

The Multiplayer Colonel Blotto Game

By Enric Boix-Adserà, Ben Edelman, and Siddhartha Jayanti

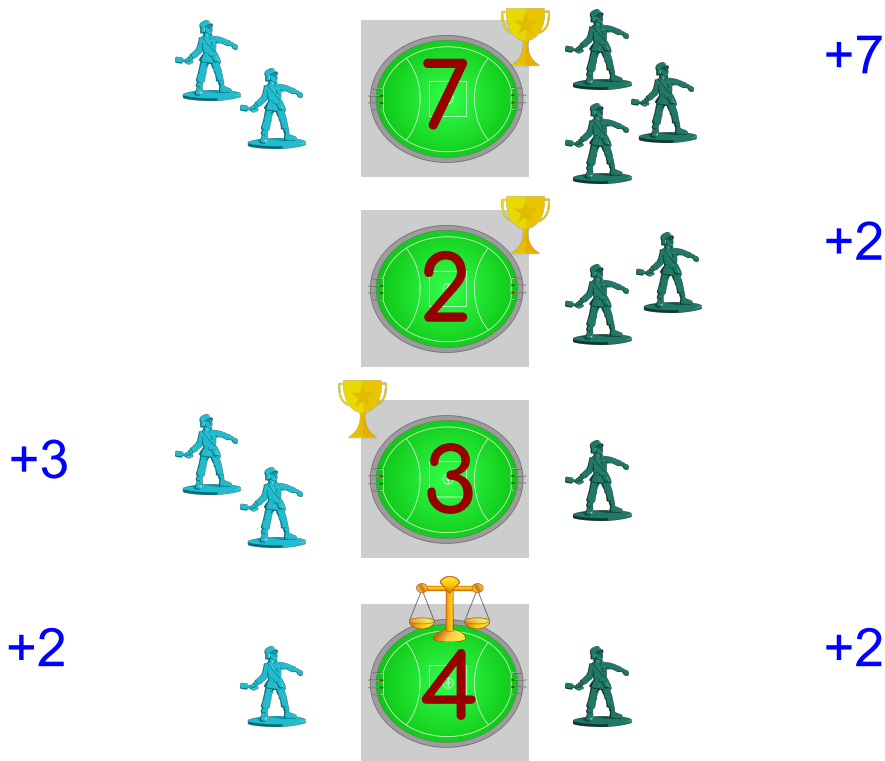
$$\text{Budget}_{\text{Alice}} = 5$$



ALICE

UTILITY: 5

Battlefields
With Values



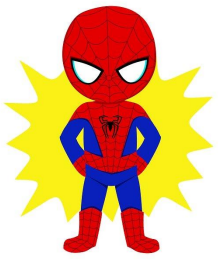
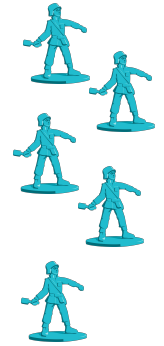
$$\text{Budget}_{\text{Blotto}} = 7$$



BLOTTO

UTILITY: 11

Our Contribution: Multiplayer Blotto



PLURALITY WINS!

Applications

Elections: k parties compete over n winner-take-all districts. Campaign resources need to be allocated.

R&D: k companies have the ability to use their fixed R&D budgets to research and develop n potential drugs.

Monopoly: k competing companies in the same industry want to become the dominant player in each of n new local markets.

Ads: k companies compete to advertise a substitute good to n consumers.

Ecology: k species in a habitat compete to fill n distinct ecological niches.

Main Results

Algorithm 1: for 3-player symmetric Blotto, we give an $O(n)$ time algorithm for sampling a strategy in Nash Equilibrium. (assuming no item is worth more than $\frac{1}{3}$ of the whole value.)

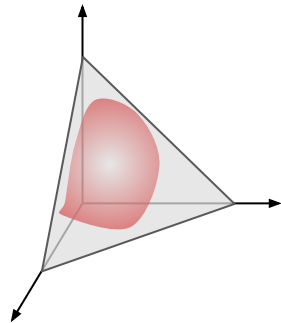
Algorithm 2: for k -player symmetric Blotto, if the battlefields can be partitioned into k equal-value parts, we give an $O(n)$ time algorithm for sampling a strategy in Nash Equilibrium.

Algorithm 3: we give an Fully Polynomial Time Approximation Scheme for sampling equilibria of Boolean Blotto games for **any** number of players.

Our Techniques

1) **Derive marginal bid distributions:**

Requirement: budget constraint holds in expectation



2) **Couple marginal bid distributions:** Requirement: budget constraint holds almost surely

3-player (Alg 1): rotate the uniform distribution on the 2-sphere in \mathbf{R}^3 into hyperspace; water-filling algorithm to construct the rotation

k-player (Alg 2): use properties of Dirichlet distribution

Boolean (Alg 3): greedy algorithm

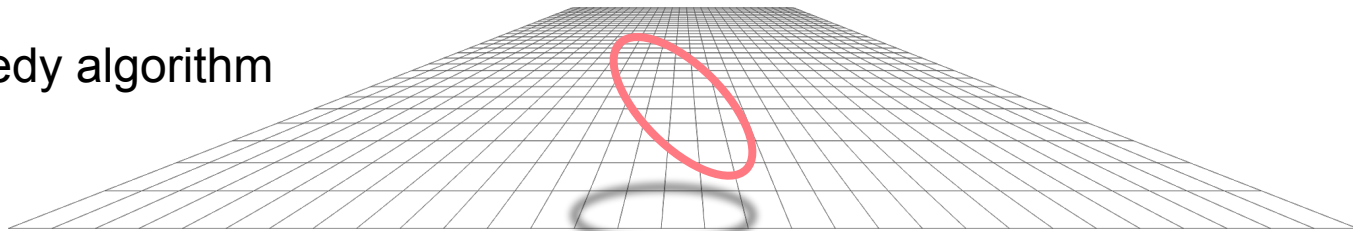


ILLUSTRATION FOR **ALG 1** IN LOWER DIMENSION