Editor's Puzzle: Strategically Choosing Products to Release

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As usual, you are allowed to use computers to help you solve this puzzle, but the final argument should be human-readable and as strong as possible.

Solutions should be sent to the editor at `conitzer@cs.duke.edu` with subject header **SIGecom Exchanges Puzzle**. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish his or her or their proof in the next issue of the Exchanges (ties broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. The editor will not give any feedback on submitted solutions and ignore any requests for hints, *etc.*

Also, the previous issue's puzzle on combinatorial auction winner determination was solved by each of Daniel Lehmann, Peter Stone, and Mukund Sundararajan, who submitted very similar solutions, but they did not consider their solutions elegant enough to publish. If someone submits a more elegant solution to that puzzle, I will be happy to publish it in the next issue. Hint (for *that* puzzle): think about prices and duality, though integrality plays a role... Now, let us get to this issue's puzzle:

There are three firms (1, 2, and 3) that must each decide on a product to release. 1 must release (exactly) one of products A, B, and C; 2 must release (exactly) one of D, E, and F; and 3 must release (exactly) one of G, H, I.

Each firm's profit depends on how well its released product interacts with the other products that are released. If product X *benefits from* product Y, then this adds a profit of 1 to the maker of product X (if X and Y are both chosen as products); if X *strongly benefits from* Y, then this adds a profit of 2 to the maker of product X; and if X *suffers from* Y, then this adds a profit of -1 to the maker of product X. (Effectively, benefit corresponds to one-directional complementarity, and suffering to one-directional substitutability.) For example, if X strongly benefits from U and V, benefits from W, and suffers from Y and Z, then the maker of X receives a total profit of 2 + 1 - 1 = 2 if U, W, and Y are also present in the market, and 2 + 2 - 1 - 1 = 2 if U, V, Y, and Z are also present.

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For the game that we are interested in, we have the following relationships (any relationship that is not specified is neutral, that is, it adds 0):

—A strongly benefits from D and G.
—B strongly benefits from E and suffers from F.
—C suffers from E and benefits from F.
—D strongly benefits from G.
—E suffers from H and strongly benefits from I.
—F benefits from H and suffers from I.
—G has no relationship to any other product.
—H suffers from A and benefits from B and C.
—I strongly benefits from A and suffers from B and C.

The question is: which products should we expect to see in the market? To determine this, you should make the following assumptions:

(1) Firms are completely rational;
(2) Firms must make their decisions simultaneously;
(3) Each firm's goal is to maximize its *expected* profit;
(4) Firms cannot make any commitments to each other, cannot transfer profit to each other, *etc.*;
(5) The game occurs only once (it is not a repeated game);
(6) All of the above is common knowledge.

Of course, a good solution does not only give the answer to the above question, but also gives a clear and strong argument why we should expect to see these products.