The Equilibrium Existence Duality: Equilibrium with Indivisibilities & Income Effects

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Motivation

- if goods are indivisible, when does competitive equilibrium exist?
  - unlike the case of divisible goods, doesn't exist in general (Henry, 1970)
- most previous work assumes that utility is quasilinear in money

$$U^j(\text{money}, \text{goods}) = \text{money} + V^j(\text{goods})$$

  - restrictive: entails that endowments don't affect demand for goods; e.g., wealth or cash on hand don't affect demand for houses, spectrum, ...
- this paper: analyzes economies with indivisible goods and *non*-quasilinear utility by using duality to "quasilinearize" preferences

Starting point: a standard duality from consumer theory

(economics textbook material)

consumer problem: fix endowments of money and goods, maximize utility

dual problem: fix a utility level, minimize expenditure of obtaining it

- defines *Hicksian demand*

two kinds of changes to Hicksian demand:

- ones induced by changes in prices, holding the utility level fixed
  - called *substitution effects*
- ones induced by changes in the utility level, holding prices fixed
  - called *income effects*; not present when preferences are quasilinear

Our contribution

1. interpret the dual consumer problems as quasilinear maximization problems to form quasilinear *Hicksian economies* in which utility levels are fixed instead of endowments
    - each Hicksian economy reflects only substitution effects
2. derive an *Equilibrium Existence Duality*: equilibrium always exists in the original economy iff it always exists in each Hicksian economy

key consequences of the Equilibrium Existence Duality:

- substitution effects fundamentally determine whether equilibria exist
    - income effects don't matter for existence
- get new domains for existence from previous quasilinear results
    - *net* substitutability (which is weaker than gross) implies existence
    - *Unimodularity Theorem* (Baldwin & Klemperer, 2019) w/income effects