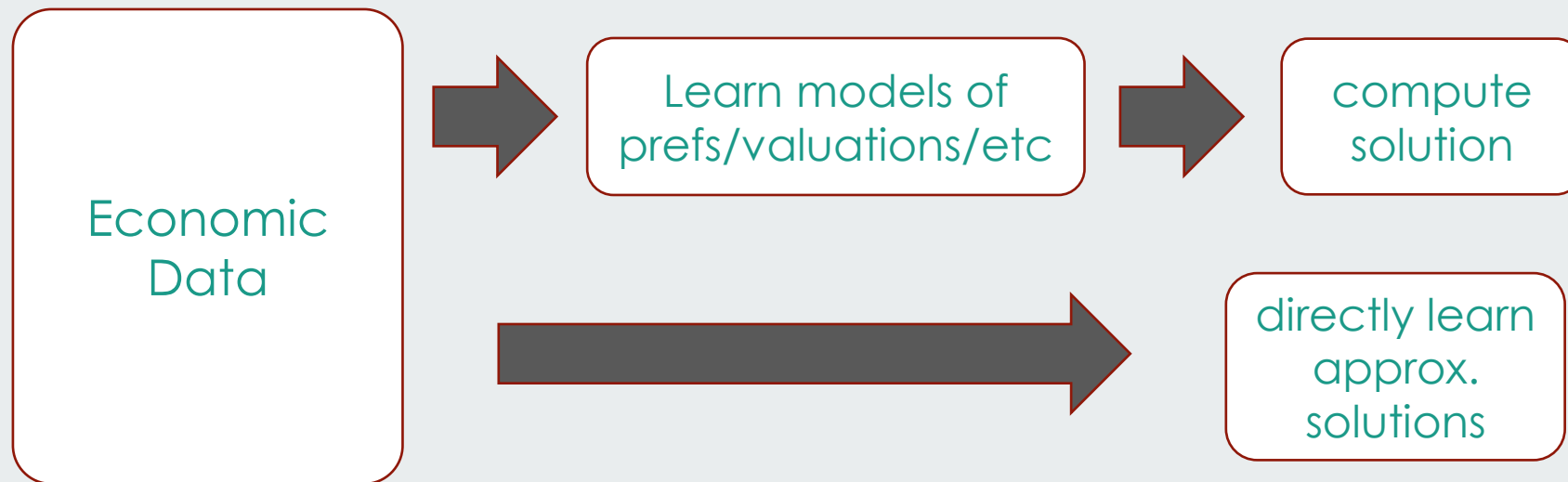


# A Learning Framework for Distribution-Based Game-Theoretic Solution Concepts

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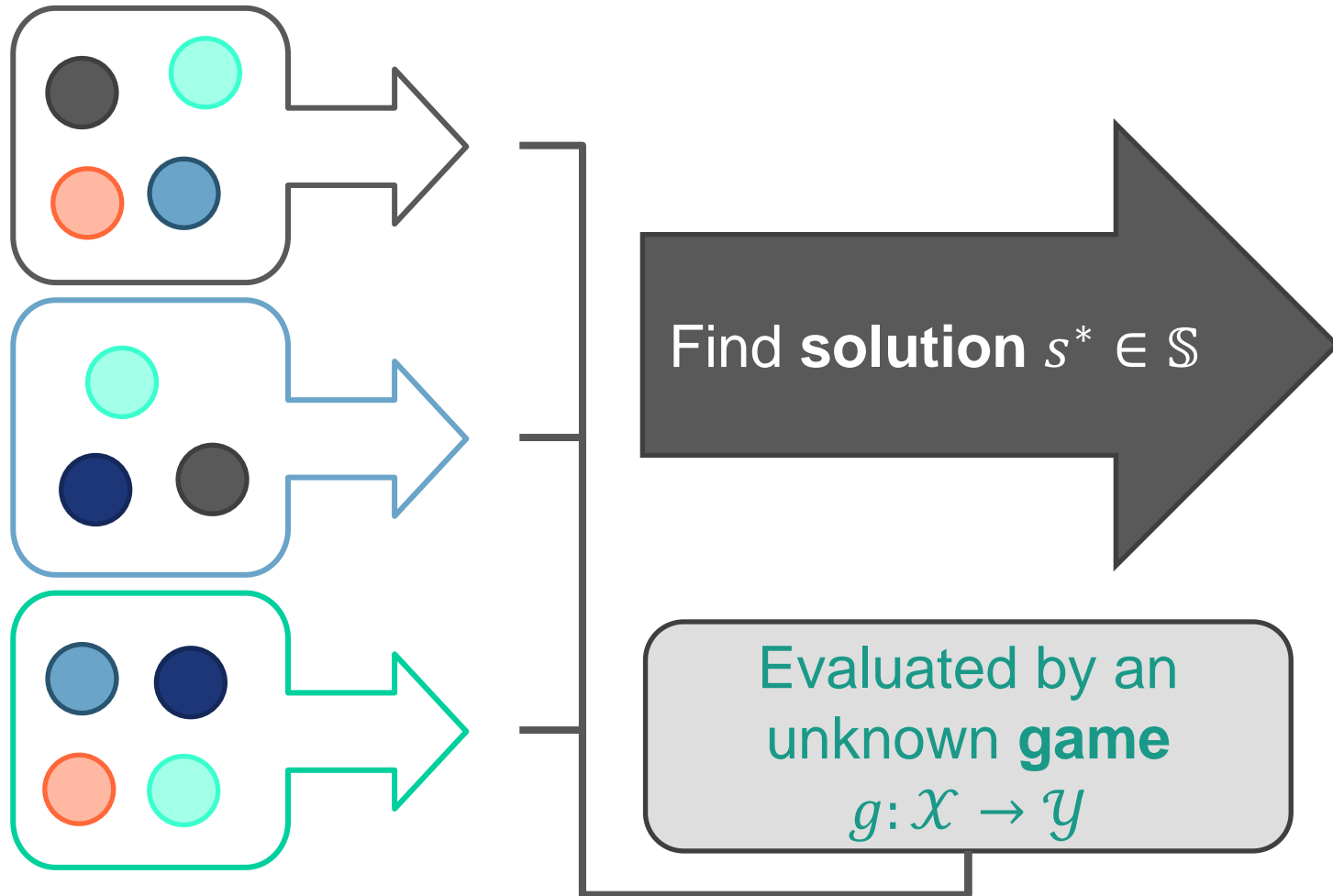
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UMass Amherst



(This work was conducted while both authors were at National University of Singapore, and was supported by Singapore National Research Foundation Fellowship.)

# General Model of PAC Solution Concepts

Observations from  $\mathcal{D}$  over **game space**  $\mathcal{X}$



probability  $\geq 1 - \delta$ ,  
over training samples

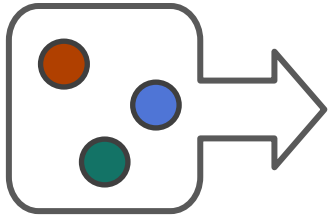
The probability that we  
observe a **loss** is low

$$\Pr_{X \sim \mathcal{D}} [\lambda(g, X, s^*)] < \varepsilon$$

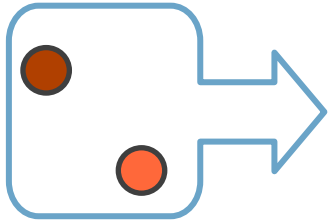
- Dimension theory for economic solution concepts
- Sample complexity bounds
- Are consistent algorithms enough?

# Example: PAC Solutions for Markets

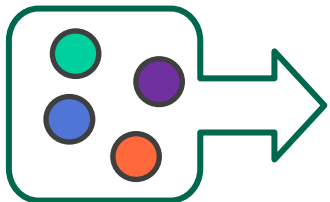
Observations, of item bundles and the utilities of  $n$  players for them, sampled i.i.d. from  $\mathcal{D}$



$v_1(s_1), v_2(s_1) \dots v_n(s_1)$



$v_1(s_2), v_2(s_2) \dots v_n(s_2)$



$v_1(s_3), v_2(s_3) \dots v_n(s_3)$

⋮

Evaluated by  
unknown player  
preferences  $(v_i)_{i \in N}$

Find an  
allocation  
and prices  
 $\langle A^*, \vec{p}^* \rangle$

A sampled set of goods  
 $S \subseteq G$  is not demanded  
by any player  $i \in N$

$$\Pr_{S \sim \mathcal{D}} [\text{Demands}_i(S; A^*, \vec{p}^*)] < \varepsilon$$

- We prove linear solution dimension.
- Therefore, prove polynomial sample complexity bound for PAC solvability.
- Also, we show existence of PAC solutions for Fisher markets (ie. budget constrained) and markets with endowments.

# Main Takeaway

Recovers previous results  
such as:

- i) PAC Core Stability in TU Cooperative Games (Balcan et al. IJCAI 2015),
- ii) in Hedonic Games (Sliwinski et al. IJCAI 2017; Igarashi et al. AAI 2019),
- iii) And others...



Enables  
generalization of  
approach to other  
domains like Markets,  
Voting, Auctions, etc.

Provides a unified formal approach for proving learnability in economics, and associated distribution-agnostic sample complexity bounds.