Two-Sided Random Matching Markets:

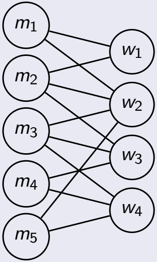
Ex-Ante Equivalence of the Deferred Acceptance Procedures

Simon Mauras
simon.mauras@irif.fr

Université de Paris, IRIF

Two-Sided Matching Market

Acceptability graph



Agents order their neighbours uniformly at random.

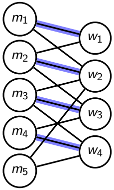
You are an **evil** decision maker

You need to chose the procedure:

- **MPDA**: Men Proposing Deferred Acceptance
- **WPDA**: Women Proposing Deferred Acceptance

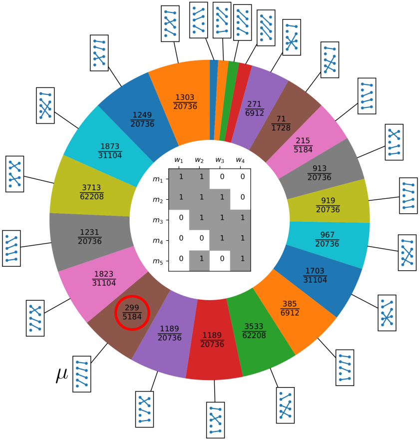
You really like this matching:

μ



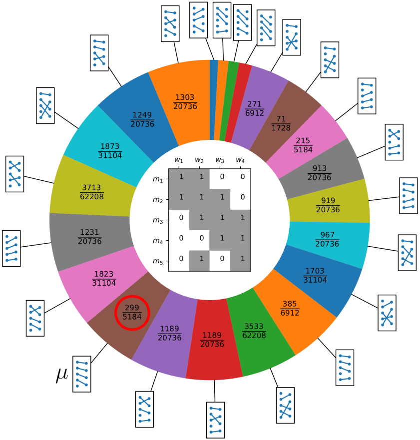
What should you do to maximize the probability of chosing μ?

Output distribution of **MPDA**.



$\mathbb{P}[\text{MPDA outputs } \mu] = 299/5184$

Output distribution of **WPDA**.



$\mathbb{P}[\text{WPDA outputs } \mu] = 299/5184$

Theorem. "Ex-ante equivalence"

MPDA and **WPDA** have the **same output distribution**.

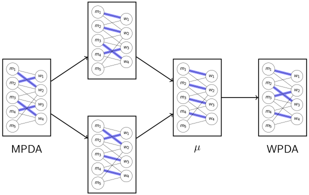
If you don't know the preferences, you cannot "manipulate"...

In the talk/paper:

1. Proof of the Theorem.
2. Non-uniform distributions.

In the proof...

Lattice of stable matchings.



Probability of stability.

$$\int_0^1 \cdots \int_0^1 \mathrm{d}x_1 \cdot \mathrm{d}x_2 \cdot \mathrm{d}x_3 \cdot \mathrm{d}x_4 \cdot \mathrm{d}y_1 \cdot \mathrm{d}y_2 \cdot \mathrm{d}y_3 \cdot \mathrm{d}y_4$$
$$\cdot (1 - x_1 y_2) \cdot (1 - x_2 y_1)$$
$$\cdot (1 - x_2 y_3) \cdot (1 - x_3 y_2)$$
$$\cdot (1 - x_3 y_4) \cdot (1 - x_4 y_3)$$
$$\cdot (1 - y_2) \cdot (1 - y_4)$$
$$= \frac{1391}{20736}$$