Tight Revenue Gaps among Simple and Optimal Mechanisms

YAONAN JIN
Columbia University
and
PINYAN LU
Shanghai University of Finance and Economics
and
QI QI
Hong Kong University of Science and Technology
and
ZHIHAO GAVIN TANG
Shanghai University of Finance and Economics
and
TAO XIAO
Shanghai Jiao Tong University

Consider a fundamental problem in microeconomics: selling a single item to a number of potential buyers, who independently draw their values from regular and publicly known distributions. There are four mechanisms widely studied in the literature and widely used in practice: Myerson Auction (OPT), Sequential Posted Pricing (SPM), Second-Price Auction with Anonymous Reserve (AR), and Anonymous Pricing (AP).

OPT is revenue-optimal but complicated, which also experiences several practical issues such as fairness. AP is the simplest mechanism, but also generates the lowest revenue among these four mechanisms. SPM and AR are of intermediate complexity and revenue. A quantitative approach to comparing the relative power of these mechanisms is to study their revenue gaps, each of which is defined as the largest ratio between the revenues from a pair of mechanisms. This letter surveys some recent developments on establishing tight revenue gaps, and highlights some open questions.

Categories and Subject Descriptors: [**Theory of Computation**]: Algorithmic Mechanism Design

General Terms: Economics, Theory

Additional Key Words and Phrases: Revenue Maximization, Approximation Ratio

1. INTRODUCTION

How to maximize the expected revenue of a seller, who wants to sell an indivisible item to a number of buyers, is a central problem in microeconomics. The simplest mechanism is Anonymous Pricing (denoted by AP). Such a mechanism simply posts a price of $p \in \mathbb{R}_{\geq 0}$ to all buyers, and the item is sold out iff at least one buyer

Authors' addresses: jin.yaonan@columbia.edu, lu.pinyan@mail.shufe.edu.cn, kaylaqi@ust.hk, tang.zhihao@mail.shufe.edu.cn, xt_1992@sjtu.edu.cn

has a value no less than this price. If the seller knows the value distributions of the buyers, he can leverage a proper price to maximize the revenue (among this family of mechanisms). Although widely-used, this is not the revenue-optimal selling method – the optimal mechanism is the prominent Myerson Auction (denoted by OPT) [Myerson 1981]. In comparison, OPT is far more complex than AP, due to two reasons:

(a) It discriminates different buyers with different value priors. Conceivably, this may incur some fairness issues, and is not feasible in some markets.

(b) It is an *auction scheme* instead of a *pricing scheme*, thus requiring more seller-to-buyer communication. This may also raise privacy concerns to the buyers, since they need to report their private values rather than make take-it-or-leave-it decisions.

These complications and other undesirable features hinder the prevalence of Myerson Auction. To address these issues, two mechanisms with intermediate complexities (compared to OPT and AP) are widely studied in the literature, and are widely adopted in practice: (a) to avoid the price discrimination, the seller can use Second-Price Auction with Anonymous Reserve (denoted by AR) [Hartline and Roughgarden 2009]; and (b) to reduce communication, the seller can employ Sequential Posted Pricing (denoted by SPM) [Chawla et al. 2010].

These four mechanisms together form the lattice structure in Figure 1, in terms of both revenue-dominance and complexity. It is well known that there is a revenue gap between any pair of mechanisms. Naturally, the reader might query that how large these gaps can be.

Indeed, quantitative analysis of these gaps is also a striking theme in algorithmic economics. To this end, the notion of approximation ratio (originated from the TCS community) turns out to be a powerful language. There is a rich literature on studying the revenue gaps/approximation ratios among various mechanisms [Bulow and Klemperer 1994; Goldberg et al. 2001; Bar-Yossef et al. 2002; Guruswami et al. 2005; Koutsoupias and Pierrakos 2013; Chen et al. 2014; 2015; Fu et al. 2015; Dütting et al. 2016; Alaei et al. 2015; Correa et al. 2017].

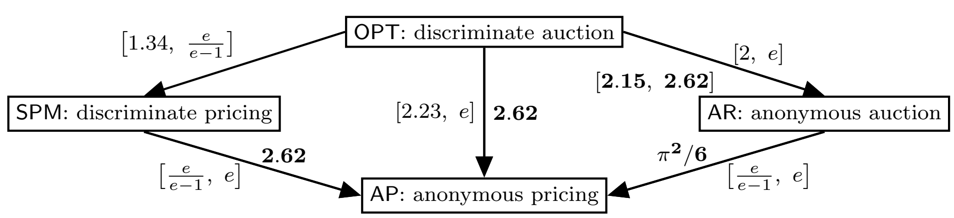


Fig. 1. The revenue gaps among the four basic mechanisms OPT, SPM, AR and AP in the asymmetric regular setting. Our results are marked in bold.

For the above four mechanisms, we establish in [Jin et al. 2019; Jin et al. 2019] three tight bounds and an improved bound in the canonical setting with asymmetric[1] and regular distributions. Prior to our work, no tight revenue gap between any pair of the four mechanisms was known.

SPM vs. AP. This comparison measures the power of discrimination in pricing schemes. We establish the tight ratio of constant $\mathcal{C}^* \approx 2.62$. Prior to our work, tight ratios were known to be (a) n in the asymmetric general[2] setting [Alaei et al. 2015]; (b) $\frac{e}{e-1} \approx 1.58$ in the i.i.d. regular setting; and (c) 2 in the i.i.d. general setting [Hartline 2013; Dütting et al. 2016]. Actually, we can also get the last two ratios by combining the results in [Myerson 1981; Krengel and Sucheston 1978; Hill et al. 1982], which was first observed by [Hajiaghayi et al. 2007].

OPT vs. AP. The same tight ratio of $\mathcal{C}^* \approx 2.62$ is achieved by the revenue gap between SPM and AP. Indeed, in our worst case instance of the SPM vs. AP problem, the optimal Sequential Posted Pricing mechanism coincides with Myerson Auction. This result answers a central question in the "simple versus optimal" research program. Previously, an upper bound of $e \approx 2.72$ was proved by [Alaei et al. 2015].

AR vs. AP. This comparison concerns the relative power between auction scheme and pricing scheme, when no price discrimination is allowed. We first (a) prove an upper bound of $\frac{\pi^2}{6} \approx 1.64$ in the asymmetric general setting, and then (b) construct matching lower-bound instances respectively in the asymmetric regular setting and the i.i.d. general setting. Before our work, (c) in the i.i.d. regular setting, where AR is identical to OPT, an upper-bound of $\frac{e}{e-1} \approx 1.58$ was first obtained by [Chawla et al. 2010], and then was shown to be tight by [Hartline 2013].

OPT vs. AR. This comparison studies the power of price discrimination in auction schemes. Prior to our work, the tight ratios were known in all settings [Myerson 1981; Hartline 2013; Alaei et al. 2015] except for the asymmetric regular setting. [Hartline and Roughgarden 2009] first tackled the problem in this setting – they (a) proved an upper-bound of 4, and (b) provided a 2-approximation lower-bound instance. Although this lower bound of 2 has never been broken (for a decade) and is widely believed to be the final answer, we establish a sharper 2.15-approximation instance in [Jin et al. 2019].

More concretely, we settle all of the SPM vs. AP problem, the OPT vs. AP problem and the AR vs. AP problem by formulating each revenue gap as the objective function of a math program. This methodology was initiated by [Chen et al. 2014] and [Alaei et al. 2015]. Employing a similar approach, [Birmpas et al. 2017] recently obtained a tight *price of anarchy* for multi-unit auctions. Our work further supports the power of this framework in proving tight bounds. En route, we have developed an abundance of tools to handle these math programs, which may find extra applications in the future.

In the remainder of this letter, we shall focus on the SPM vs. AP problem, elaborating on how to formulate this optimization problem and then reduce it to a

[1]Throughout the letter, asymmetric distributions refer to the setting where different buyers may have distinct value distributions, as opposed to identical value distributions.
[2]Here, the general setting refers to the case where the distributions are not necessarily regular.

clear math program. For the other claimed results, we invite the readers to read the full papers for more details.

2. PRELIMINARIES

We will consider the single-item Bayesian mechanism design environment, where $n \in \mathbb{N}_{\geq 1}$ buyers independently draw their values $\mathbf{b} = \{\mathrm{b}_i\}_{i=1}^n \in \mathbb{R}_{\geq 0}^n$ from known distributions $\mathbf{F} = \{\mathrm{F}_i\}_{i=1}^n$. We begin with some necessary definitions and notations.

2.1 Value Distribution

Regular Distribution. For any CDF F and the corresponding PDF f:

(a) The *virtual value function* is defined as $\varphi(\mathrm{p}) \stackrel{\mathrm{def}}{=} \mathrm{p} - \frac{1-\mathrm{F}(\mathrm{p})}{\mathrm{f}(\mathrm{p})}$.

(b) The *revenue-quantile curve* is defined as $\mathrm{R}(\mathrm{q}) \stackrel{\mathrm{def}}{=} \mathrm{q} \cdot \mathrm{F}^{-1}(1-\mathrm{q})$.

The distribution F is *regular* iff the virtual value function φ is non-decreasing, or equivalently, iff the revenue-quantile curve R is a concave function. We will work more with the second definition, since it is more convenient for our use. Denote by Reg the family of all regular distributions.

Triangular Distribution. This family of distributions is introduced in [Alaei et al. 2015], named in view of the shapes of their revenue-quantile curves (as Figure 2 illustrates). With two parameters $\mathrm{v}_i \in \mathbb{R}_{\geq 0}$ and $\mathrm{q}_i \in [0,\ 1]$, a triangular distribution $\textsc{Tri}(\mathrm{v}_i,\ \mathrm{q}_i)$ has a CDF of $\mathrm{F}_i(\mathrm{p}) = \frac{(1-\mathrm{q}_i)\cdot \mathrm{p}}{(1-\mathrm{q}_i)\cdot \mathrm{p} + \mathrm{v}_i \mathrm{q}_i}$ when $\mathrm{p} \in [0,\ \mathrm{v}_i)$ and $\mathrm{F}_i(\mathrm{p}) = 1$ when $\mathrm{p} \in [\mathrm{v}_i,\ \infty)$. This distribution family Tri in some sense lies on the boundary of the regular distribution family Reg, and plays an important role in our results.

Particularly, when $\mathrm{H} \to \infty$, the distribution $\textsc{Tri}(\mathrm{H},\ \frac{1}{\mathrm{H}})$ has a limitation CDF of $\mathrm{F}(\mathrm{p}) = \frac{\mathrm{p}}{\mathrm{p}+1}$, for all $\mathrm{p} \in \mathbb{R}_{\geq 0}$. Denote this special limitation distribution by $\textsc{Tri}(\infty)$, which will be involved in the worst-case instance of the SPM vs. AP problem.

2.2 Mechanisms

Anonymous Pricing (AP). The seller posts a price of $\mathrm{p} \in \mathbb{R}_{\geq 0}$ to all buyers, and the item is sold out iff at least one buyer values the item no less than this price. For brevity, we denote by $\mathsf{AP}(\mathrm{p}) \stackrel{\mathrm{def}}{=} \mathrm{p} \cdot \big(1 - \prod_{i=1}^n \mathrm{F}_i(\mathrm{p})\big)$ the resulting revenue.

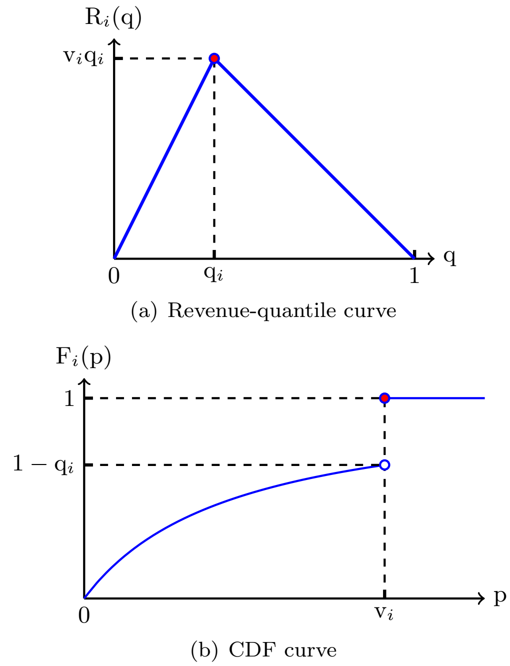
Sequential Posted-Pricing (SPM). Such a mechanism involves a price vector $\mathbf{p} = \{\mathrm{p}_i\}_{i=1}^n \in \mathbb{R}_{\geq 0}^n$, with each price p_i posted sequentially to the i-th coming buyer. Without loss of generality, we assume the i-th coming buyer is exactly the index-i buyer. Then, the first coming buyer with a value of $\mathrm{b}_i \geq \mathrm{p}_i$ will win the item. We denote by $\mathsf{SPM}(\mathbf{p})$ the resulting revenue.

3. SEQUENTIAL POSTED PRICING VS. ANONYMOUS PRICING

We interpret this SPM vs. AP problem as the following math program, and safely drop constraint (C1) on interval $\mathrm{p} \in [0,\ 1]$ as it trivially holds.

$$\max_{\mathbf{F} \in \textsc{Reg}^n} \quad \mathsf{SPM} = \max_{\mathbf{p} \in \mathbb{R}_{\geq 0}^n} \big\{\mathsf{SPM}(\mathbf{p})\big\} \tag{P1}$$

$$\text{subject to:} \quad \mathsf{AP}(\mathrm{p}) = \mathrm{p} \cdot \big(1 - \textstyle\prod_{i=1}^n \mathrm{F}_i(\mathrm{p})\big) \leq 1 \qquad \forall \mathrm{p} \in (1,\ \infty) \tag{C1}$$



(a) Revenue-quantile curve

(b) CDF curve

Fig. 2. Demonstration for the triangular distribution $\textsc{Tri}(\mathrm{v}_i,\ \mathrm{q}_i)$.

Now, consider a certain regular instance $\mathbf{F} = \{\mathrm{F}_i\}_{i=1}^n$ and an optimal Sequential Posted Pricing mechanism for it. Let $\mathbf{p}^* = \{\mathrm{p}_i^*\}_{i=1}^n$ be the involved posted prices. For each buyer $i \in [n]$, define $\mathrm{v}_i \stackrel{\text{def}}{=} \mathrm{p}_i^*$ and $\mathrm{q}_i \stackrel{\text{def}}{=} 1 - \mathrm{F}_i(\mathrm{p}_i^*)$. These two parameters are crucial, in the sense that the revenue from the concerning mechanism (using the prices $\mathbf{p}^* = \{\mathrm{v}_i\}_{i=1}^n$) is fully captured by the parameters $\{\mathrm{q}_i\}_{i=1}^n$. In other words, if another instance $\overline{\mathbf{F}} = \{\overline{\mathrm{F}}_i\}_{i=1}^n$ satisfies that $\overline{\mathrm{F}}_i(\mathrm{v}_i) = \mathrm{F}_i(\mathrm{v}_i)$ for each $i \in [n]$, then adopting the same prices $\mathbf{p}^* = \{\mathrm{v}_i\}_{i=1}^n$ to this new instance results in the same amount of expected revenue. Indeed, the revenue is given by

$$\mathsf{SPM}(\mathbf{p}^*) = \mathsf{SPM}(\{\mathrm{v}_i\}_{i=1}^n) = \textstyle\sum_{i=1}^n \mathrm{v}_i\mathrm{q}_i \cdot \prod_{j=1}^{i-1}(1 - \mathrm{q}_j).$$

As Figure 3 illustrates, let us squeeze each distribution F_i to the triangular distribution $\textsc{Tri}(\mathrm{v}_i,\ \mathrm{q}_i)$. Based on the above arguments, the objective function of our math program (namely the SPM revenue) remains the same, while the constraint gets relaxed – due to the stochastic dominance $\mathrm{F}_i \succeq \textsc{Tri}(\mathrm{v}_i,\ \mathrm{q}_i)$ for each $i \in [n]$. As mentioned, the triangle distributions lie on the boundary of the regular distribution family $\textsc{Reg}$, which means we cannot squeeze the instance any further.

In addition, we can verify that any instance admits an optimal Sequential Posted Pricing mechanism $\mathsf{SPM}(\mathbf{p}^*)$ that uses decreasing prices $\mathrm{p}_1^* \geq \mathrm{p}_2^* \geq \cdots \geq \mathrm{p}_n^*$. This allows us to concentrate merely on those triangle instances $\{\textsc{Tri}(\mathrm{v}_i,\ \mathrm{q}_i)\}_{i=1}^n$ with $\mathrm{v}_1 \geq \mathrm{v}_2 \geq \cdots \geq \mathrm{v}_n$. Since such an instance has explicit CDF's, plugging these CDF's into the above $\mathsf{AP}(\mathrm{p})$ and SPM revenue formulas and rearranging the constraint, we can deduce a clearer math program as follows:

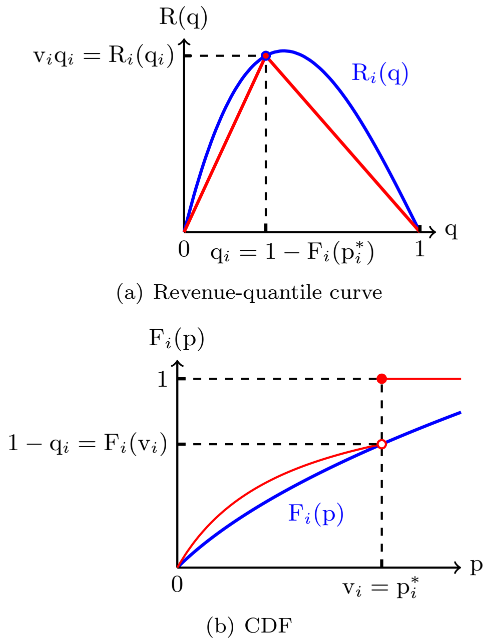


Fig. 3. Reduction from a regular instance $\{F_i\}_{i=1}^n$ to a particular triangular instance $\{\text{TRI}(v_i,\ q_i)\}_{i=1}^n$.

$$\max_{\{\text{TRI}(v_i,\ q_i)\}_{i=1}^n} \quad \mathsf{SPM} = \sum_{i=1}^n v_i q_i \cdot \prod_{j=1}^{i-1}(1 - q_j) \tag{P2}$$

$$\text{subject to:} \quad \sum_{i:\ v_i \geq p} \ln\left(1 + \frac{v_i q_i}{1 - q_i} \cdot \frac{1}{p}\right) \leq -\ln(1 - p^{-1}) \quad \forall p \in (1,\ \infty) \tag{C2}$$

$$v_1 \geq v_2 \geq \cdots \geq v_n$$

The next step is to show the existence of two special buyers $\text{TRI}(\infty)$ and $\text{TRI}(1,\ 1)$ in a worst case instance. Intuitively, $\text{TRI}(\infty)$ is the most effective buyer in extracting SPM revenue – he gives an expected revenue of $\lim_{H \to \infty} H \cdot (1 - \frac{1}{H+1}) = 1$ but with a *negligible* winning probability of $\lim_{H \to \infty} \frac{1}{H+1} = 0$. In addition, the other special buyer $\text{TRI}(1,\ 1)$ has a *deterministic* value of 1 – the seller can acquire a promised revenue of 1 from him, even if all the other buyers refuse their posted-price offers. For ease of presentation, we omit those technical details here.

To conclude, we write down the following math program, and solve it optimally. Notably, the optimal solution (i.e., the worst-case instance) turns out to be reached when constraint (C3) is tight everywhere for any price $p \in (1,\ \infty)$.

$$\max_{\{\text{TRI}(v_i,\ q_i)\}_{i=1}^n} \quad \mathsf{SPM} = 2 + \sum_{i=1}^n (v_i - 1) \cdot q_i \cdot \prod_{j=1}^{i-1}(1 - q_i) \tag{P3}$$

$$\text{subject to:} \quad \sum_{i:\ v_i \geq p} \ln\left(1 + \frac{v_i q_i}{1 - q_i} \cdot \frac{1}{p}\right) \leq -\ln(1 - p^{-2}) \quad \forall p \in (1,\ \infty) \tag{C3}$$

$$v_1 > v_2 > \cdots > v_n > 1$$

4. OPEN QUESTIONS

This letter reports our recent works on studying the revenue gaps among different families of single-item mechanisms, mostly in the setting with asymmetric regular distributions. Despite the exciting progress, two important revenue gaps about the concerning mechanisms have yet to be understood.

OPT vs. AR. A most attractive research direction is to pin down the exact ratio of Second-Price Auction with Anonymous Reserve against Myerson Auction. Our result certifies that the final answer sits in the range of [2.15, 2.62]. On the other hand, we highly believe that neither of the upper and lower bounds are tight.

Interestingly, the previous lower-bound instance of [Hartline and Roughgarden 2009] consists of two buyers, yet our sharper instances in [Jin et al. 2019] involve three or four buyers. Arguably, when there are more buyers, the ratio may further increase. We thus conjecture that the worst-case instance contains an *infinite* number of buyers, and the optimization techniques from [Alaei et al. 2015; Jin et al. 2019; Jin et al. 2019] might be the right tool to attack it.

OPT vs. SPM. Due to the reduction in [Hajiaghayi et al. 2007; Chawla et al. 2010; Correa et al. 2019], designing the optimal Sequential Posted Pricing mechanisms is equivalent to finding the optimal stopping rules against a *prophet* (in a certain n-choose-1 game). Namely, the OPT vs. SPM problem has the same tight ratio as the *ordered prophet inequality*. For a full survey on that topic, the interested readers can turn to [Lucier 2017; Correa et al. 2018].

Other Comparisons. In the literature, some other practical mechanisms have been investigated as well, mostly measuring their revenue gaps against Myerson Auction. E.g., [Beyhaghi et al. 2018; Ma and Sivan 2019] studied Second-Price Auction with Personalized Reserves, and obtained lower and upper bounds of [1.29, 1.50]. Apart from that ratio, the revenue gap between this family of mechanisms and SPM or AR or AP is also interesting to explore.

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