random deadlines. 

A key tradeoff in the design of these protocols is that the collection \(D\) should be large enough to recover the entire belief structure—a complex, infinite dimensional object—while \(D\) should also be small enough for the existence of an associated \(\mu\) that induces a strict best response. Our first main result shows that, for a carefully selected class \(D\), and with appropriate probability measures \(\mu\), the protocol is “proper” in that it elicits dynamic beliefs of the forecaster as a strict best response at every instant. Our second main result shows that any protocol that elicits dynamic beliefs delivers scores or payoffs that can be approximated arbitrary closely using some \((D, \mu)-\)protocol in our class.

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Approximating the Nash Social Welfare with Indivisible Items

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In this letter we briefly discuss our main result from [Cole and Gkatzelis 2015]. Given a set of indivisible items and a set of agents having additive valuations, our goal is to allocate the items to the agents in order to maximize the geometric mean of the agents’ valuations, i.e., the Nash social welfare. This problem is known to be NP-hard, and our main result is the first efficient constant-factor approximation algorithm for this objective.

Categories and Subject Descriptors: F.2.2 [Analysis of Algorithms and Problem Complexity]: Nonnumerical Algorithms & Computations on discrete structures
General Terms: Algorithms, Theory, Economics
Additional Key Words and Phrases: Nash social welfare, approximation algorithms, fair division

1. INTRODUCTION

Imagine that we need to allocate a collection of \( m \) indivisible items in a fair and efficient manner among a set of \( n \leq m \) agents having additive valuations. That is, each agent \( i \) has a non-negative value \( v_{ij} \) for each item \( j \), and her value for an allocation \( x \) that assigns to her some bundle of items \( B_i \), is \( v_i(x) = \sum_{j \in B_i} v_{ij} \).

Since the items are indivisible, an allocation \( x \) assigns each item to a single agent, and different allocations lead to different distributions of value \( v_i(x) \) among the agents. How should we allocate the items to ensure a distribution of value that balances fairness and efficiency?

Maximizing efficiency is commonly associated with maximizing the utilitarian social welfare objective, i.e., the total value across the agents: \( \max_x \sum_i v_i(x) \). It is easy to verify though that this objective can often lead to some agents being allocated nothing, hence neglecting fairness considerations. On the other extreme, maximizing fairness is often associated with maximizing the egalitarian social welfare objective, i.e., the minimum value across all agents: \( \max_x \min_i v_i(x) \). But, just as the utilitarian objective disregards fairness, the egalitarian objective can lead to really inefficient outcomes, allocating many items to hard to satisfy agents.

A well-studied objective that lies between these two extremes is the Nash social welfare (NSW), which maximizes the geometric mean of the agents’ values, i.e.,

\[
\max_x \left( \prod_i v_i(x) \right)^{1/n}
\]

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The Nash social welfare objective goes back to the fifties [Nash 1950; Kaneko and Nakamura 1979], and it satisfies several appealing properties [Moulin 2003]. As we already mentioned, it strikes a natural balance between fairness and efficiency since maximizing the geometric mean favors more uniform value distributions, but without sacrificing the utilitarian social welfare too much. Furthermore, this objective is scale-free: its optimal allocation $x^*$ is independent of the scale of each agent’s valuations, so choosing the desired allocation does not require interpersonal comparability of the individual’s preferences. This property is particularly useful in settings where the agents are not paying for the items that they are allocated, in which case the scale in which their valuations are expressed may not have any real meaning (if they were paying, $v_{ij}$ could be interpreted as the amount that agent $i$ is willing to pay for item $j$). These are properties that neither the utilitarian nor the egalitarian objective satisfy.

2. PRELIMINARIES

Given a set $M$ of $m$ items and a set $N$ of $n$ agents ($n \leq m$) with additive valuations, our optimization problem can be expressed as the following integer program, IP:

$$\begin{align*}
\text{maximize:} & \quad \left( \prod_{i \in N} u_i \right)^{1/n} \\
\text{subject to:} & \quad \sum_{j \in M} x_{ij} v_{ij} = u_i, \quad \forall i \in N \\
& \quad \sum_{i \in N} x_{ij} = 1, \quad \forall j \in M \\
& \quad x_{ij} \in \{0, 1\}, \quad \forall i \in N, j \in M
\end{align*}$$

Note that, for any fixed number of agents $n$, the objective of IP is equivalent to maximizing $\sum_{i \in N} \log u_i$. As a result, our optimization problem can be expressed as a convex integer program. In fact, the fractional relaxation of this convex program is an instance of the well studied Eisenberg-Gale program [Eisenberg and Gale 1959], whose solutions can be interpreted as the equilibrium allocation for the linear case of Fisher’s market model [Nisan et al. 2007, Chapter 5]. In this model, each agent has a certain budget, and she is using this budget in order to buy fractions of the available items. Although the agents in our setting are not using money, this market-based interpretation of the optimal solution of IP’s fractional relaxation provides some very useful intuition, which we use in order to design our approximation algorithm.

In the Fisher market corresponding to our problem the items are divisible and the valuation of agent $i$ who is receiving a fraction $x_{ij} \in [0, 1]$ of each item $j$ is $\sum_{j \in M} x_{ij} v_{ij}$. Each agent has the same budget of, say, $1$ to spend on items and each item $j$ has a price $p_j$. If agent $i$ spends $b_{ij}$ on an item whose price is $p_j$, then she receives a fraction $x_{ij} = b_{ij}/p_j$ of that item (there is one unit of each item, so $\sum_{i \in N} b_{ij} \leq p_j$). A vector of item prices $p = (p_1, \ldots, p_m)$ induces a market equilibrium if every agent is spending all of her budget on her “optimal” items given these prices, and the market clears, i.e., all of the items are allocated fully. The “optimal” items for agent $i$, given prices $p$, are the ones that maximize the ratio $v_{ij}/p_j$, also known as the maximum bang per buck (MBB) items.
Example 2.1. The graph of Figure 1 comprises 4 vertices on the left (the agents) and 5 vertices on the right (the items). The valuations of each agent appear to the left of her vertex, so the first agent values only the first item. The second agent values this item at 15 and the second item at 2, while she has no value for the other items, etc. The price to the right of each item’s vertex corresponds to its price in the market equilibrium for this problem instance, and the directed edges point from each agent to its MBB items at the given prices. To verify that these are market clearing prices, Figure 2 shows how each agent could spend all of her budget on MBB items. The label of each edge (i, j) denotes how much agent i is spending on item j. Since the total money spent on each item is exactly equal to its price, all of the items are fully allocated, which implies that this equilibrium is the optimal solution of the fractional relaxation of IP: the first three agents each get a 1/3 fraction of the first item, and the fourth agent gets all of the other items.

3. OUR MAIN RESULT

Let $x^*$ denote the integral allocation that maximizes the Nash social welfare. Our main result in [Cole and Gkatzelis 2015] is the Spending-Restricted Rounding (SRR) algorithm, an efficient algorithm which computes an integral allocation $\tilde{x}$ guaranteed to be within a constant factor of the optimal one. The best previously known approximation factor was $\Omega(m)$ [Nguyen and Rothe 2014].

Theorem 3.1. The allocation $\tilde{x}$ computed by the SRR algorithm satisfies

$$\left( \prod_{i \in N} v_i(x^*) \right)^{1/n} \leq 2.889 \left( \prod_{i \in N} v_i(\tilde{x}) \right)^{1/n}.$$

Since we can solve the fractional relaxation of IP using the Eisenberg-Gale program, a standard technique for designing an approximation algorithm would be to take the fractional allocation and “round” it in an appropriate way to get an integral one. The hope would be that the fractional allocation provides some useful information regarding what a good integral allocation should look like, as well as an upper bound for the geometric mean of $x^*$. Unfortunately, we show that the integrality gap of IP, i.e., the ratio of the geometric mean of the fractional solution and that of $x^*$, is $\Omega(2^m)$, which implies that the fractional solution of IP cannot be used for proving a constant-factor approximation guarantee using standard techniques.
To circumvent the integrality gap, we introduce an interesting new constraint on the fractional solution. In particular, we restrict the total amount of money spent on any item to be at most $1, i.e., at most the budget of a single agent. For any item \( j \), the solution needs to satisfy \( \sum_{i \in N} x_{ij}p_j \leq 1 \); a constraint which combines both the primal \( (x_{ij}) \) and the dual \( (p_j) \) variables of the Eisenberg-Gale program.

**Definition 3.2.** A spending-restricted (SR) outcome is a fractional allocation \( x \) and a price vector \( p \) such that every agent spends all of her budget on her MBB items at prices \( p \), and the total spending on each item is equal to \( \min\{1, p_j\} \).

**Example 3.3.** In the unrestricted equilibrium of the instance considered in Example 2.1, the price of the highly demanded first item was $3, and three agents were spending all of their budgets on it. In a spending-restricted outcome this would not be acceptable, so the price of this item would need to be increased further until only the first agent, who has no other alternative, is spending her budget on it. The prices and the spending of the SR outcome can be seen in Figure 3. Note that, unlike the fractional solution of the unrestricted market equilibrium, this fractional solution reveals more information regarding the preferences of Agents 2 and 3.

Our SRR algorithm begins by computing an SR allocation \( x \) and prices \( p \) and then appropriately allocates each item to one of the agents who is receiving some of it in \( x \). Each one of the items \( j \) with a price \( p_j > 1/2 \) is assigned to a distinct agent, while the rest of the items are allocated in a way that ensures a balanced distribution of value. To prove the approximation guarantee, we also show that the SR outcome implies an interesting upper bound for the optimal Nash social welfare.

<table>
<thead>
<tr>
<th>Agents</th>
<th>Items</th>
<th>Prices</th>
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<td>$1</td>
<td>$10</td>
</tr>
<tr>
<td>[15,2,0,0,0] 2</td>
<td>$1</td>
<td>$4/3</td>
</tr>
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<td>$2/3</td>
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</tr>
<tr>
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<td>$1/3</td>
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</tr>
<tr>
<td>[3,2,1,1,1] 5</td>
<td>$2/3</td>
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Fig. 3: The spending-restricted outcome.

4. CONCLUSION
Allocating indivisible items in a fair and efficient manner is a long standing problem that has received a lot of attention. Our algorithm provides a non-trivial approximation guarantee with respect to a well motivated objective that strikes a natural balance between fairness and efficiency. In designing this algorithm we use a fractional allocation that satisfies a novel constraint motivated by a market-theoretic interpretation of the fractional relaxation. In particular, this constraint restricts the amount of money that can be spent on any given item, thus forcing some agents to spend on less demanded items, thereby revealing useful information regarding the agents’ preferences. This constraint combines both the primal and the dual variables of the Eisenberg-Gale convex program. How to compute the allocation that satisfies it is not obvious, but we show that it can, in fact, be computed in polynomial time.
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Incentives are more likely to elicit desired outcomes when they are derived based on accurate models of agent behavior. A growing literature—from behavioral economics, as well as online user studies—suggests, however, that people do not always behave like standard economic agents in a variety of environments, both online and offline. What consequences might such differences have for the optimal design of these environments?

In this note, we summarize our results from Easley and Ghosh [2015] which explores this question of behavioral design—how departures from standard economic models of agent behavior affect mechanism design—via the problem of the optimal choice of contract structure in crowdsourcing markets where workers make decisions according to prospect theory preferences [Kahneman and Tversky 1979] rather than classical expected utility theory: we show that a principal might indeed choose a fundamentally different kind of mechanism—an output-contingent contest versus a ‘safe’, output-independent, fixed-payment scheme—and do better as a result, if he accounts for deviations from the standard economic models of decision-making that are typically used in theoretical design.

Categories and Subject Descriptors: J.4 [Social and Behavioral Sciences]: Economics
General Terms: Mechanism design, Behavioral economics, Game theory, Crowdsourcing

1. INTRODUCTION

The vast range of systems with outcomes that depend on the choices made by economic agents has led to a rich and large literature on mechanism design, which regards designing incentives so that agents make choices resulting in ‘good’ outcomes. Incentives are more likely to elicit desired outcomes when they are derived based on accurate models of agent behavior. A growing literature, however, suggests that people do not always behave like the standard economic agents in the mechanism design literature. Can such differences have significant consequences for the optimal design of incentive mechanisms?

Ghosh and Kleinberg [2014] investigate this question of ‘behavioral’ mechanism design in the context of the optimal design of contests, a commonly used incentive mechanism in crowdwork, for simple agents: unlike agents who strategically choose the quality of their submission to the contest, simple agents only strategize about whether or not to participate in a contest, but not over the quality of their submission. But why, or rather when, a contest? In Easley and Ghosh [2015], we explore another dimension of deviation from the standard economic models of behavior via this question: what kind of incentive structure should a principal in a crowdsourcing environment use—a contest, a fixed payment scheme, and so on—and how does the answer to this question depend on how agents make decisions under uncertainty?
Decision making under uncertainty: Prospect theory. Many environments to which the mechanism design literature has been applied involve risky choice—agents who must make decisions between a set of choices, each of which yields associated payoffs with some probability. The standard economic model for choice under uncertainty is expected utility theory [Neumann and Morgenstern 1944; Savage 1954], whereby agents make choices by comparing the expected utility of outcomes resulting from each possible choice. A substantial body of work in behavioral economics, however, demonstrates that individuals display systematic deviations from the standard expected utility model, including overweighting low-probability events and under-weighting high-probability ones, as well as displaying a greater disutility for losses compared to utilities from a gain of the same amount.

Prospect theory, introduced in Kahneman and Tversky [1979] and further refined to cumulative prospect theory in Tversky and Kahneman [1992], is a descriptive model that describes much empirically observed decision-making behavior more accurately than expected utility theory, and can explain a wide range of experimentally documented behavioral biases, including the status quo bias and endowment effects. One of the best-known achievements of behavioral economics, prospect theory led to the award of the 2002 Nobel prize in economics to Kahnemann for his “...contributions to behavioral economics, in particular the development of cumulative prospect theory”.

Yet the literature on mechanism design almost uniformly models agents as making choices according to the tenets of the classical expected utility theory. While the expected utility model may accurately model choice-making for the applications addressed by classical mechanism design—such as auction theory, where large firms, which are possibly exempt from these behavioral biases, are the decision-making agents—a number of newer online systems such as crowdsourcing and labor markets, peer-to-peer economies, and online auctions with small bidders, involve individual agents to whom these behavioral biases do apply. What consequences might such biases in decision-making have for the design of these environments?

2. OPTIMAL CONTRACT STRUCTURE IN CROWDSOURCING MARKETS

In Easley and Ghosh [2015], we explore this question in the context of principal-agent problems in online labor and crowdsourcing markets. Consider a principal with a single task that he wants to complete by hiring workers via an online platform. The principal has many options available in terms of what incentive structures he can use, such as those supported by various online crowdsourcing platforms—fixed-price contracts on platforms such as Amazon Mechanical Turk (MTurk) or their analog with price discovery, an auction to determine the payment for the task as on oDesk, as well as contests of various kinds on platforms like TopCoder or Kaggle. How should the principal choose between these various kinds of incentive structures—and does the answer to his question depend on whether his population of potential workers behaves according to the classical model of expected utility theory, or whether they deviate, behaving instead according to prospect theory?

We consider a simple model with a principal with a single task and a population
of potential workers or agents, each of whom incurs an opportunity cost $c$ to undertaking the task. Agents make endogenous participation choices, but quality is exogenous\footnote{While the central issue in most of the principal-agent literature centers around effort elicitation—providing the right incentives for workers to elicit high effort (as a means to high output quality) in situations with imperfectly observable effort, incentives for participation, rather than quality, are the central issue in several online labor and crowdsourcing settings. This is due to differences in both the nature of available monitoring tools (various online labor platforms (including oDesk) offer virtual office applications for monitoring of workers) as well as intrinsic motivation for the task at hand: a pattern discovered by multiple experimental studies investigating worker behavior in online crowdsourcing platforms is that a change in the level of offered incentives affects participation rates, but not the degree of effort that workers exert conditional on participation (Mason and Watts [2009], Jeon et al. [2010], Kraut and Resnick [2012]).} i.e., participation, but not quality, is a strategic choice: each agent who strategically decides to undertake the task produces output with quality $q_i \sim F(q)$ at a cost $c$. Agents evaluate each possible action choice according to the value of the corresponding prospect it induces: a prospect which offers payoffs $x_k$ with probabilities $p_k$ is evaluated according to $\sum_k u(x_k)p_k$, where $u$ is the utility function for payoffs and $\pi$ the decision weights that map probabilities into the weights with which they appear in the evaluation of the prospect.

To allow addressing the broader question of what kind of incentive contract the principal might prefer—as opposed to optimizing within a specific class of contracts (as, for instance, in an optimal contest design problem)—we use a general model for contracts encompassing a broad variety of incentive schemes, including those observed on major crowdsourcing platforms such as fixed prices to a fixed number of workers (MTurk), base payments with bonuses (oDesk), and open-entry tournaments with prizes to a small number of contestants (Kaggle, Topcoder).

### 2.1 Optimal contracts: Expected utility maximizers

We first investigate the nature of the optimal contract that a principal should use when the population of workers behaves according to the standard model of expected utility theory—namely, agents with decision weights that equal probabilities, $\pi(p) = p$, and risk-neutral or risk-averse preferences corresponding to concave functions $u(x)$ (risk neutrality corresponds to the special case of linear $u$). We show that the optimal contract, when agents are expected utility maximizers, always takes the form of a fixed price mechanism where the principal pays an optimally chosen number of agents a fixed price\footnote{This price is $c$; if the value of $c$ is private information to agents it can be elicited using an auction mechanism} independent of output quality (Theorem 3.1 in Easley and Ghosh [2015]). The proof hinges on agents' not being risk-seeking, and proceeds by arguing that for any total expected payout $W$ the principal might make, that payout $W$ incentivizes the highest participation when it is disbursed as fixed payments rather than via any output-contingent scheme.

### 2.2 Contract design: Prospect theory preferences

We then ask whether a principal would ever choose a fundamentally different contract for agents with prospect theory preferences than that for expected utility maximizing agents. To address this question, we compare the incentives created by contests, which are widely used in a variety of crowdsourcing applications, against...
the fixed payment contracts which are optimal for expected utility maximizing agents: for a given population of agents with preferences described by some value and decision weight functions, are there any values of the total amount the principal wants to spend on his task, $W$, for which he is better off using a winner-take-all contest with prize $W$ instead of a fixed-payment contract with total payment $W$?

Contests are a very different kind of contract than fixed-payment schemes, in a qualitative sense: in contrast with fixed-payment schemes, an agent’s reward in a contest is output-contingent, depending not only on her own output quality $q_i$ but also on the draws of all other participants in the contest. So when might a contest, which is inherently a riskier prospect, provide stronger incentives for participation than the fixed-payment contract with its output-independent payoffs?

**Contests and prospect theory preferences.** If agents used actual outcome probabilities as their decision weights, Theorem 3.1 in Easley and Ghosh [2015] can be extended to show that a principal would never use output-contingent payments for any risk-neutral or risk-averse population of agents, no matter what the nature of his task. With prospect theory preferences, however, a small chance of winning a large prize might contribute more than its ‘true’ share of utility due to the overweighting of small probabilities, potentially creating a larger perceived payoff for the same expected payout to the principal.

To understand the intuition behind why contests might provide stronger incentives with overweighting of small probabilities, assume for a moment that an agent will ‘win’ the prize $W$ if her quality draw beats an exogenous threshold $q^*$ (imagine, for example, that the principal derives a value $W$ if the output quality is greater equal the current state of the art $q^*$, and no value otherwise). This event $q_i \geq q^*$ has probability $\epsilon = 1 - F(q^*)$, and the perceived payoff to the agent from the contest is $u(W)\pi(\epsilon)$. A different payment scheme that has the same expected payout to the principal is the following: rather than wait to see if the output quality exceeds $q^*$ before making the payment, the (risk-neutral) principal commits to paying his expected value, which is $W\epsilon$, yielding payoff $u(W\epsilon)$ to the worker.

To develop intuition, imagine that the agent is risk-neutral so that $u(x) = \alpha x$; the agent then perceives respective payoffs $\alpha W\pi(\epsilon)$ and $\alpha W\epsilon$ from the output-contingent contest and fixed-payment schemes respectively. Now if the chance of winning the contest $\epsilon = 1 - F(q^*)$ is small enough for the overweighting of small probabilities to become relevant, the agent overweight the outcome of winning $W$ ($\pi(\epsilon) > \epsilon$), leading to a larger perceived benefit ($\alpha W\pi(\epsilon)$) from the contest than from the fixed-payment scheme ($\alpha W\epsilon$), and correspondingly stronger incentives for participation. Of course, this reasoning is oversimplified—it ignores risk aversion, corresponding to non-linear $u$; more importantly, it ignores the fact that the threshold $q^*$ that an agent’s output $q_i$ needs to beat in a contest is not exogenous but rather is endogenously determined in equilibrium by the choices of all other workers: $q^*(N^*) = \max_{j=2,\ldots,N^*} \{q_j\}$ where $N^*$ is the equilibrium number of participants in the contest. However, it illustrates why overweighting of small probabilities can skew incentives in favor of ‘riskier’ gambles over ‘safer’ prospects, under the right circumstances.\(^3\)

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\(^3\)This same reasoning can be used to understand why an output-dependent bonus payment, while
Despite the overweighting of small probabilities, however, a contest nonetheless does not always dominate a fixed-payment scheme: if the prize from winning the contest is not much larger than the cost of undertaking the associated task, few enough agents participate in equilibrium so that the probability of winning the prize becomes too large to be overweighted. This occurs even with risk-neutral agents; when agents display risk and loss aversion, the overweighting of probabilities—favoring the contest—needs to also compensate for this aversion to uncertainty if a contest is to dominate a fixed-payment contract.

Comparing contests against fixed-payment contracts. We first derive a necessary and sufficient condition under which a contest will dominate the fixed-payment contract for general preferences \( (u, \pi) \). Our first application of this result shows when this condition cannot be satisfied: we show that, for any prospect theory preferences, a principal who does not have a large enough budget to spend on the task should not conduct a contest, even for agents who overweight small probabilities—for such small prizes, risk and loss aversion beat out the benefits from overweighting of small probabilities.

More specific structure on the preference functions is required to say when the condition for a contest to dominate a fixed-payment contract can be satisfied. For this, we use the functional forms for \( u \) and \( \pi \) from the literature on econometric estimation of the cumulative prospect theory model [Tversky and Kahneman 1992; Bruhin et al. 2010]. Since these functional forms are derived from fitting the prospect theory model to extensive experimental data, they yield the best answer we can hope to have to our question—without conducting a measurement of the functions \( u \) and \( \pi \) in the specific marketplace of interest—for ‘real’ population preferences \( u, \pi \).

Our main results, Theorems 4.4 and 4.5 in Easley and Ghosh [2015], together with the estimated parameter values from [Tversky and Kahneman 1992; Bruhin et al. 2010], provide an affirmative answer to our central question: we find that if the parameter describing the degree of risk aversion in \( u \) is larger than the parameter describing the degree of probability weighting in \( \pi \) (as is the case in the estimated model), a contest will eventually dominate a fixed payment scheme for large enough total payouts \( W \). To the extent that these parameters indeed describe the decision-making behavior of agents in online crowdsourcing environments, therefore, a principal who values the output from crowdsourcing his task sufficiently highly (compared to the cost to a worker to produce that output) might indeed choose a different kind of mechanism, and do better as a result, if he accounts for deviations from the standard economic model of expected utilities that are typically used in theoretical design.

3. CONCLUSION AND FURTHER DIRECTIONS

Easley and Ghosh [2015] explores the idea that behavioral biases can indeed make a fundamental difference to a principal’s choice of mechanism, via the problem of strictly dominated for risk-averse agents with linear weighting of probabilities, may similarly improve incentives if agents overweight small probabilities for the same expected payout to the principal.
optimal contract structure in online crowdsourcing markets with participation-only strategic choices. While our model is a reasonable description of the specific crowdsourcing markets we use to motivate it, it is deliberately chosen to be the simplest, most parsimonious possible model that illustrates this idea—that deviations from expected utility theory, according to prospect theory preferences, are potentially quite significant to theoretical design—in a realistic setting. This means that there are a number of complexities of crowdsourcing markets that our model does not capture, presenting promising, and important, directions for further work, most interesting amongst which is when agents have heterogeneous preferences.

More generally, however, the question of how choice according to prospect theory versus expected utility affects equilibrium analysis and optimal design extends to domains beyond principal-agent problems in online labor: principal-agent problems are merely one amongst many applications of mechanism design where agents make decisions under uncertainty. A number of other domains to which mechanism design applies, especially those motivated by online systems where individuals rather than large firms are decision makers—such as online auctions with small bidders, or reputation systems in peer-to-peer economies—depend on the decisions of individual agents to whom these behavioral biases do apply. The issue of whether agents choose according to prospect theory versus expected utility preferences is a fundamental component of equilibrium analysis and the optimal design question in each of these environments, and as such, lays open a vast field of problems for further exploration.

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A New Approach to Measure Social Capital using Game-Theoretic Techniques

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Although the notion of social capital has been extensively studied in various bodies of the literature, there is no universally accepted definition or measure of this concept. In this article, we discuss a new approach for measuring social capital which builds upon cooperative game theory. The new approach not only turns out to be a natural tool for modeling social capital, but also captures various aspects of this phenomenon that are not captured by other approaches.

Categories and Subject Descriptors: J.4 [Social and Behavioral Sciences]: Economics

General Terms: Social capital, Social computing, Cooperative game theory
Additional Key Words and Phrases: Social capital, Shapley value, network centrality

1. INTRODUCTION

Social capital is one of the fundamental concepts in sociology [Coleman 1990]. Intuitively, we can think of social capital as the ability of individuals to profit from their position in the society, i.e., from their connections in the social network. Although it has been extensively studied, there is no single definition or measure that captures all facets of this concept. In this article, we comment on recent works that build upon cooperative game theory to develop advanced measures of social capital. According to Borgatti et al. [1998], social capital is understood in the literature in two conceptually different ways, called group and individual social capital. The former interprets social capital as the quality or performance of a given group of individuals in a social network [Putnam 1995]. The latter interprets social capital as the value of an individual's social connections, which are seen as potential
sources of information, power, or opportunities [Burt 1992]. Moreover, Borgatti et al. argue that the analysis of individual social capital can be done externally, while for group social capital it can be done either internally and externally. For instance, the internal analysis of group social capital would consider any given group as a universe that consists of connections, norms and trust, but nothing outside the group is modeled. In contrast, external analysis of group social capital would focus on how the members of the group are connected to the outside world. Since social capital depends on how the different actors are positioned in their social network, existing measures of social capital are usually built upon methods pertaining to social network analysis [Borgatti et al. 1998]. The following are a few such measures, categorized according to the type of social capital they capture:

—(A): Individual-centric, External Measures—those focus on the connections of an individual. Some key examples include the ego-network measures [Burt and Minor 1983], the structural hole measures [Burt 1992], and the standard centrality measures such as closeness, betweenness, and eigenvector centrality.

—(B): Group-centric, Internal Measures—those focus on internal connections of a group and are often referred to as internal measures for collective actors. They include the average or maximum distance and homophily [Borgatti et al. 1998].

—(C): Group-centric, External Measures—those focus on internal connections of a group and are often referred to as external measures for collective actors. Among the most important examples are group-degree (i.e., the number of the group’s neighbors), group-closeness, and group-betweenness (defined similarly) [Everett and Borgatti 1999].

Although the above measures are able to capture some aspects of social capital, they cannot capture others. Below are two particular deficiencies that we are especially interested in:

(i) While existing measures quantify each of the above types of social capital separately, none of them sheds light on the interactions between those types;

(ii) Since the aforementioned measures originate from social network analysis, they are limited to the aspects of social capital related solely to network topology.

In this article, we consider a more advanced approach to the measurement of social capital, which builds upon recent literature that uses concepts rooted in cooperative game theory to analyze social networks. We argue that this new approach is able to address both problems (i) and (ii) outlined above.

In the next section, we present the theoretical underpinnings of this approach.

2. GAME-THEORETIC CENTRALITY

In a cooperative game, players are allowed to form coalitions and share the payoff generated by cooperation. Formally, given a set of players \( N \), the payoff generated by a coalition \( C \subseteq N \) is denoted by \( \nu(C) \). There exist well-established schemes to divide the payoff attainable by all the players working together, \( \nu(N) \). The most prominent one is the Shapley value [Shapley 1953], the theoretical properties of which have been extensively studied for decades. To define the Shapley value, let us denote by \( MC(C, i) \) the marginal contribution of player \( i \) to coalition \( C \), i.e.,
MC(C, i) = ν(C \cup \{i\}) − ν(C). Then, the payoff of i according to the Shapley value is:

$$SV_i(ν) = \sum_{C \subseteq N \setminus \{i\}} \frac{|C|!(|N| - |C| - 1)!}{|N|!} MC(C, i).$$

Game-theoretic centrality borrows the above apparatus and applies it to quantify the role played by nodes in a network. To this end, the nodes are considered to be players in a cooperative game, whereby the payoff of any coalition of nodes is assumed to capture in some way the performance of these nodes in the network. By drawing this analogy, the game-theoretic payoff division schemes that evaluate the role (also interpreted as power) of a player in a coalitional game can be used to evaluate the role of nodes in the network. As an example of such an approach, Keinan et al. [2004] divided an animal’s brain into several interconnected sections that form a network. After that, certain groups of nodes (i.e., sections of the brain) were temporarily lesioned, and a payoff was assigned to each such group based on how it influenced the animal’s performance. Finally, the Shapley value was used to rank the role played by individual nodes.

Game-theoretic centrality has been advocated in various domains as a network-analysis tool, delivering insights unattainable by classical measures. In this article, we argue that this tool naturally captures various aspects of social capital. However, its use poses significant computational challenges inherited from coalitional games. In particular, given a network with a set of nodes, V, and a set of edges, E, there are $2^{|V|}$ coalitions to consider. If each of them has to be considered separately then the exact value of a game-theoretic centrality can only be computed for very small networks. Interestingly, however, it has been recently shown that certain classes of game-theoretic centrality are computable in polynomial time [Michalak et al. 2013; Szczepański et al. 2012]. We will discuss one such class in the next section.

3. THE OWEN VALUE-BASED MEASURE OF SOCIAL CAPITAL

The Owen value [Owen 1977] is an extension of the Shapley value to situations in which the players are already divided into coalitions (rather than situations where players are contemplating which coalitions to form). The relevance to our social-network context becomes apparent as soon as we think of those already-formed coalitions as communities. Similarly to the Shapley value, the Owen value quantifies the role of a player, but now taking into account the role of the community that this player belongs to. Thus, a player may be weak by herself, but if she belongs to a strong community, then her role will grow stronger according to the Owen value.

To define the Owen value given the set of communities $M = \{C_1, \ldots, C_m\}$, let us first introduce the concept of the quotient game whereby the set of players is $M$, (i.e., the players are the communities themselves), and the payoff of a coalition of players (i.e., a union of communities), $R \subseteq M$ is: $ν^Q(R) = ν(\cup R)$. We are now ready to define the Owen value of player $i \in C_j \in M$. This is:

$$OV_i(ν, M) = \sum_{R \subseteq M \setminus \{C_j\}} \frac{|R|!(|M| - |R| - 1)!}{|M|!} \sum_{C \subseteq C_j \setminus \{i\}} \frac{|C|!(|C_j| - |C| - 1)!}{|C_j|!} MC(R \cup C, i).$$

Intuitively, the computation of the Owen value can be thought of as a two-step
process. In the first step, communities play the game between themselves and receive their respective Shapley values. In the second step, the values of these communities are, in turn, divided among their members, again, in spirit of the Shapley value.

Recently, Szczepański et al. [2014] proposed the game-theoretic extension of degree centrality based on the Owen value. Specifically, given a community structure $M$, this centrality is computed for each node $v \in V$ as $OV_v(\nu, M)$, where $\nu(C) = |\text{Neighbors}(C)|$. It has an interesting interpretation as a measure of social capital. In particular, the social capital of an individual is increased (decreased) if she belongs to a community that is rich (poor) in terms of social capital. For instance, if a lawyer is not particularly well connected (i.e., has low degree), her social capital is still increased by the fact that she belongs to the well-connected community of lawyers. As such, this is the first social capital measure that evaluates individual actors both in the context of their relationship with the external world as well as the context of the external role of their community, thus reflecting types (A) and (C) of social capital (see the introduction).

Interestingly, while the Owen value is computationally challenging, Szczepański et al. [2014] showed that their measure can be computed in just $O(|V| + |E|)$ time, which makes this concept practical even for large networks.

4. SOCIAL CAPITAL AND THE MYERSON VALUE

Having presented a measure of social capital based on the Owen value, we now present yet another measure, based on the Myerson value. Generally speaking, the Myerson value is a solution concept from cooperative game theory, developed by Myerson [1977] as an extension of the Shapley value to settings where cooperation is restricted via a communication graph: only if players are connected, either directly via an edge or indirectly via intermediaries, can they effectively cooperate.

Specifically, the Myerson value of player $i$ is:

$$MV_i(\nu, G) = SV_i(\nu^G),$$

where $\nu^G$ specifies the payoffs of coalitions as follows: if coalition $C$ is connected in $G$ then $\nu^G(C) = \nu(C)$. Otherwise, if $C$ is a disconnected coalition made of $m$ components, $K_1, \ldots, K_m$, then $\nu^G(C) = \sum_{K_i} v(K_i)$.

The Myerson value has a number of desirable properties, e.g., it is the only solution concept satisfying the following two intuitive axioms: (i) “efficiency”—the entire payoff of a connected coalition is divided among its members; (ii) “fairness”—the payoffs of two players should increase (decrease) by exactly the same amount when the edge between them is added (deleted) from the communication graph.

Among the various applications of the Myerson value, Moretti et al. [2010] used it to evaluate the relevance of genes in biological networks. Here, players correspond to genes, edges correspond to interaction ties, and coalitions of genes are evaluated based on the overall magnitude of the interaction between those genes and an a priori given set of key-genes (e.g., a set of genes that are involved in a certain biological condition of interest).

González-Arangüena et al. [2011] proposed a dedicated measure of social capital that builds upon the Myerson value. In particular, the authors argue that if the communication graph was complete (i.e., if every two individuals in the society...
knew each other) then such a situation can be modeled as a standard coalitional
game, and the role of an individual can be measured using the Shapley value. On
the other hand, since in reality the communication graph is incomplete (i.e., one
is typically not acquainted with every other member of the society), the role of an
individual can be measured using the Myerson value. Based on this argument, the
authors propose to measure social capital as the difference between one’s role in
the actual social network, and her role in a hypothetical, complete network. That
is, social capital is the impact of the social-network topology on the role that an
individual plays in the society.

With the above approach, the authors measure an individual’s social capital while
considering both the internal and external social capital of her community, thus
reflecting types (A), (B) and (C) of social capital (see the introduction). More
specifically, the “efficiency” axiom of the Myerson value takes into account the
internal role of the community, by protecting the internal capital of the community
from any potential changes that may occur outside of it. On the other hand, the
“fairness” axiom implies that the members of the community are influenced by their
connections to non-members, which reflects the external capital.

While it is computationally challenging to compute the Myerson value exactly
[Michalak et al. 2013; Skibski et al. 2014], approximation techniques such as Monte
Carlo sampling have been shown to be effective (see, e.g., [Narayanam et al. 2014]
and [Moretti 2014]).

5. CONCLUSIONS AND FURTHER DIRECTIONS

We discussed a new approach for measuring social capital, based on techniques
pertaining to cooperative game theory. We argued that the new measures are able
to capture aspects of social capital that are unattainable by previous approaches.

Inspired by the ever-increasing availability of data about social interactions, an
interesting direction for future research is to expand the repository of games based
on which tractable social-capital measures can be defined. Furthermore, empirical
evaluations of these measures, coupled with the new insights brought forward by the
game-theoretic approach, may well expand on our understanding of social capital,
and social substance in general.

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A Stackelberg game is played between a leader and a follower. The leader first chooses an action, and then the follower plays his best response, and the goal of the leader is to pick the action that will maximize his payoff given the follower’s best response. Stackelberg games capture, for example, the following interaction between a retailer and a buyer. The retailer chooses the prices of the goods he produces, and then the buyer chooses to buy a utility-maximizing bundle of goods. The goal of the retailer here is to set prices to maximize his profit—his revenue minus the production cost of the purchased bundle. It is quite natural that the retailer in this example would not know the buyer’s utility function. However, he does have access to revealed preference feedback—he can set prices, and then observe the purchased bundle and his own profit. We give algorithms for efficiently solving, in terms of both computational and query complexity, a broad class of Stackelberg games in which the follower’s utility function is unknown, using only “revealed preference” access to it. This class includes the profit maximization problem, as well as the optimal tolling problem in nonatomic congestion games, when the latency functions are unknown. Surprisingly, we are able to solve these problems even though the corresponding maximization problems are not concave in the leader’s actions.

Categories and Subject Descriptors: G.1.6 [Mathematics of Computing]: Optimization

General Terms: Algorithms, Theory, Economics

1. INTRODUCTION

In a revealed preferences problem, there is an economic agent and a series of decisions made by that agent. Classically, the goal is to determine whether those decisions are rationalizable—i.e., to determine if they can be explained, ex-post, as the result of the agent maximizing some fixed utility function. The revealed preferences problem dates back to the work of Samuelson [Samuelson 1938] and there is a large literature on this topic [Mas-Colell et al. 1995; Rubinstein 2012; Varian 2006]. However, even when the agents’ decisions can be rationalized in hindsight as maximizing some fixed utility function, that utility function may not generalize to predict future decisions. This deficiency was first observed by Beigman and Vohra [Beigman and Vohra 2006] and has led to a more recent interest in studying
the problem of learning from revealed preferences [Zadimoghaddam and Roth 2012; Balcan et al. 2014; Amin et al. 2015].

In this note, we describe our work on this topic from [Roth et al. 2015]. We study a family of problems best exemplified by the problem of maximizing profit using revealed preferences:

A retailer, who sells $d$ goods, repeatedly interacts with a buyer. In each interaction, the retailer decides how to price the $d$ goods by choosing $p \in \mathbb{R}_{\geq 0}^d$, and in response the buyer purchases the bundle $x \in \mathbb{R}_{\geq 0}^d$ that maximizes her utility $v(x) - \langle x, p \rangle$, where $v$ is an unknown concave valuation function. The retailer observes the bundle purchased, and therefore his profit, which is $\langle x, p \rangle - c(x)$, where $c$ is a convex cost function. The retailer would like to set prices that maximize his profit after only a small number of interactions with the buyer.

This problem fits naturally into the well studied model of zeroth-order or bandit optimization—the retailer is trying to maximize an unknown objective function given only query access to the function. That is, he can set prices and observe the value of his objective. Unfortunately, when this problem is posed as a bandit optimization problem, the objective function is not concave. This can be seen in the following very simple instance.

**Example 1.** Consider a setting with one good ($d = 1$). The buyer’s valuation function is $v(x) = \sqrt{x}$, and the retailer’s cost function is $c(x) = x$. The buyer’s utility for buying $x$ units at price $p$ is $\sqrt{x} - xp$. Thus if the price is $p$, a utility-maximizing buyer will purchase $x^*(p) = 1/4p^2$ units. The profit of the retailer is then $\text{Profit}(p) = px^*(p) - c(x^*(p)) = 1/4p - 1/4p^2$. Unfortunately, this profit function is not concave.

Since the retailer’s profit function is not concave in the price, it cannot be optimized efficiently using generic methods for concave maximization. Surprisingly, despite this lack of concavity we show that this problem and others can be solved efficiently subject to certain mild conditions.

The key idea is that the retailer’s objective is concave when written as a function of the follower’s action, which we can demonstrate by returning to the previous example.

**Example 2.** Recall that if the buyer’s valuation function is $v(x) = \sqrt{x}$, then when she faces a price $p$, she will buy the bundle $x^*(p) = 1/4p^2$. In this simple case, we can see that setting a price of $p^*(x) = 1/2\sqrt{x}$ will induce the buyer to purchase $x$ units. In principle, we can now write the retailer’s profit function as a function of the bundle $x$. In our example, the retailer’s cost function is simply $c(x) = x$. So we have $\text{Profit}(x) = p^*(x) \cdot x - c(x) = \sqrt{x}/2 - x$, which is concave.

This example shows that when written in terms of $x$, the profit function is concave! As we show, this phenomenon continues in higher dimensions for arbitrary convex cost functions $c$, and for a wide class of economically meaningful concave valuation functions including the well studied families of CES and Cobb-Douglas utility functions.
Therefore, if the retailer had access to an oracle for the concave function $\text{Profit}(x)$, we could use an algorithm for bandit concave maximization to optimize the retailer’s profit. Unfortunately, the retailer does not directly get to choose the bundle purchased by the buyer and observe the profit for that bundle. Instead, he can only set prices to observe the buyer’s chosen bundle $x^*(p)$ at those prices and the resulting profit $\text{Profit}(x^*(p))$.

Nevertheless, we have reduced the retailer’s problem to a possibly simpler one. In order to find the profit maximizing prices, it suffices to give an algorithm which simulates access to an oracle for $\text{Profit}(x)$ given only the retailer’s query access to $x^*(p)$ and $\text{Profit}(x^*(p))$. Specifically, if for a given bundle $x$ the retailer could find prices $p$ such that the buyer’s chosen bundle $x^*(p) = x$, then he could simulate access to $\text{Profit}(x)$ by setting prices $p$ and receiving $\text{Profit}(x^*(p)) = \text{Profit}(x)$.

The next key ingredient is a “tâtonnement-like” procedure that efficiently finds prices that approximately induce the buyer to purchase a target bundle $x^*(p)$. The procedure works as long as the buyer’s valuation function is Lipschitz and strongly concave on the set of feasible bundles. Specifically, given a target bundle $x$, our procedure finds prices $p$ such that $|\text{Profit}(x^*(p)) - \text{Profit}(x)| \leq \varepsilon$. Thus, we can use our procedure to simulate approximate access to the function $\text{Profit}(x)$. Our procedure requires only $\text{poly}(d, 1/\varepsilon)$ queries to $x^*(p)$. Fortunately, it turns out that recent algorithms for noise tolerant bandit optimization due to [Belloni et al. 2015] can maximize the retailer’s profits efficiently even with only approximate access to $\text{Profit}(x)$.

Our tâtonnement procedure was inspired by an elegant recent paper of [Bhaskar et al. 2014], who gave an Ellipsoid-based procedure for the similar problem of finding tolls that induce a target flow in a routing game where the latency functions are unknown. Our procedure is rather general, and applies to a broad family of Stackelberg games in which the leader wishes to optimize his objective function, without knowing what the follower’s utility function is, and has access only to observations of the follower’s best responses. In particular, our techniques can also be applied to the flow problem studied by [Bhaskar et al. 2014], yielding incomparable guarantees. (Our results hold for a much broader class of latency functions, but our convergence time is slower).

We view our work as one of the first steps in a broader agenda of studying “revealed preferences problems” from a computational perspective. There are many interesting problems in this space, and we will now highlight one. In our profit maximization application, it would be very natural to consider a “Bayesian” version of our problem. At each round the producer sets prices, and then a new consumer, with valuation function drawn from an unknown prior, purchases her utility maximizing bundle. The producer’s goal is to find the prices that maximize her expected profit, over draws from the unknown prior. Under what conditions can we solve this problem efficiently? The main challenge (and the reason why it likely requires new techniques) is that the expected value of the purchased bundle need not maximize any well behaved utility function, even if each individual consumer is maximizing a concave utility function.
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Algorithmic Game Theory and Econometrics

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The traditional econometrics approach for inferring properties of strategic interactions that are not fully observable in the data, heavily relies on the assumption that the observed strategic behavior has settled at an equilibrium. This assumption is not robust in complex economic environments such as online markets where players are typically unaware of all the parameters of the game in which they are participating, but rather only learn their utility after taking an action. Behavioral models from online learning theory have recently emerged as an attractive alternative to the equilibrium assumption and have been extensively analyzed from a theoretical standpoint in the algorithmic game theory literature over the past decade. In this letter we survey two recent works, [Nekipelov et al. 2015, Hoy et al. 2015], in which we take a learning agent approach to econometrics, i.e. infer properties of the game, such as private valuations or efficiency of observed allocation, by only assuming that the observed repeated behavior is the outcome of a no-regret learning algorithm, rather than a static equilibrium. In both works we apply our methods to datasets from Microsoft’s sponsored search auction system.

Categories and Subject Descriptors: J.4 [Social and Behavioral Sciences]: Economics; K.4.4 [Computers and Society]: Electronic Commerce; F.1.2 [Modes of Computation]: Online computation

General Terms: Theory, Economics, Econometrics, Experimental

Additional Key Words and Phrases: Econometrics, no-regret learning, sponsored search, value inference, welfare guarantees

1. INTRODUCTION

One of the main goals of the econometric analysis of strategic interactions is the inference of the private parameters of participants based solely on their observed actions. For instance, by observing a sequence of bids of a set of bidders participating repeatedly in an auction for a single item, one aims to infer the private value each player has for the item. Another quantity of interest in such environments is the efficiency of the outcome of the strategic interaction, i.e. was the item sold to the player with the highest or approximately highest valuation.

Any such task requires an assumption on how the players make decisions in a repeated game setting. One of the main assumptions that has been overwhelmingly used in traditional econometrics is that the actions that we observe in the data are the product of a Nash equilibrium behavior of the participants, i.e. a state of mutual best-responses [Athey and Nekipelov 2010; Bajari et al. 2013]. Such an assumption is rather strong, especially in complex environments such as online sponsored search auctions, where the players do not even know who they are competing against and do not even know all the parameters of the auction rule.

In such settings, players typically only observe periodic aggregate feedback of what their utility would have been for any possible action they could have taken in the last period. Therefore, models of strategic behavior should be better suited...
to such feedback structures and allow for bounded rationality of the players. One such model of behavior that has been proposed in the game theory literature [Foster and Vohra 1997; Freund and Schapire 1999] and which has been extensively analyzed in the algorithmic game theory literature in the past decade is that of no-regret learning [Blum et al. 2008; Roughgarden 2009; Syrgkanis and Tardos 2013].

No-regret learning simply assumes that players use a learning algorithm which, over-time, guarantees them that their utility is at least as good as the best fixed action in hindsight. Thus unlike the Nash equilibrium assumption, no-regret learning allows for dynamic player behavior and requires only an approximate best-response property and only on average over a time period. Moreover, there exist many learning algorithms that achieve this property and which work even in the aforementioned utility feedback model.

Fig. 1. Normalized bid for listings of a single advertiser over the course of a week.

Dynamic behavioral models, such as learning agent models, seem of practical importance, since in many real sponsored search datasets we observe bidders changing their bids very frequently. For instance, in Figure 1 we depict the bids of a subset of the listings of a single advertiser in Microsoft’s sponsored search auction system over the period of a week.

2. VALUE INFERENCE FOR LEARNING AGENTS

In [Nekipelov et al. 2015] we address the problem of inferring player valuations from a sequence of bid observations in a repeated sponsored search auction environment. We propose an approach that solely assumes that the sequence of bids is the outcome of a vanishing regret learning algorithm.

In the setting that we analyzed, advertisers submit a bid for being allocated a position in the sponsored section of a search page. Advertisers are allocated positions based on some quality score and their bid. When an advertiser is clicked we assume that she receives some value \( v \) which is private and known only to her. This per-click valuation is the parameter that we want to infer. We also assume that the utility of an advertiser is quasi-linear in money, i.e., her utility is her value minus her payment.

Assuming that the sequence of bids of an advertiser is an \( \epsilon \)-regret sequence implies that the utility that she derived over the entire period that we observe must be at least as high as what any fixed bid would have achieved less some \( \epsilon \). This condition gives a set of inequalities that the value of a player must satisfy, one inequality per fixed bid. The intersection of these inequalities, is the set of values that are rationalizable under the assumption of \( \epsilon \)-regret. Varying \( \epsilon \), we get a set of pairs \((v, \epsilon)\), such that value \( v \) is rationalizable under the \( \epsilon \)-regret assumption. We refer to this set as the rationalizable set. We show that the rationalizable set is convex and
characterize its statistical learning properties. We show that the statistical learning rate of the rationalizable set is remarkably comparable with the statistical learning rates of methods that make the stronger equilibrium behavior assumption [Athey and Nekipelov 2010].

If one wants to make a point prediction on the value of a player, then a selection rule is needed, to select among the points in the rationalizable set. We analyze the point that corresponds to the smallest multiplicative regret (the sequence has multiplicative regret $\lambda$ if the current utility of the bidder is at least $(1 - \lambda)$ times the utility of any fixed bid). We apply this point-prediction approach to a dataset from Microsoft’s sponsored search system. Figure 2 depicts the results of our analysis when applied to all the listings of a single account. For the inferred values, we depict the distribution of how much a player shades his value on average and the distribution of the smallest rationalizable error across listings. We find that on average for many accounts, advertisers bid around 60% of their inferred valuation and that the smallest rationalizable error, though small, is bounded away from zero for almost 70% of the listings (i.e. doesn’t satisfy the exact best response property).

Fig. 2. Distribution of bid shade ratio and smallest multiplicative regret across listings of a single advertiser.

3. DATA-DRIVEN ROBUST EFFICIENCY GUARANTEES

In [Hoy et al. 2015], we give an econometric approach for directly inferring a lower bound on the efficiency of the resulting allocation in a repeated auction setting, without even inferring first the valuations of the players. Our approach is an empirical analogue of the smoothness approach on quantifying the worst-case inefficiency in games [Roughgarden 2009; Syrgkanis and Tardos 2013; Hartline et al. 2014] and therefore inherits several robustness properties of smoothness. For instance, the lower bound on the efficiency that is derived via our method holds regardless of whether the data that we observe are the product of a Bayes-Nash equilibrium where the player valuations are stochastic, or whether they are the product of a learning process employed by an advertiser with a fixed valuation. Moreover, our method enjoys fast statistical learning rates when only a sub-sample of the strategic interactions is observed.

The smoothness approach of [Syrgkanis and Tardos 2013] and its refinement for single-parameter mechanism design environments, via the revenue and value covering formulation of [Hartline et al. 2014] is based on the following argument: at any
outcome of the game that satisfies a best-response or approximate best-response property either the player is getting high utility and hence high allocation probability, or the payment that he needs to make in order to achieve a high allocation probability given the competition must be high. The latter quantity is typically referred to as the threshold payment. Subsequently, if this threshold payment is closely related to the revenue that the auction receives then we can attribute this term to the current welfare of some other bidder. Combining these two arguments gives a lower bound on the efficiency of the allocation.

The crucial observation in [Hoy et al. 2015] is that both the threshold payment quantity and the revenue are observed in the data. Thereby we do not need to theoretically prove a relation between the two quantities. One simply needs to analyze the relation of the two quantities from the data. This can potentially lead to better efficiency guarantees than the theoretically provable ones. More importantly, our approach can be used to infer efficiency lower bounds even in auctions where no worst-case theoretical relation is known between the two quantities and therefore no worst-case efficiency lower bound can be inferred simply from the rules of the auction without observing the data. This is the case with the actual complex sponsored search auction that is being used in Microsoft’s sponsored search system. For instance, the application of our approach to real datasets for a selection of high-revenue keywords yielded significant efficiency guarantees, ranging from 30% to 70% of the efficiency of the optimal allocation.

REFERENCES


1The actual quantity that goes into the formulation in order to produce tight efficiency results is slightly more involved and the reader is referred to the paper for a full exposition.