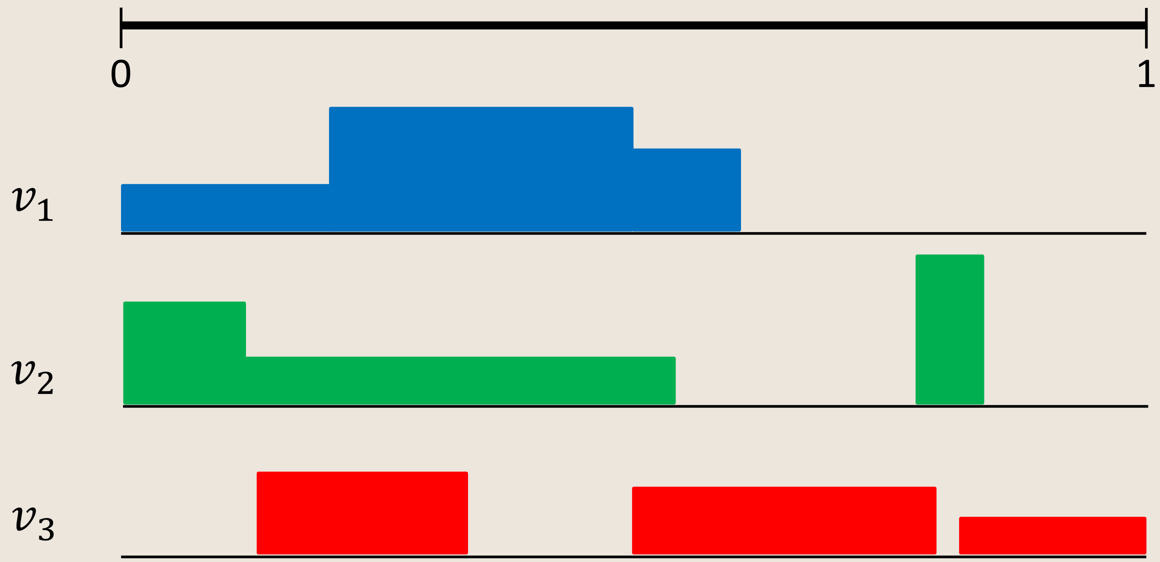
Consensus-Halving

- resource: the interval $I = [0,1]$
- n agents

  valuations $v_1, v_2, \dots, v_n$



Consensus-Halving:

A partition of $I = I_+ \cup I_-$, such that *all agents agree that the two pieces have the same value*

for all agents $i \in [n]$: $v_i(I_+) = v_i(I_-)$

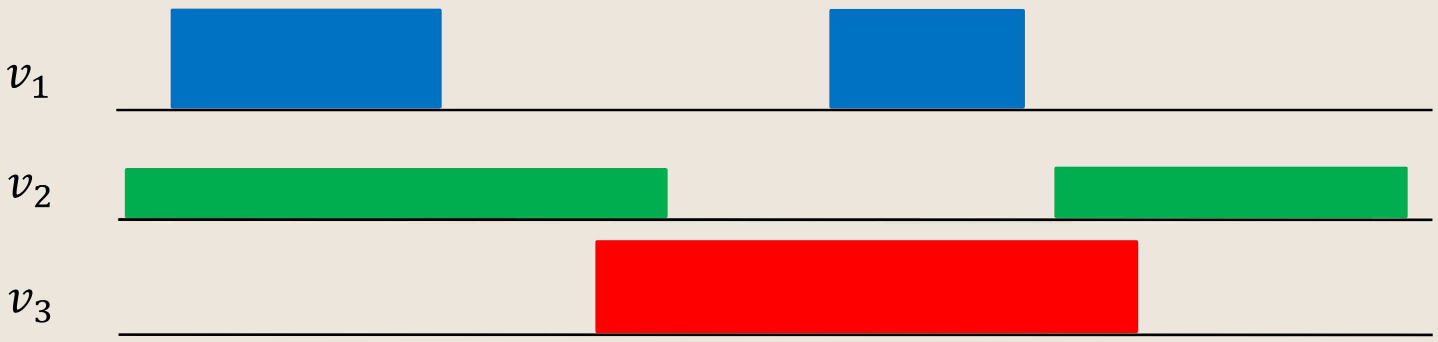
The Consensus-Halving Problem

Theorem [Hobby-Rice 1965, Simmons-Su 2003]:
There always exists a Consensus-Halving using at most n cuts.

Computational Problem: "Compute a Consensus-Halving that uses at most n cuts"

Theorem [Filos-Ratsikas, Goldberg 2018-19]:
Consensus-Halving is PPA-complete for piecewise-constant valuations.

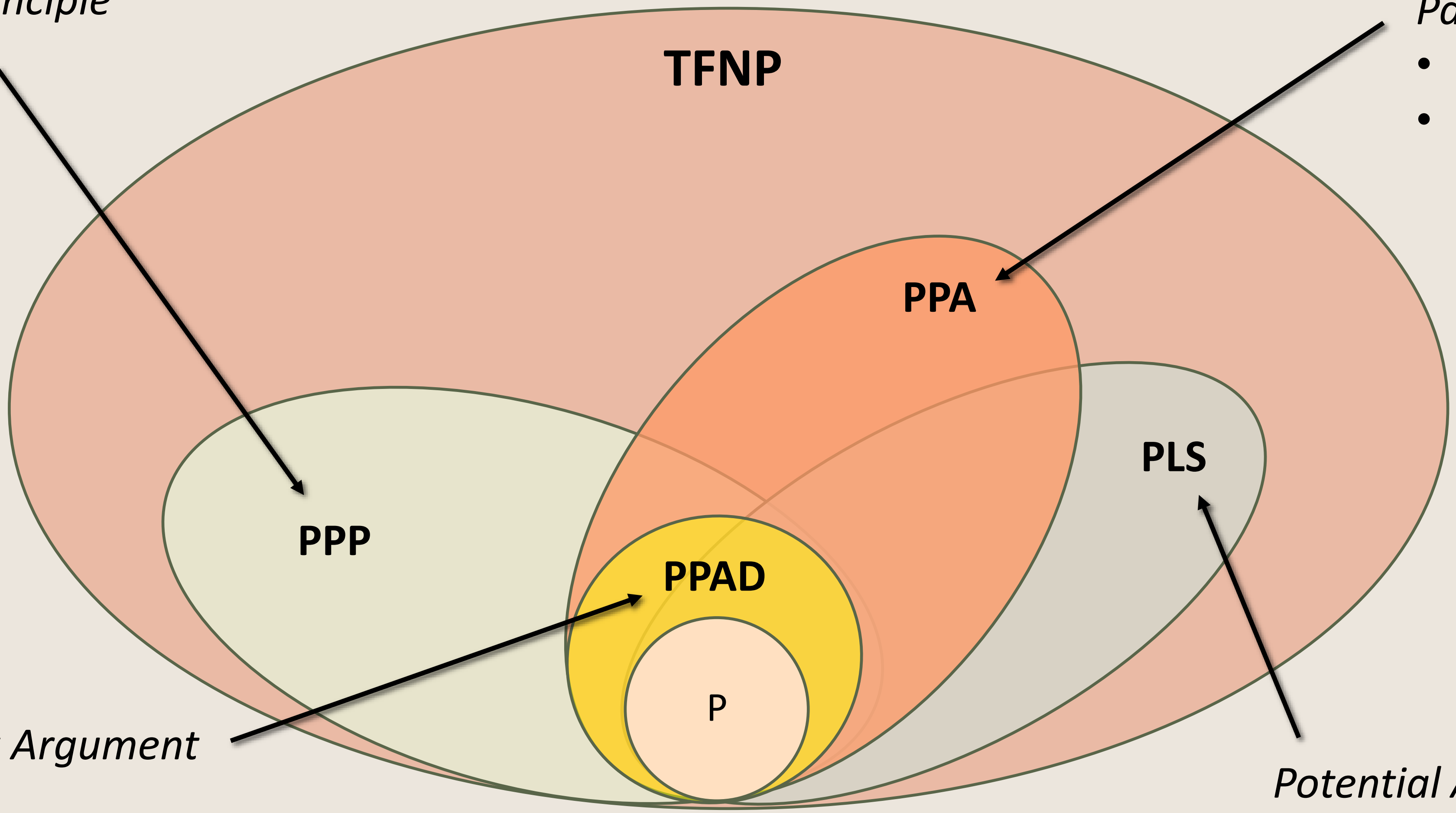
Theorem: Consensus-Halving is PPA-complete, even for *2-block uniform valuations*.



The TFNP landscape

Pigeonhole Principle

Parity Argument
- Borsuk-Ulam
- Consensus-Halving



Directed Parity Argument
- Brouwer
- Nash

Potential Argument/Local Search
- Local Max-Cut

Single-block valuations: Positive Results

Theorem:
ε-Consensus-Halving for single-block valuations can be solved in poly-time in the following cases:

- $\varepsilon = \frac{1}{2}$

  → technique: greedy algorithm
- $2n - k$ cuts allowed, for any constant k

  → technique: polynomial number of LPs
- the *maximum overlap number* d is constant

  → technique: dynamic programming