

# Assortment Optimization: An Annotated Reading Assortment

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Which varieties or brands of a product should a retailer stock on its shelf? Carrying a large variety caters to more customers' needs, but could cannibalize the sales of high-end brands and also cause an inventory nightmare. Assortment optimization aims to formalize these tradeoffs, with the basic problem being as follows. There is a universe of brands  $j \in U$ , each with a market-accepted price  $r_j$ . For any  $S \subseteq U$ , a function  $\phi(j, S)$  indicates the probability that a representative customer from the population would purchase  $j$  when given the choice from *assortment*  $S$ , under the market prices. The optimization problem is to maximize the average revenue per customer, i.e.

$$\max_S \sum_{j \in S} r_j \phi(j, S), \quad (1)$$

possibly with constraints on  $S$  due to shelf size. Assortment optimization started out by showing how to efficiently find the optimal  $S$  from the exponentially many possibilities, under well-established parametric forms for the function  $\phi$  that are called (*discrete*) *choice models*. Since then, the literature has developed choice models of its own that are specialized for assortment optimization. The basic problem has also been extended, and connected with topics such as online algorithms, machine learning, and mechanism design that are mainstream in the Economics and Computation community, with a vast horizon for future directions.

This is an annotated reading list about assortment optimization, that aims to provide broad coverage while facing a “cardinality constraint” on the number of papers in the assortment.

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## 1. INITIAL THEORY ON EXISTING CHOICE MODELS

- (1) [Talluri and Van Ryzin 2004] An early work to maximize revenue on discrete choice models. Although this paper focuses on the online revenue management problem, a key result attributed to it is that even in the basic problem (1), the optimal assortment has a *revenue-ordered* structure when  $\phi$  falls under the *Multi-Nomial Logit (MNL)* choice model. To elaborate, revenue-ordered means that  $S = \{j \in U : r_j \geq \tau\}$  for some price threshold  $\tau$ . Meanwhile, MNL imposes that  $\phi(j, S) = w_j / \sum_{j' \in S \cup \{0\}} w_{j'}$  for some popularity weights  $w_{j'}$ . MNL is the simplest choice model to capture a non-trivial form of *cannibalization*, where inserting  $j'$  into the assortment  $S$  decreases the probability of the customer choosing any other  $j$ , and 0 represents the “no-purchase” option whose weight is often normalized to 1. This paper also discusses how to estimate the popularity weights in MNL.
- (2) [Rusmevichientong et al. 2010] An early paper to consider constraints on the assortment  $S$ , motivated by limited shelf size. The authors develop a search technique that solves cardinality-constrained assortment optimization for MNL. Importantly, they show that the optimal assortment may no longer be revenue-ordered if there is a cardinality constraint, contrasting the structural result of

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[Talluri and Van Ryzin 2004]. This paper also studies how to learn the optimal cardinality-constrained assortment when the function  $\phi$  is initially unknown.

- (3) [Gallego and Topaloglu 2014] Nested Logit is a more general parametric choice model that increases the expressiveness of MNL by allowing for products to be categorized into nests, and having more cannibalization within nest than across nests. This paper makes substantial progress on assortment optimization for Nested Logit, developing a Linear Program to solve the problem even with a cardinality constraint on  $S$ . It is also one of the early papers to take an approximation algorithms approach to assortment optimization, by developing a 2-approximation under the more general knapsack constraints. Finally, this paper shows how pricing decisions can be captured using assortment optimization, by creating copies of products with different prices  $r_j$  and adding constraints that only one price level for each product can be offered.

## 2. NEW CHOICE MODELS FOR ASSORTMENT OPTIMIZATION

Parametric choice models have nice analytical forms for the function  $\phi$ , that are also explained by customers drawing random valuations  $V_{j'}$  for each  $j' \in U \cup \{0\}$  and defining  $\phi(j, S) = \Pr[j = \operatorname{argmax}_{j' \in S \cup \{0\}} V_{j'}]$ . However, model selection can be difficult—too few parameters and your demand is misspecified; too many parameters and you overfit. These papers propose non-parametric choice models that focus on prescribing assortment decisions, without worrying about how explainable  $\phi$  is from random-utility theory.

- (4) [Farias et al. 2013] This paper proposes a paradigm for choice modeling where customers have a latent distribution of *ordinal* rankings over  $U \cup \{0\}$ , motivated by the fact that transaction data in practice only involves a customer making comparisons. The authors take a robust approach to estimating the ranking distribution and solving the assortment optimization problem, that automatically tunes model complexity based on the data.
- (5) [Blanchet et al. 2016] Also motivated by the challenge in model selection, this paper proposes to only capture a customer’s first two choices in a Markov chain, and shows how this can simultaneously approximate all random-utility discrete choice models. This surprising insight gives birth to the Markov Chain choice model, and the authors also show how to solve assortment optimization on it.

## 3. ONLINE VARIANTS

These papers consider the dynamics of multiple sales from different assortments.

- (6) [Golrezaei et al. 2014] An assortment is classically interpreted as a deliberate set of products carried by a brick-and-mortar retailer. This paper considers the personalized assortments that can be offered by an online retailer, and shows how to adjust these based on remaining inventories. It introduces primal-dual analysis and competitive ratios to the assortment optimization literature, and initiated a large body of work that incorporates assortment optimization into online algorithms.
- (7) [Goyal et al. 2016] This paper revitalizes the *dynamic substitution model* where one must jointly decide the assortment  $S$  and how much of each  $j \in S$  to stock,

which was one of the original motivations of assortment optimization stemming from inventory theory. A stochastic sequence of customers arrives, choosing from the subset of  $j$  within  $S$  that have remaining inventory. In this model the decision is initial inventory after which the assortment cannot be controlled, contrasting the model of [Golrezaei et al. 2014] where initial inventories are given but the decision is how to dynamically control assortments. This paper develops NP-hardness and PTAS type results for a new formulation where the objective is to maximize revenue subject to a constraint on the total number of units initially stocked, which would become the subject of several follow-ups.

- (8) [Agrawal et al. 2019] This paper considers the joint learning and optimization problem facing a stream of customers who choose according to the same unknown MNL model. It introduces a neat analysis for exploring and exploiting at the same time, illustrating the richness that choice models bring to the classical exploration-exploitation tradeoff and spawning a whole literature on “MNL-Bandit”.

#### 4. FUTURE DIRECTIONS

- (9) [Aouad and Désir 2022] Motivated by the success of deep learning in prediction, this paper proposes an architecture that uses neural networks to accurately predict customer choices. It leverages high-dimensional contextual information about customers and products, while preserving the random-utility structure underlying most choice models. Although this paper does not directly optimize assortments, it plants the seeds for a potentially rich literature that relates assortments to the optimization of inputs to trained machine learning models.
- (10) [Ma 2023] Assortment optimization has been fixated on solving problem (1), which can be interpreted as posting a single assortment for the customer to choose from. But what about more general selling methods? This paper captures assortment optimization as part of a more general Bayesian mechanism design problem, in which customers have ordinal preferences over fixed-price items. It characterizes choice models for which the optimal mechanism is or is not a posted assortment. More generally, assortment optimization motivates the design of Bayesian mechanisms without arbitrary payments, which is potentially a large ground for innovation in the Economics and Computation community.

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