Myerson (1996) argued informally for a new refinement concept that he termed sustainable equilibria.

In this line of argument:

- **Strict** Nash equilibria are sustainable.
- Battle of sexes: only strict equilibria are sustainable.
- If a game has a unique equilibrium, it is sustainable.
- Every generic game has a sustainable equilibrium.
Hofbauer conjecture

Hofbauer (2000) expanded on Myerson’s idea and formalised the notion of sustainable equilibria.

- He defines an equivalence relation among pairs \((G, \sigma)\) where \(G\) is a game and \(\sigma\) is an equilibrium of \(G\).

- \((G, \sigma) \sim (\hat{G}, \hat{\sigma})\) if \(\sigma = \hat{\sigma}\) (up to a relabelling) and the restriction of \(G\) and \(\hat{G}\) to the best replies to \(\sigma\) and \(\hat{\sigma}\), resp., are the same game (up to a relabelling).

- An equilibrium \(\sigma\) of a game \(G\) is sustainable iff \((G, \sigma) \sim (\hat{G}, \hat{\sigma})\) and \(\hat{\sigma}\) is the unique equilibrium of \(\hat{G}\).
Example: Battle of the sexes

3 Nash equilibria: 2 strict $\sigma = (t, l)$ and $\theta = (b, r)$, and 1 mixed.

\[
G = \begin{array}{cc}
  & l & r \\
t & (3, 2) & (0, 0) \\
b & (0, 0) & (2, 3) \\
\end{array}
\]

By adding two strategies, $\sigma$ is the unique equilibrium of $\hat{G}$:

\[
\hat{G} = \begin{array}{ccc}
  & l & r & y \\
t & (3, 2) & (0, 0) & (0, 1) \\
b & (0, 0) & (2, 3) & (−2, 4) \\
x & (1, 0) & (4, −2) & (−1, −1) \\
\end{array}
\]

- Hence, the strict equilibrium $\sigma$ is sustainable in $G$.
- The mixed equilibrium is not sustainable (prove it?).
- This is in line with Myerson requirements.
Hofbauer conjecture

**Hofbauer conjecture:** A regular equilibrium is sustainable if and only if it has index $+1$.

- We prove it for $N$-player games using algebraic topology.
- **Corollary 1:** since the sum of the indices of equilibria is $+1$, any regular game has a sustainable equilibrium.
- **Corollary 2:** Since the set of regular games is open and dense, almost every game has a sustainable equilibrium.