An Improved Approximation Algorithm For MMS Allocation

Input
- Agents: \( N = \{1, 2, \ldots, n\} \)
- Indivisible items: \( M = \{1, 2, \ldots, m\} \)
- Additive valuation functions
  \[ v_i(S) = \sum_{j \in S} v_{ij} \quad \text{for all } i \in N, S \subseteq M \]

Output
- A 3/4-Maximin Share (MMS) allocation \( A_1, A_2, \ldots, A_n \) where
  \[ v_i(A_i) \geq \frac{3}{4} MMS_i \] (aka maximin value)
### MMS value / partition / allocation

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<th>Agents\items</th>
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Finding MMS value is hard!

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<thead>
<tr>
<th>value</th>
<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>MMS value</td>
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MMS allocation: \( v_i(A_i) \geq MMS_i \)

MMS allocation might not exist, but 3/4-MMS allocation always exist.
Algorithm Big Picture

To show the existence of $3/4$-MMS allocation:

We assume $\text{MMS}_i$ is known for all $i$

$$\implies \text{Scale valuations such that } \text{MMS}_i = 1 \text{ for all } i \Rightarrow v_i(M) \geq n$$

- Step 1: Valid Reductions
  - Exist $S \subseteq M$ and $i^* \in N$ such that $v_{i^*}(S) \geq (3/4)\text{MMS}_{i^*}^n(M)$
  - $\text{MMS}_{i^*}^{n-1}(M \setminus S) \geq \text{MMS}_{i^*}^n(M)$ for all $i \neq i^*$

- Step 2: Generalized Bag Filling
Results

Existence of \( \frac{3}{4} \)-MMS allocation

Strongly Polynomial-time Algorithm for \( \frac{3}{4} \)-MMS allocation

More careful analysis

Existence of \( \left( \frac{3}{4} + \frac{1}{(12n)} \right) \)-MMS allocation

PTAS for \( \left( \frac{3}{4} + \frac{1}{(12n)} \right) \)-MMS allocation

MMS values are known

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