

# Characterization of Group-Strategyproof Mechanisms for Facility Location in Strictly Convex Space

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# Settings

- Facility location game of  $n$  agents
  - Profile:  $\mathbf{x} = (x_1, \dots, x_n)$
  - Mechanism:  $f(\mathbf{x})$ ; possibly random
- Strategyproofness / Group-strategyproofness
- Strictly convex space
  - $x \neq y, \|x\| = \|y\| \implies \|x\| + \|y\| > \|x + y\|$
  - E.g., Euclidean space,  $L^p$ -norm vector space ( $p \in (1, \infty)$ )
- Unanimity:  $x_1 = \dots = x_n = x \implies f(\mathbf{x}) = x$  (the same as Arrow's)
- Translational invariance:  $f(\mathbf{x}) + a = f(\mathbf{x} + a)$  ( $a$  is any constant)

# Our Results

- **Theorem 1 (Informal).** Deterministic, unanimous, group-strategyproof  $\implies$  **dictatorial**.
  - Dictatorship:  $f(\mathbf{x}) = x_i$  for any  $\mathbf{x}$ ; agent  $i$  is the dictator.
- **Theorem 2 (Informal).** Unanimous, translation-invariant, group-strategyproof  $\implies$  **2-dictatorial**.
  - 2-Dictatorship:  $f(\mathbf{x})$  always lies between  $x_i$  and  $x_j$ ; agents  $i$  and  $j$  are the 2-dictators.

# Our Results

- Theorem 1 and 2 implies almost tight bounds of approximately optimal mechanisms under both maximum and social cost.

|              | Deterministic    | Randomized   |
|--------------|------------------|--|
| Maximum cost | $[2, 2]$         | $[3/2^a, 3/2]$ when $n = 2$ , $[2^b, 2]$ when $n \geq 3$ |
| Social cost  | $[n - 1, n - 1]$ | $[n/2 - 1^b, n/2]$                                       |

<sup>a</sup> Proved by (Procaccia and Tennenholtz, 2009) in the one-dimensional case.

<sup>b</sup> Requires translational invariance.