Best of Both Worlds: Ex-Ante and Ex-Post Fairness in Resource Allocation

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Implementing Fractional Allocations

\[
\begin{align*}
\text{Ex-ante Fair} & \quad \frac{1}{3} \times \emptyset \\
\text{Ex-post Unfair} & \quad \frac{1}{3} \times \text{candies} \\
\text{Ex-post Unfair} & \quad \frac{1}{3} \times \emptyset \\
\text{Ex-post Unfair} & \quad \text{candies}
\end{align*}
\]
Implementing Fractional Allocations

\[
\text{Ex-ante Fair} = \frac{1}{3} \times \text{Ex-post Fair} + \frac{1}{3} \times \text{Ex-post Fair} + \frac{1}{3} \times \text{Ex-post Fair}
\]
Can we always achieve ex-ante fairness by randomizing over ex-post fair allocations?

(Efficiently) possible: ex-ante EF + ex-post EF1

(Efficiently) possible: ex-ante GF + ex-post Prop1 + ex-post EF1

Impossible: ex-ante Prop + ex-post EF1 + ex-post fPO

Extensions to division of chores