PROPHET INEQUALITIES

- n values drawn **independently** from **known** distributions:

$$X_i \sim F_i$$

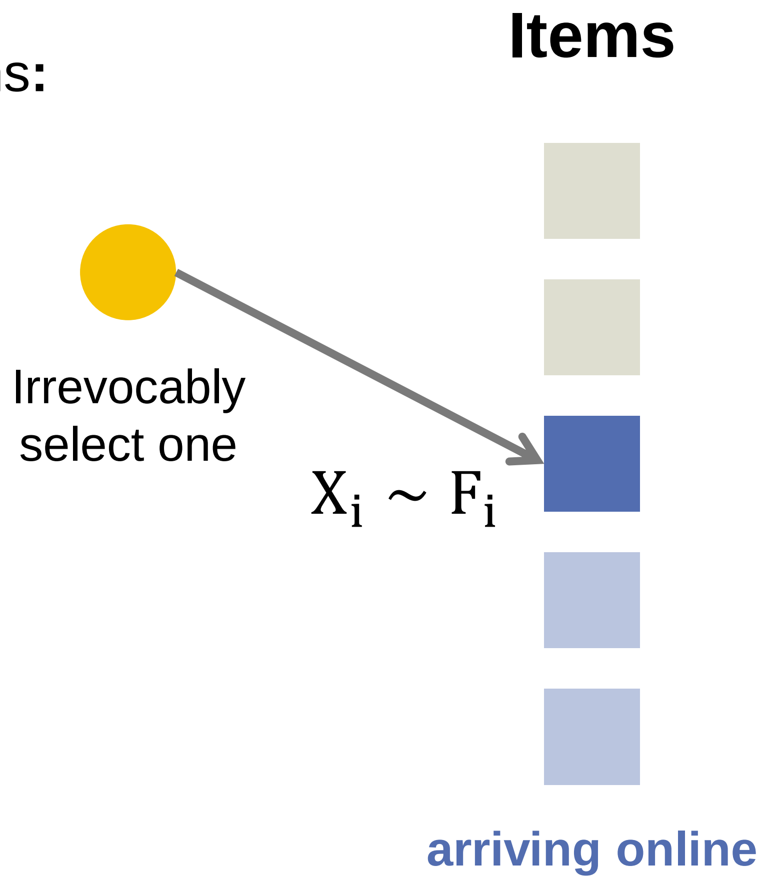
> **α-approx** means: $E[\text{reward}] \geq 1/\alpha \cdot E[\max X_i]$

- **τ-threshold** mechanism: Take first item with $X_i \geq \tau$

Theorem: The following give **2-approximation**

- Median: $\Pr[\max X_i \geq \tau] = 1/2$ Samuel-Cahn'84
- Mean: $\tau = E[\max X_i]/2$ Kleinberg-Weinberg'12

- **What if values Correlated?**
  - $\Omega(n)$-approx Hill-Kertz'92
  - What about "mild" correlations?



LINEAR CORRELATIONS MODEL

Bateni-Dehghani-Hajighayi-Seddighin'15
Chawla-Malec-Sivan'15

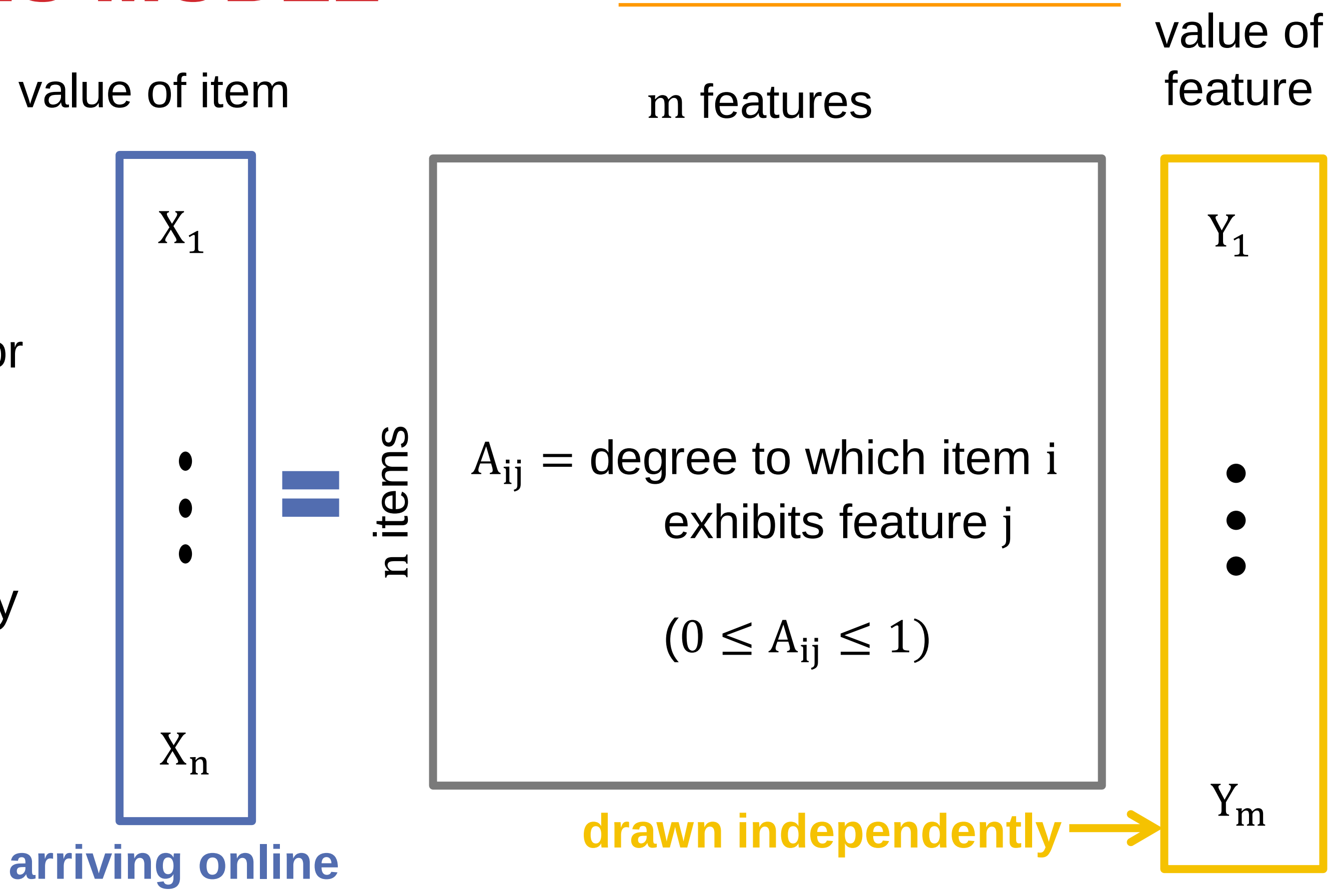
- **VALUES**

$$\vec{\mathbf{X}} := A \cdot \vec{\mathbf{Y}}$$

Known matrix $A \in [0,1]^{n \times m}$ and $Y_i \sim F_i$ for known distributions F_i.

- **E.g.,** $A =$ identity matrix gives classical prophet inequality

- s_{row}: row sparsity of A
- s_{col}: column sparsity of A



THM 1 (Single Item): $\Theta(\min\{s_{row}, s_{col}\})$ **approx**

THM 2 (Multiple Items): Selecting r items
- **FOR** $r \gg s_{col}$: $(1 + o(1))$ **approximation**
- **FOR** $r \gg s_{row}$: $\Theta(s_{row})$ **approximation**

MAIN SUBPROBLEM

Augmentation Problem

positive "noise"

1. Think of X_i's = independent part + dependent part: $\underbrace{X_i = Z_i}_{\text{Independent}} + \underbrace{W_i}_{\text{Correlated with past}}$
2. Can we **recover** $E[\max Z_i]$ given only Z_i distributions?

Note: Prophet inequality for $W_i = 0$

Illustrative Example

- X_1 drawn uniformly from $[0,1]$ $+2$
- X_2 is 10^4 w.p. $1/100$; zero otherwise

AFTER ADDING SOME POSITIVE NOISE

Median threshold: $\tau \approx 1/2$, picks X_1 ~~half the time~~ **all the time**.

Mean threshold: $\tau \approx 50$ never picks X_1.

AUGMENTATION LEMMA: Threshold $\tau = E[\max Z_i]/2$ guarantees $\mathbf{E[ALG_\tau] \geq E[\max Z_i]/2}$

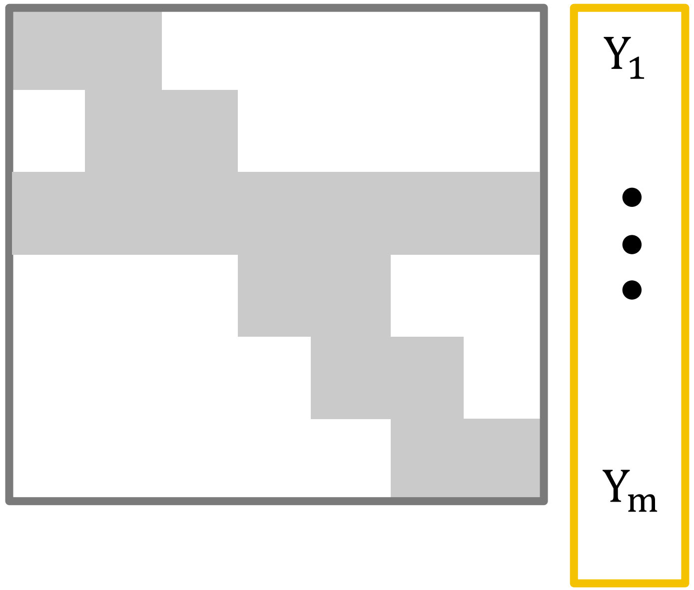
COLUMN SPARSITY

τ-threshold mechanisms have $\Omega(n)$ approximation

THM (Single Item): $O(s_{col})$ **approximation**

Inclusion-Threshold Mechanism: Run τ-threshold on a ``random subset''

- **Ignore** each X_i independently w.p. $(1 - 1/s_{col})$
- Assign Y_j to first **surviving** X_i that contains it
- Define $Z_i = \sum_{j \to i} A_{ij} Y_j$ and use **Augmentation Lemma**



Proof Idea: Show $E[\max Z_i] \approx \frac{E[\max X_i]}{e \cdot s_{col}}$

1. Max X_i survives with $1/s_{col}$ probability
2. Pr[Y_j in Max X_i assigned to Z_i] $\geq (1 - 1/s_{col})^{s_{col}-1} \approx 1/e$