MENU-SIZE COMPLEXITY AND REVENUE CONTINUITY OF BUY-MANY MECHANISMS

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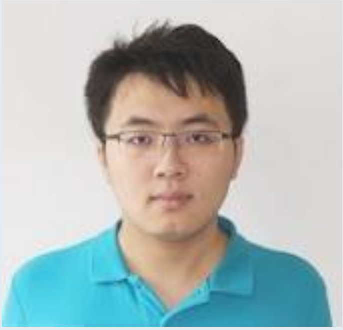
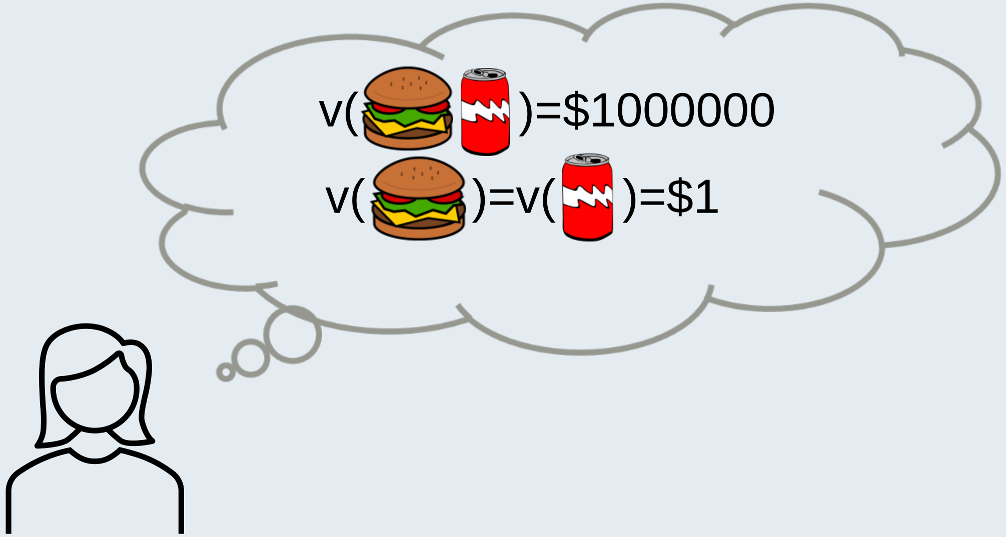
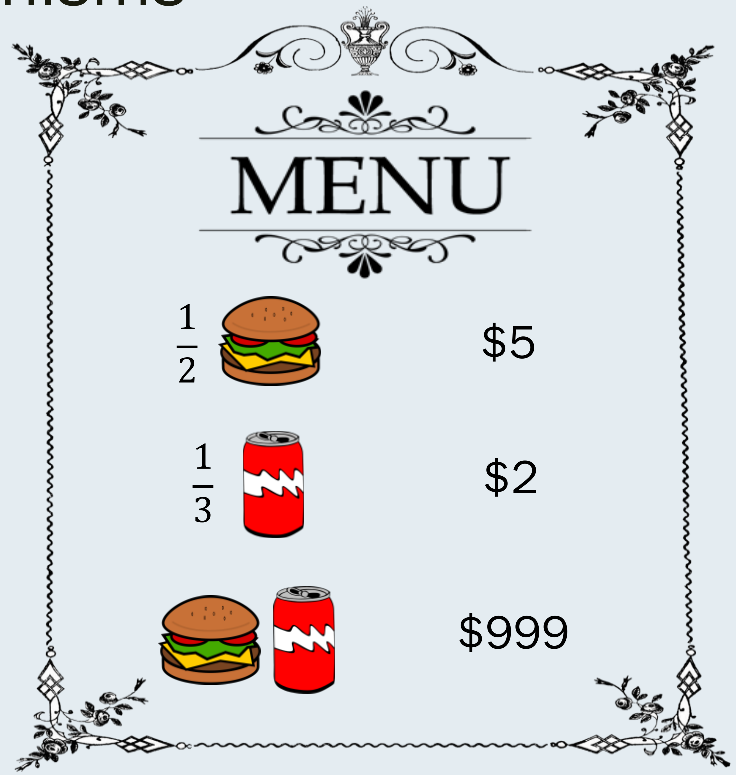


Joint work with Shuchi Chawla and Christos Tzamos (UW-Madison)

Buy-one mechanisms and buy-many mechanisms





- A seller has n heterogeneous items to sell to a single buyer.
- Typical buy-one mechanisms: buyer interact with the seller once.
  - *Optimal strategy: purchases the third menu option, pay $999.*
- Buy-many mechanisms: buyer interact with the mechanism multiple times.
  - *Optimal strategy: repeatedly purchase*  *, then repeatedly purchase*  *, pay $16 in expectation.*

Menu-size complexity for near-optimal revenue

- How many menu options are needed for $(1-\epsilon)$-approx in revenue?
  - *Buy-one mechanisms: infinite [Hart Nisan'13].*
  - *Buy-many mechanisms: finite.*

Theorem 1. For any distribution D and $\epsilon \in [0,1]$, exists mechanism M with finite menu size $f(n,\epsilon)$, such that

$$Rev_D(M) \geq (1-\epsilon) BuyManyRev_D.$$

- $f(n,\epsilon) = (1/\epsilon)^{2^{O(n)}}$.
- The doubly-exponential dependency of n is tight.

Theorem 2. There exists D being a distribution over XOS functions, such that for any mechanism M with description complexity $2^{2^{o(n^{1/4})}}$,

$$BuyManyRev_D \geq o(\log n) Rev_D(M).$$

Revenue Continuity

- When the buyer's values for the sets of items perturb multiplicatively slightly, how much does the revenue change?
  - *Any $v \sim D$ is perturbed to $v' \sim D'$, such that $v'(S) \in [(1-\epsilon)v(S), (1+\epsilon)v(S)], \forall S \subseteq [n]$.*
- Buy-one mechanisms: revenue may change significantly [Psomas et al.'19].
  - *Continuity only holds for weaker additive perturbation [Rubinstein Weinberg'15] [Brustle et al.'20].*
- Buy-many mechanisms: revenue changes slightly.

Theorem 3. For any value distribution D and any $1 \pm \epsilon$ multiplicative perturbation D',

$$BuyManyRev_{D'} \geq \left(1 - poly(n, \epsilon)\right) BuyManyRev_D.$$

- Note: such dependency on n is necessary.

Theorem 4. There exists D over unit-demand functions and a $1 \pm \epsilon$ multiplicative perturbation D', such that

$$BuyManyRev_{D'} \leq \frac{1}{\epsilon n} BuyManyRev_D.$$

- Full paper: https://arxiv.org/abs/2003.10636