Our Results

- Value of the buyer for the item is drawn from some distribution F.
- **Data-poor regime** : Seller has only two samples from F.

Theorem 1.
A rounded version of ERM from two samples attains revenue $\sim 0.56 \cdot \textsc{Opt}$ when F is regular.

Theorem 2.
There exists a regular distribution F under which ERM from two samples cannot do better than $0.642 \cdot \text{OPT}$.

Abstract Optimization Program Formulation

$$\alpha_\rho^\star = \min_{F,\textsc{Opt},p^\star} \quad \frac{1}{\textsc{Opt}} \text{(revenue of } \mathrm{ERM}_\rho \text{ for distribution } F\text{)}$$

$$\text{s.t.} \quad (F \text{ satisfies the regularity constraints})$$

$$(\text{the revenue for any price is at most } \textsc{Opt})$$

$$(\text{the revenue for price } p^\star \text{ is exactly } \textsc{Opt})$$

Semidefinite Programming Relaxation

- Instead of the vector of variables f_i define the 3-d tensor of variables $f_{i,j,k}$.
- **Replace $f_i \cdot f_j \cdot f_k$ with $f_{i,j,k}$.**
- Replace $f_i \cdot f_j$ with $f_{i,j} = \sum_k f_{i,j,k}$.
- Replace f_i with $\sum_k \sum_j f_{i,j,k}$.
- Let $F_i = (f_{i,j,k})_{j,k}$, **add the constraint $F_i \succeq 0$.**

Implementation Details

We implemented $\overline{\alpha}_{\rho,n}(i^\star)$ for all $i^\star \in [n/2, n]$ in python using `cvxpy` that calls the solver `SCS` [O'Donoghue Chu Parikh Boyd '16].

The solver `SCS` produces not only a solution to the primal but also a solution to the dual problem. Hence, the values of the dual variables provide **a proof** for a lower bound for the value of $\overline{\alpha}_{\rho,n}(i^\star)$!