Editor’s Puzzle: Identifying the Champion

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Solutions should be sent to the editor at conitzer@cs.duke.edu with subject header SIGecom Exchanges Puzzle. The author(s) of the most elegant solution (as judged by the editor) will be allowed to publish his or her or their proof in the next issue of the Exchanges (ties broken towards earlier submissions). To make the solution accessible to a wide audience, try to minimize technical jargon in the proof. The editor will not give any feedback on submitted solutions and ignore any requests for hints, etc.

A Japanese prophecy foretells that a champion warrior will emerge, a warrior that is strictly more skillful than any other warrior. A large (but finite) group of warriors has assembled to determine who the champion warrior is. It is common knowledge among these warriors that the prophecy is true, and, moreover, that the champion is among them.

To determine the champion, an infinite sequence 1, 2, . . . of rooms with challenges has been set up for them, each one more difficult than the previous. For every warrior $i$, there is a number $n_i \in \{0, 1, \ldots\}$ such that the warrior’s performance will be satisfactory in the first $n_i$ rooms, and unsatisfactory in the remaining rooms. The champion, of course, has the strictly highest $n_i$.

Each warrior spends every day in one of the rooms, trying to pass the challenge. All warriors start in the first room. Every day exactly at midnight each warrior can choose to move on to the next room, or stay in the current room. A warrior is said to qualify for a room if his performance in the previous room was satisfactory. If a warrior ends up in a room for which he is not qualified then his performance will be not only unsatisfactory, but humiliating. This will cast shame on the warrior’s family name, which each warrior wishes to avoid at any cost. Therefore, a warrior will move on to the next room if and only if he is completely sure that his performance in the current room is satisfactory.

To make matters more difficult, each warrior is unable to (directly) assess whether his own performance in a room is satisfactory. But, each warrior can easily assess whether the performance of any other warrior in the same room is satisfactory (which also implies that they can see who is in the same room). The warriors are quite stern and do not communicate with each other, but it is common knowledge that the warriors are highly intelligent.

Will the champion be identified? How long does the process take? Where does each warrior end up? Why is the prophecy Japanese?

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