Sustainable Equilibria

- Myerson (1996) argued informally for a new refinement concept that he termed sustainable equilibria.
- In this line of argument:
  - **Strict** Nash equilibria are sustainable.
  - Battle of sexes: **only strict equilibria are sustainable**.
  - If a game has **a unique equilibrium**, it **is sustainable**.
  - Every generic game has a sustainable equilibrium.

Hofbauer conjecture

Hofbauer (2000) expanded on Myerson's idea and formalised the notion of sustainable equilibria.

- He defines **an equivalence relation** among pairs (G, σ) where G is a game and σ is an equilibrium of G.
- $(G, \sigma) \sim (\hat{G}, \hat{\sigma})$ if $\sigma = \hat{\sigma}$ (up to a relabelling) and the **restriction of** G **and** $\hat{G}$ **to the best replies** to σ and $\hat{\sigma}$, resp., **are the same game** (up to a relabelling).
- An equilibrium σ of a game G is sustainable iff $(G, \sigma) \sim (\hat{G}, \hat{\sigma})$ and $\hat{\sigma}$ is the unique equilibrium of $\hat{G}$.

Example: Battle of the sexes

3 Nash equilibria: 2 strict $\sigma = (t, l)$ and $\theta = (b, r)$, and 1 mixed.

$G =$

	l	r
t	$(3, 2)$	$(0, 0)$
b	$(0, 0)$	$(2, 3)$

By adding two strategies, σ is the unique equilibrium of $\hat{G}$:

$\hat{G} =$

	l	r	y
t	$(3, 2)$	$(0, 0)$	$(0, 1)$
b	$(0, 0)$	$(2, 3)$	$(-2, 4)$
x	$(1, 0)$	$(4, -2)$	$(-1, -1)$

- Hence, the strict equilibrium σ is sustainable in G.
- The mixed equilibrium is not sustainable (**prove it?**).
- This is in line with Myerson requirements.

Hofbauer conjecture

Hofbauer conjecture: A regular equilibrium is sustainable if and only if it has index $+1$.

- von Schemde & von Stengel (2008) proved the conjecture for 2-player games using polytopial geometry.
- We prove it for N-player games using algebraic topology.
- **Corollary 1**: since the sum of the indices of equilibria is $+1$, any regular game has a sustainable equilibrium.
- **Corollary 2**: Since the set of regular games is open and dense, almost every game has a sustainable equilibrium.