Motivation and Definition of Optimin in Normal-Form Games

	L	R
U	2, 2	0, -1
D	1, 2	1, -1

Figure: Maximin strategies may be implausible in non-zero-sum games: D is the maximin strategy for Row, whereas U is not because of the payoff of 0, if Column plays R. But R is an implausible strategy. I define the following optimin notion to extend the maximin strategy from zero-sum games to non-zero-sum games by ruling out similar implausible strategies.

An "agreement" $p \in \Delta X$, which is a mixed strategy, is said to satisfy the *optimin criterion*, if it solves for every i

$$\max_{q \in \Delta X} \inf_{p'_{-i} \in B_{-i}(q)} u_i(q_i, p'_{-i}),$$

where $B_i(p) = \{p'_i \in \Delta X_i | u_i(p'_i, p_{-i}) > u_i(p_i, p_{-i})$ (better-response correspondence), $B_{-i}(p) = \times_{j \in N \setminus \{i\}} (B_j(p) \cup \{p_j\})$.

Illustrative Example

	A	B	C
A	100,100	100,105	0,0
B	105,100	95,95	0,210
C	0,0	210,0	5,5

	A	B	C
A	[100,100]	[100,0]	[0,0]
B	[0,100]	[0, 0]	[0, 5]
C	[0,0]	[5, 0]	[5,5]

Figure. A game in pure strategies (left) and its minimal payoffs (right). The optimin point is (A, A) because its minimal payoffs, [100,100], are maximal under unilateral profitable deviations, whereas every strategy is a maximin strategy. The minimal payoffs attached to (B,B) is 0 for each player because there is a unilateral profitable deviation to C, which leaves the non-deviator with a payoff of 0. The Nash equilibrium is (C, C).

Results

1. Optimin criterion can explain the direction of non-Nash deviations towards cooperation in noncooperative games.
2. Every game with compact strategy sets and continuous utility has an optimin point.
3. When restricted to pure strategies, pure optimin exists in finite games.
4. The optimin criterion generalizes Nash equilibrium in *n*-person constant-sum games.
5. Coincides with Wald's maximin criterion when the Nature is antagonistic.
6. General equilibrium: Competitive equilibrium satisfies the optimin criterion.
7. Cooperative games: Optimin criterion is equivalent to the core when core exists, but it exists even when the core is empty.

A Preview into The Optimin in Cooperative Games

Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Utility	35	30	25	90	80	70	110

Table: A cooperative game with an empty core

- The intuition of the optimin for cooperative games is similar.
- The Shapley value is (44.16, 36.66, 29.16), and the nucleolus of the game is (46.66, 36.66, 26.66).
- Optimin points: $\{x \in \mathbb{R}^3 | x_1 = 40, x_2 + x_3 = 70, x_2 \geq 30, x_3 \geq 25\}$.
- Under the both Shapley value and the nucleolus, player 2 and 3 can rationally deviate, so player 1's minimal payoff is 35; whereas, player 1's minimal payoff is 40 under the optimin because player 2 and 3 have no incentive to deviate.

The paper is available at: arXiv:1912.00211
Mehmet Ismail, King's College London
mehmet.s.ismail@gmail.com