The Multiplayer Colonel Blotto Game

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**Budget** \(_{Alice} = \) 5

**Budget** \(_{Blotto} = \) 7

**Battlefields**

- **Alice**: Utility: 5
  - Battlefields 1: Budget = 7, Value = +7
  - Battlefields 2: Budget = 2, Value = +2
  - Battlefields 3: Budget = 3, Value = +3
  - Battlefields 4: Budget = 2, Value = +2

- **Blotto**: Utility: 11
  - Battlefields 1: Budget = 7, Value = +7
Our Contribution: Multiplayer Blotto

**Applications**

**Elections:** $k$ parties compete over $n$ winner-take-all districts. Campaign resources need to be allocated.

**R&D:** $k$ companies have the ability to use their fixed R&D budgets to research and develop $n$ potential drugs.

**Monopoly:** $k$ competing companies in the same industry want to become the dominant player in each of $n$ new local markets.

**Ads:** $k$ companies compete to advertise a substitute good to $n$ consumers.

**Ecology:** $k$ species in a habitat compete to fill $n$ distinct ecological niches.
Main Results

Algorithm 1: for 3-player symmetric Blotto, we give an $O(n)$ time algorithm for sampling a strategy in Nash Equilibrium. (assuming no item is worth more than $\frac{1}{3}$ of the whole value.)

Algorithm 2: for $k$-player symmetric Blotto, if the battlefields can be partitioned into $k$ equal-value parts, we give an $O(n)$ time algorithm for sampling a strategy in Nash Equilibrium.

Algorithm 3: we give an Fully Polynomial Time Approximation Scheme for sampling equilibria of Boolean Blotto games for any number of players.
Our Techniques

1) **Derive marginal bid distributions:**
   
   Requirement: budget constraint holds in expectation

2) **Couple marginal bid distributions:** Requirement: budget constraint holds almost surely

   3-player (**Alg 1**): rotate the uniform distribution on the 2-sphere in $\mathbb{R}^3$ into hyperspace; water-filling algorithm to construct the rotation

   **k-player (**Alg 2**): use properties of Dirichlet distribution

   **Boolean (**Alg 3**): greedy algorithm