

Diversification in the Internet Economy: The Role of For-Profit Mediators

SUDHIR KUMAR SINGH

University of California, Los Angeles

VWANI ROYCHOWDHURY

University of California, Los Angeles & NetSeer, Inc.

HIMAWAN GUNADHI

NetSeer, Inc.

and

BEHNAM REZAEI

NetSeer, Inc.

We investigate market forces that would lead to the emergence of new classes of players in the sponsored search advertising market. We report a multi-fold diversification triggered by an inherent feature of the sponsored search market, namely, *capacity constraints*, arising from the fact that there is a limit on the number of available advertisement slots, especially for the *popular keywords*. As a result, a significant pool of interested advertisers are left out. We present a comparative study of two scenarios motivated by capacity constraints - one where the additional capacity is provided by for-profit agents (or, mediators), who compete for slots in the original auction, draw traffic, and run their *own sub-auctions*, and the other, where the additional capacity is provided by the auctioneer herself, by essentially acting as a mediator and running a *single combined auction*. The quality of the additional capacity is measured by its *fitness* factor. We observe that the single combined-auction model seems inferior to the mediator-based model and market becomes more *capacity efficient* in the latter. For instance, the revenue of the auctioneer always increases when mediators are involved, unlike the auctioneer based scenario where often there is a tradeoff between the revenue and the capacity. Further, the social value (i.e. efficiency) always increases when mediators are involved. Thus, our analysis indicates that there are *significant opportunities for diversification* in the internet economy and we should expect it to continue to develop richer structure, with room for different types of market entities and mechanisms to coexist.

Categories and Subject Descriptors: H.4.m [Information Systems]: Miscellaneous

General Terms: Algorithms, Economics, Theory

Additional Key Words and Phrases: Capacity Constraints, Diversification, Internet Economy, Mechanism Design, Mediators, Sponsored Search

A preliminary version of this work was presented in WINE 2007 wherein the mediator-based model was studied [Singh et al. 2007].

Authors' Address: Department of Electrical Engineering, University of California, Los Angeles, CA 90095.

Netseer Inc., 11943 Montana Ave. Suite 200, Los Angeles, CA 90049.

Emails: {sudhir, vwani, gunadhi, behnam}@netseer.com

Permission to make digital/hard copy of all or part of this material without fee for personal or classroom use provided that the copies are not made or distributed for profit or commercial advantage, the ACM copyright/server notice, the title of the publication, and its date appear, and notice is given that copying is by permission of the ACM, Inc. To copy otherwise, to republish, to post on servers, or to redistribute to lists requires prior specific permission and/or a fee.

© 2008 ACM 1529-3785/2008/0700-0001 \$5.00

1. INTRODUCTION

Sponsored search advertising (SSA), where advertisers pay to appear alongside the algorithmic/organic search results, is a significant growth market and is largely responsible for the success of Internet Search giants such as Google and Yahoo!. The statistics show that the growth of the overall online advertising market has been around 30% every year, as compared to the 1-2% of the traditional media, and is expected to increase to \$35.4 billion in 2012 from around \$20 billion in 2007.

In this form of advertising, the Search Engine allocates the advertising space using an auction. Advertisers bid upon specific keywords (i.e. query words). When a user searches for a keyword, the search engine (i.e. the auctioneer) allocates the advertising space to the bidding merchants based on their bid values and *quality scores*, and their ads are listed accordingly. Usually, the sponsored search results appear in a separate section of the page designated as “sponsored links” above/below or to the right of the organic/algorithmic results and have similar display format as the algorithmic results. Each position in such a list of sponsored links is called a *slot*. Whenever a user clicks on an ad, the corresponding advertiser pays an amount specified by the auctioneer; hence, the term *Cost Per Click* (CPC). Generally, users are more likely to click on a higher ranked slot, and therefore, advertisers prefer to be in higher ranked slots and compete for them. The auction currently used by Google and Yahoo! is a generalization of the Vickrey auction [Vickrey 1961], and is referred to as the *GSP* (Generalized Second Price) mechanism. GSP is tailored to the unique requirements of SSA, and has quite different incentive properties than the original Vickrey auction, and has been extensively studied in recent years [Edelman et al. 2007; Varian 2007; Lahaie 2006; Aggarwal et al. 2006; Lahaie and Pennock 2007].

The analysis of the underlying SSA models has so far primarily focused on the scenario where advertisers/bidders interact directly with a primary auctioneer or AdNetwork, e.g., they bid for ad-space at leading search engine and publisher portals. The market, however, has evolved rapidly, and is already witnessing the spontaneous emergence of several categories of companies who are trying to mediate or facilitate the auction process. The main focus of such entities is to generate relevant leads or traffic for the advertisers. For example, a whole genre of companies, collectively referred to as the *online lead generation market*, specialize in aggregating traffic to their sites by bidding for keywords on major portals and search engines. Then, instead of selling services and products themselves, they have advertisers signed up on their sites to capture the funneled traffic. The exact pricing model for the leads sold at these sites varies a great deal, including cost-per-thousand impressions (CPM), CPC and Cost per Action (CPA), where “action” could imply completion of a certain transaction by the lead at the advertiser’s site. Examples of such companies include, Oversee.net, LeadClick Media, ad pepper, ValueClick etc. and, according to IDC¹, the lead-generation market is the fastest growing segment of online advertising and in 2007 raked in more than \$1.5 billion in revenues.

There are several *unexplored fundamental issues* that come up when one considers the *combined system* comprising both the primary auctioneers and the mediators. For example, in the above mentioned lead-generation scenario, advertisers have the choice to either

¹<http://www.gpbullhound.com/research.php>, <http://tmginteractive.com/Sector%20Report%20Online%20Lead%20Generation%20March%202007.pdf>

directly place their ads on the search portals (i.e., the primary auctioneers), or buy leads from the mediators, or pursue both avenues. Then, are there inherent unmet demands or inefficiencies in the SSA model that enable mediators to fill an economic need and survive, or would in the long term the primary auctioneers simply change mechanisms or improve efficiency and take over the services provided by the mediators? How are the revenues of the primary auctioneers effected by the presence of the mediators? What are some of the mechanisms that the middlemen can use so as to carve out an efficient niche? In other words, we ask whether the primary auctioneers and the for-profit mediators can *coexist in an economic and game theoretic sense*, and if they do coexist, then what ramifications would it have on the overall efficiency and the utility of the advertisers.

In the present work, we adopt a fundamental approach, where we first identify an inherent feature of the sponsored search advertising market, namely *capacity constraints*, which could be key to the emergence of new market entities. This natural constraint in the SSA framework arises from the fact that there is a limit on the number of available advertisement slots (or *slots that receive any clicks from users*), especially for the *popular keywords*, and as a result, a significant pool of advertisers are left out. Consequently, new market mechanisms, as well as, new for-profit agents are likely to emerge to combat or to make profit from the opportunities created by scarcity in ad-space inventory. We show that this unmet need, i.e., the need of a large pool of advertisers to get leads, triggers a 3-fold diversification in the terms of

- the emergence of new market mechanisms
- the emergence of new for-profit agents, and
- the participation of a wider pool of bidders/advertisers.

First, we propose a model where the additional capacity is provided by for-profit agents (or, mediators), who compete for slots in the original auction, draw traffic, and run their own sub-auctions. We show that the revenue of the auctioneer, as well as the social value (i.e. efficiency), always increase when mediators are involved. Next, we ask the question—what if the auctioneer wants to provide the additional capacity herself by essentially acting herself as a mediator and running a *single combined auction*? Do the revenue of the auctioneer and overall efficiency improve or do they degrade in such a model? We show that, unlike the mediator-based model, there is often a tradeoff between the revenue and the capacity, and there is a phase transition from possibly a gain in terms of revenue to a loss as the *fitness* (a measure of the quality of the additional capacity) increases, meaning that there is a critical fitness value beyond which the auctioneer always loses in revenue. However, there exist scenarios where the revenue of the auctioneer could indeed increase by increasing capacity. In the case of efficiency, the result is more in consonance with the mediator-based model, i.e., the efficiency increases as fitness increases. However, unlike the mediator-based model, the efficiency could indeed decrease by increasing capacity.

Our results and analysis indicate that for-profit mediators that can increase capacity and ad inventory space can indeed coexist along with primary auctioneers. In fact, they add significantly to the overall efficiency and the utility of the advertisers. Thus, the SSA market becomes more *capacity efficient* by the involvement of such mediators, and we should expect such entities to proliferate and continue to thrive. The market, however, has had concerns about certain other kinds of mediators, particularly those that abuse inefficiencies present in the market, and both Google and Yahoo! have taken measures to actively discourage and eliminate such entities. As discussed in the following, the mediators that we

have analyzed are very different in nature and can survive only if they enhance both user experience, and quality of traffic for the advertisers.

- (1) Our model is motivated by a few examples from *Google Adwords*, provided in the *APPENDIX*. For example, the mediator “business.com” bids for the keyword “888 number” and then sells the leads at its own site via a second auction. Similarly, the mediator “personalloans.com” does the same for the keyword “easy loans”. Of course, the real world mediators use mechanisms other than secondary auctions to sell the leads that they aggregate. The analysis of such combined systems (i.e., keyword auctions at the primary site and different pricing mechanisms at the mediators’ sites) can also be carried out in a manner similar to the approach adopted in this paper.
- (2) The success of a mediator depends on how well she creates the additional capacity, and how efficiently she sells them. The first aspect is captured by a *fitness* factor in our model, which is essentially a measure of the quality of the additional capacity provided by the mediator. The second aspect is captured by the value she derives by selling the additional capacity. Both these quantities are formally defined later in the paper. Intuitively, it is important that the fitness of the mediator be very good so that she can ensure a better value (i.e. revenue from selling additional capacity) and be competent in bidding for and obtaining a slot in the primary auction (i.e. at the search portal). Consequently, a mediator with poor fitness will not be able to survive in the market. Thus, the kind of mediators we study in this paper are specifically the ones who can efficiently create extra capacity (i.e., increase ad inventory) while enhancing user experience.
- (3) In general, there could be several other inefficiencies in the SSA framework, and the market may naturally see the emergence of different kinds of for-profit agents as a result of these inefficiencies. For example, the tail queries or infrequent keywords, can easily comprise 40% or more of the total query volume at any search portal. Individually, each such keyword is difficult to identify, and even if identified, it does not have high enough volume to be attractive enough for advertisers to place bids on. Consequently, a significant fraction of queries are never matched to any advertisement, even though the users may have specific and well-defined commercial intent behind such queries. The existence of such high-volume but poorly monetized query traffic has led to the emergence of a *separate class* of mediators, collectively referred to as the search engine or *click arbitrage* sector. In an ideal world, such mediators could enhance user experience by better capturing *user intention*, i.e., by buying a large number of infrequent keywords with similar intent or from a particular vertical sector (e.g., health, finance, or travel), and then funneling the traffic to a site that shows relevant ads (i.e., on topics related to the users’ original queries), but based on high-priced keywords. Since the infrequent keywords are cheaper to buy, the mediators can turn a profit by showing ads for keywords that are more popular, and hence more expensive. *In reality, however*, capturing user intention in a large-scale fashion is a very difficult problem, and many such click arbitrageurs end up buying infrequent keywords at a cheap price and selling them at a higher price by taking the users to pages full of ads that are not necessarily related to the original keywords (and may be put by a different auctioneer), without regard to and often compromising user experience. Most of the time, users just click on these pricey but irrelevant ads to make their way out of those pages. Given that the pricing mechanism is PPC (pay-per-click) based, the advertisers

do pay for all such junk clicks, making fortunes for these arbitrageurs. In the long run, however, such an abuse of the inefficiency (i.e., the inability of the primary auctioneer to capture commercial user intention for a large enough fraction of query volume) is automatically eliminated as advertisers figure out the diminishing conversion rates, thereby decreasing their bids and paying much less or nothing for these junk clicks. Also, the auctioneers may take smart actions to ban such arbitrageurs (since the traffic from these arbitrage companies are of poor quality or being funneled to a competitor), and the company Geosign being banned by Google is a prime example of this². *It is important to reiterate* that the mediators we discuss in this paper, however, *do not fall in this category of short term profiteers*.

Now we discuss the formal setup for the standard sponsored search auctions which will be helpful in the presentation of our model for creating additional capacity. Formally, in the current models, there are K slots to be allocated among N ($\geq K$) bidders (i.e. the advertisers). A bidder i has a true valuation v_i (known only to the bidder i) for the specific keyword and she bids b_i . The expected *click through rate* (CTR) of an ad put by bidder i when allocated slot j has the form $CTR_{i,j} = \gamma_j e_i$ i.e. separable in to a position effect and an advertiser effect. γ_j 's can be interpreted as the probability that an ad will be noticed when put in slot j and it is assumed that $\gamma_j > \gamma_{j+1}$ for all $1 \leq j \leq K$ and $\gamma_j = 0$ for $j > K$. e_i can be interpreted as the probability that an ad put by bidder i will be clicked on if noticed and is referred to as the *relevance* of bidder i . The payoff/utility of bidder i when given slot j at a price of p per-click is given by $e_i \gamma_j (v_i - p)$ and they are assumed to be rational agents trying to maximize their payoffs.

As of now, Google as well as Yahoo! use schemes closely modeled as RBR(rank by revenue) with GSP(generalized second pricing). The bidders are ranked in the decreasing order of $e_i b_i$ and the slots are allocated as per these ranks. For simplicity of notation, assume that the i th bidder is the one allocated slot i according to this ranking rule, then i is charged an amount equal to $\frac{e_{i+1} b_{i+1}}{e_i}$ per-click. This mechanism has been extensively studied in recent years [Edelman et al. 2007; Varian 2007; Lahaie 2006; Aggarwal et al. 2006; Lahaie and Pennock 2007]. The solution concept that is widely adopted to study this auction game is a refinement of Nash equilibrium independently proposed by Varian [Varian 2007] and Edelman et al [Edelman et al. 2007]. Under this refinement, the bidders have no incentive to change to another positions even at the current price paid by the bidders currently at that position. Edelman et al [Edelman et al. 2007] calls it *locally envy-free equilibria* and argue that such an equilibrium arises if agents are raising their bids to increase the payments of those above them, a practice which is believed to be common in actual keyword auctions. Varian [Varian 2007] called it *symmetric Nash equilibria(SNE)* and provided some empirical evidence that the Google bid data agrees well with the SNE bid profile. In particular, an **SNE** bid profile b_i 's satisfy

$$(\gamma_i - \gamma_{i+1})v_{i+1}e_{i+1} + \gamma_{i+1}e_{i+2}b_{i+2} \leq \gamma_i e_{i+1} b_{i+1} \leq (\gamma_i - \gamma_{i+1})v_i e_i + \gamma_{i+1}e_{i+2}b_{i+2} \quad (1)$$

for all $i = 1, 2, \dots, N$. Now, recall that in the RBR with GSP mechanism, the bidder i pays an amount $\frac{e_{i+1} b_{i+1}}{e_i}$ per-click, therefore the expected payment i makes per-impression

²<http://www.techdirt.com/articles/20080319/020719583.shtml>, <http://www.techcrunch.com/2008/03/18/how-geosign-blew-160-million/>

is $\gamma_i e_i \frac{e_{i+1} b_{i+1}}{e_i} = \gamma_i e_{i+1} b_{i+1}$. Thus the best **SNE** bid profile for advertisers (worst for the auctioneer) is minimum bid profile possible according to Equation 1 and is given by

$$\gamma_i e_{i+1} b_{i+1} = \sum_{j=i}^K (\gamma_j - \gamma_{j+1}) v_{j+1} e_{j+1} \quad (2)$$

and therefore, the revenue of the auctioneer at this minimum *SNE* is

$$\sum_{i=1}^K \gamma_i e_{i+1} b_{i+1} = \sum_{i=1}^K \sum_{j=i}^K (\gamma_j - \gamma_{j+1}) v_{j+1} e_{j+1} = \sum_{j=1}^K (\gamma_j - \gamma_{j+1}) j v_{j+1} e_{j+1}. \quad (3)$$

For the comparative analysis, in the present work, we assume that the auction used to sell the original slots (i.e., without any additional capacity), the single combined auction in the auctioneer-based-model run by the primary auctioneer to sell the original slots together with the additional slots created by him, as well as, the two auctions in the mediator-based model (one run by the primary auctioneer to sell the original slots and the other run by the mediator to sell the additional slots created by her), are all run via RBR with GSP i.e. the mechanism currently being used by Google and Yahoo!. The solution concept we use is Symmetric Nash Equilibria(SNE)/locally envy-free equilibria [Edelman et al. 2007; Varian 2007]. Nevertheless, as evident from the intuition behind the proofs provided later in the paper, the results hold true for other interesting allocation and pricing mechanisms as well.

2. THE MODEL

We will refer to the scenario where the additional capacity is created by a for-profit mediator as **MDC** (**M**ediator **D**riven additional **C**apacity) and the scenario where the additional capacity is created by the auctioneer as **ADC** (**A**uctioneer **D**riven additional **C**apacity).

—Additional/Secondary Slots:

—*How are the slots created?* In **MDC**, the mediator participates in the original auction run by the search engine (called *p-auction*) and competes with advertisers for slots (called *primary slots*). Suppose that in the *p-auction*, the slot assigned to the mediator is l , then effectively, the additional slots are obtained by forking this *primary slot* in to L additional slots, where $L \leq K$. By forking we mean the following: on the associated landing page the mediator puts some information relevant to the specific keyword associated with the *p-auction* along with the space for additional slots. Let us call these additional slots as *secondary slots*. In **ADC**, similarly, the additional slots are obtained by forking *one* of the original slots. Here, the auctioneer puts her own ad/link in that slot, and on the associated landing page, she puts some information relevant to the specific keyword along with space for additional slots. We consider the single fork case in **ADC** and single mediator case in **MDC** for the sake of simplicity of presentation and so that the calculations do not get unwieldy, but the results can be extended to the case where the auctioneer forks multiple slots (in **ADC**) and adds additional capacity, or there are more than one mediators involved (in **MDC**).

—*Fitness and New position based CTRs:* The quality of the additional/secondary slots is measured by a *fitness factor*. Let the probability associated with the ad put by the auctioneer (in **ADC**) or the mediator (in **MDC**) for creating additional capacity to be clicked, if noticed, be denoted as \tilde{f} . In **MDC**, this is actually the relevance score of the mediator in the *p-auction*. Moreover, the position-based CTRs for the additional slots

in the landing page will in general be different than on the main page, and it might actually improve, say by a factor of α . This means that the position based CTR for the j th additional slot on the associated landing page is modeled as $\alpha\gamma_j$. Therefore, we can define a fitness factor f to indicate the effective quality of additional slots being created, which is equal to $\tilde{f}\alpha$. Thus, if the original slot being forked is l , and there are L additional slots being created on the landing page, then the *effective* position based CTRs for the additional slots thus obtained are $\gamma_l f \gamma_1, \gamma_l f \gamma_2, \dots, \gamma_l f \gamma_L$ respectively. Clearly, $f\gamma_1 < 1$; however, f itself could be greater than 1.

—**A single combined auction vs two uncoupled auctions:** The major difference in the two scenarios **MDC** and **ADC** is that in **MDC** there are two uncoupled auctions- the one run by the auctioneer to sell the primary slots where the mediator also competes for a slot (i.e. p -auction), and the other run by the mediator to sell the secondary slots (called s -auction), however in **ADC** there is a *single* auction run by the auctioneer to sell primary as well as secondary slots. Thus there is no s -auction in **ADC**. For the comparative analysis, we assume that the single combined auction in **ADC** as well as the two auctions in **MDC**, are all run via RBR with GSP (i.e. the mechanism currently being used by Google and Yahoo!) and the solution concept we use is Symmetric Nash Equilibria(SNE)/locally envy-free equilibria [Edelman et al. 2007; Varian 2007].

— **p -auction:** In **MDC**, the mediator participates in the original auction run by the search engine and compete with advertisers for a primary slot. For the i th agent (an advertiser or a mediator), let v_i^p and b_i^p denote her true valuation and the bid for the p -auction respectively. Further, let us denote $v_i^p e_i^p$ by s_i^p where e_i^p is the relevance score of i th agent for p -auction. There are still K slots for this p -auction, and the position based CTRs are still the same as in the case without additional capacity.

In **ADC**, in the combined auction there are now $\tilde{K} = K + L - 1$ slots and for each slot there will be a probability of being noticed if an advertiser is assigned to that slot i.e. its position based CTR. We rename the slots in the decreasing order of their CTRs. That is, the j th slot is the one having j th maximum of the elements from the set $\{\gamma_1, \gamma_2, \dots, \gamma_{l-1}, \gamma_{l+1}, \dots, \gamma_K\} \cup \{\gamma_l f \gamma_1, \gamma_l f \gamma_2, \dots, \gamma_l f \gamma_L\}$ and its CTR is denoted by $\tilde{\gamma}_j$. For the sake of simplicity, we assume that there are no ties i.e. no two slots have the same position based CTRs. Therefore, like γ_j 's we have $\tilde{\gamma}_j > \tilde{\gamma}_{j+1}$ for all $1 \leq j \leq \tilde{K} - 1$ and $\tilde{\gamma}_j = 0$ for all $j \geq \tilde{K}$. Further note that, $\tilde{\gamma}_j = \gamma_j$ for $j \leq l - 1$, and $\tilde{\gamma}_l < \gamma_l$. Therefore $\tilde{\gamma}_j - \tilde{\gamma}_{j+1} = \gamma_j - \gamma_{j+1}$ for $j < l - 1$, $\tilde{\gamma}_{l-1} - \tilde{\gamma}_l > \gamma_{l-1} - \gamma_l$, and $\tilde{\gamma}_j - \tilde{\gamma}_{j+1}$ could be greater than or less than $\gamma_j - \gamma_{j+1}$ for $l \leq j \leq K$ depending on how the new position based CTRs are distributed among the old ones.

— **s -auction:** In **ADC**, there is no s -auction. In **MDC**, the mediator runs her individual sub-auction for selling the secondary slots. For an advertiser there is another type of valuations and bids, the ones associated with s -auctions. For the i th agent, let v_i^s and b_i^s denote her true valuation and the bid for the s -auction respectively. In general, the two types of valuations or bids corresponding to p -auction and the s -auctions might differ a lot. We also assume that $v_i^s = 0$ and $b_i^s = 0$ whenever i is a mediator. Further, for the advertisers who do not participate in one auction (p -auction or s -auction), the corresponding true valuation and the bid are assumed to be zero. Also, for notational convenience let us denote $v_i^s e_i^s$ by s_i^s , where e_i^s is the relevance score of i th agent for the s -auction. Further, the s -auction is not coupled to the p -auction, meaning

that the corresponding auctions are independent of each other in the sense that for a player who participates in both the auction games- the problem of maximizing the combined payoff from the two auctions is same as the problems of maximizing the payoffs from the individual auctions independently. This is indeed very reasonable in practice because the conversion rates (and consequently the valuations) derived at the two auction sites would be generally different, and further they have the flexibility of reporting different bid for the two auctions.

—**Freedom of participation:** In **ADC**, since the auctioneer runs a single combined auction to sell original slots together with the additional ones, a bidder is allowed *only* to bid for all slots (original slots plus the additional ones) together and not for the two kind of slots individually. This is unlike in **MDC**, where the mediator runs her own sub-auction. Advertisers are free to bid for primary as well as secondary slots, and in general report two bid values - one to the auctioneer for the p -auction, and the other to the mediator for the s -auction.

—**True valuation of mediator:** The true valuation of the mediator (for the p -auction) is derived from the expected revenue (total payments from advertisers) she obtains from her corresponding s -auction *ex ante*. This way of deriving the true valuation for the mediator is reasonable because, the mediator can participate in the p -auction several times and run her corresponding s -auction and can estimate the revenue she is deriving from the s -auction.

—**Capacity:** The *capacity* is defined as the sum of position based CTRs. Thus the capacity in the original model without the additional/secondary slots is $\sum_{j=1}^K \gamma_j$. In **ADC** or **MDC**, it is $\sum_{i=1, i \neq l}^K \gamma_j + \gamma_l f \sum_{i=1}^L \gamma_i$. Note that for a fixed L, l , the capacity increases iff f increases, for a fixed l, f , it increases iff L increases.

3. RESULTS

3.1 The MDC Scenario

We first discuss the change in the revenue of the auctioneer due to the involvement of the mediator and our observation as noted in Result 3.1 is that it always increases. Intuitively, when the mediator participates for buying the primary slots, it increases the competition in the p -auction and therefore the revenue of the auctioneer goes up. Thus, as long as the mediator has a good enough fitness that guarantees her a slot in the p -auction, the auctioneer definitely gains in terms of revenue. Further, the better the mediator's valuation is, the better slot the mediator gets allocated, bringing forth more gain in revenue. Besides keeping a good fitness factor f , there is another smart way for the mediator to improve her true valuation. She could actually run many subauctions related to the specific keyword in question. This can be done as follows: besides providing the additional slots on the landing page, the information section of the page could contain links to other pages wherein further additional slots associated with a related keyword could be provided³.

RESULT 3.1. *Increasing the capacity via mediators improves the revenue of the auctioneer.*

For the formal proof of Result 3.1, we will first need to discuss the incentive properties of the two uncoupled auctions, the p -auction and the s -auction respectively, and in particular

³For example, the mediator "personalloans.com".

the bid profiles at their respective SNE's.

Suppose the allocations for the p -auction and s -auction are $\sigma : \{1, 2, \dots, N\} \longrightarrow \{1, 2, \dots, N\}$ and $\tau : \{1, 2, \dots, N\} \longrightarrow \{1, 2, \dots, N\}$ respectively. Then the payoff of the i th agent from the combined auction (p -auction and s -auction together) is

$$u_i = \gamma_{\sigma^{-1}(i)} \left(s_i^p - r_{\sigma^{-1}(i)+1}^p \right) + \tilde{\gamma}_{\tau^{-1}(i)} \left(s_i^s - r_{\tau^{-1}(i)+1}^s \right)$$

$$\text{where } r_j^p = b_{\sigma(j)}^p e_{\sigma(j)}^p, \quad r_j^s = b_{\tau(j)}^s e_{\tau(j)}^s.$$

From the mathematical structure of payoffs and strategies available to the bidders wherein two different uncorrelated values can be reported as bids in the two types of auctions independently of each other (i.e. since the two auctions are uncoupled), it is clear that the equilibrium of the combined auction game is the one obtained from the equilibria of the p -auction game and the s -auction game each played in isolation. In particular at minimum SNE [Edelman et al. 2007; Varian 2007],

$$\gamma_i r_{i+1}^p = \sum_{j=i}^K (\gamma_j - \gamma_{j+1}) s_{\sigma(j+1)}^p \text{ for all } i = 1, 2, \dots, K$$

and

$$\tilde{\gamma}_i r_{i+1}^s = \sum_{j=i}^L (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) s_{\tau(j+1)}^s \text{ for all } i = 1, 2, \dots, L$$

which implies that (recall that the *effective* position based CTRs for the secondary slots are $\gamma_1 f \gamma_1, \gamma_1 f \gamma_2, \dots, \gamma_1 f \gamma_L$ respectively)

$$\gamma_i r_{i+1}^s = \sum_{j=i}^{L-1} (\gamma_j - \gamma_{j+1}) s_{\tau(j+1)}^s + \gamma_L s_{\tau(L+1)}^s \text{ for all } i = 1, 2, \dots, L \quad \text{where}$$

$$s_{\sigma(l)}^p = s_M^p = f \sum_{j=1}^L \gamma_j r_{j+1}^s = f \left(\sum_{j=1}^{L-1} (\gamma_j - \gamma_{j+1}) j s_{\sigma(j+1)}^s + \gamma_L L s_{\sigma(L+1)}^s \right)$$

is the true valuation of the mediator multiplied by her relevance score as per our definition, which is the expected revenue she derives from her s -auction *ex ante* given a slot in the p -auction.

Proof of Result 3.1: The revenue of the auctioneer with the participation of the mediator is

$$R = \sum_{j=1}^K \gamma_j r_{j+1}^p = \sum_{j=1}^K (\gamma_j - \gamma_{j+1}) j s_{\sigma(j+1)}^p$$

and similarly, the revenue of the auctioneer without the participation of the mediator is

$$\begin{aligned}
R_0 &= \sum_{j=1}^K (\gamma_j - \gamma_{j+1}) j s_{\tilde{\sigma}(j+1)}^p \\
&\quad (\text{where } \tilde{\sigma}(j) = \sigma(j) \text{ for } j < l \text{ and } \tilde{\sigma}(j) = \sigma(j+1) \text{ for } j \geq l) \\
&= \sum_{j=1}^{l-2} (\gamma_j - \gamma_{j+1}) j s_{\sigma(j+1)}^p + \sum_{j=l-1}^K (\gamma_j - \gamma_{j+1}) j s_{\sigma(j+2)}^p \\
\therefore R - R_0 &= \sum_{j=\max\{1, l-1\}}^K (\gamma_j - \gamma_{j+1}) j (s_{\sigma(j+1)}^p - s_{\sigma(j+2)}^p) \geq 0
\end{aligned}$$

wherein the last inequality follows from the observation that

$$s_{\sigma(i)}^p \geq s_{\sigma(i+1)}^p \forall i = 1, 2, \dots, K+1 \text{ at SNE. } \square$$

Now let us turn our attention to the change in the efficiency and as we will note below in the Result 3.2, the efficiency always improves by the participation of the mediator. The basic intuitions behind an increase in efficiency are that the allocation at SNE is an efficient one [Edelman et al. 2007; Varian 2007], and that the mediator brings in more value by accommodating more advertisers with a high *collective* value. Further, better the fitness and higher the values of advertisers in the *s*-auction, better the efficiency gain will be. Furthermore, it is implicit in our analysis that the *user experience*, measured in terms of total clickability, also improves when the fitness is good⁴. Thus, we can indeed say that the social welfare (i.e. the total welfare of all the parties involved) improves. Moreover, even the payoffs of all the advertisers will increase if the mediator has a high enough fitness (ref. APPENDIX).

RESULT 3.2. *Increasing the capacity via mediators improves the efficiency.*

Proof: Let E and E_0 denote the efficiency with and without the participation of the mediator respectively, then we have

$$\begin{aligned}
E_0 &= \sum_{j=1}^K \gamma_j s_{\tilde{\sigma}(j)}^p = \sum_{j=1}^{l-1} \gamma_j s_{\sigma(j)}^p + \sum_{j=l}^K \gamma_j s_{\sigma(j+1)}^p, \\
E &= \sum_{j=1}^{l-1} \gamma_j s_{\sigma(j)}^p + \sum_{j=l+1}^K \gamma_j s_{\sigma(j)}^p + \gamma_l f \sum_{j=1}^L \gamma_j s_{\tau(j)}^s \\
\therefore E - E_0 &= \gamma_l f \sum_{j=1}^L \gamma_j s_{\tau(j)}^s - \sum_{l}^K (\gamma_j - \gamma_{j+1}) s_{\sigma(j+1)}^p = \gamma_l f \sum_{j=1}^L \gamma_j s_{\tau(j)}^s - \gamma_l r_{l+1}^p \geq 0
\end{aligned}$$

wherein the last inequality holds because

$$\gamma_l f \sum_{j=1}^L \gamma_j s_{\tau(j)}^s \geq \gamma_l f \sum_{j=1}^L \gamma_j r_{j+1}^s = \gamma_l s_{\sigma(l)}^p \geq \gamma_l r_{l+1}^p \text{ at SNE. } \square$$

⁴Athey and Ellison [2007] is an example of work that takes user experience explicitly into account, although not in the setting of the present paper.

3.2 The ADC Scenario

Now, we ask the question- what if the auctioneer wants to provide the additional capacity herself by essentially acting herself as a mediator and running a *single combined auction*? Does the revenue of the auctioneer/efficiency improve or does it degrade in such a model? Our observation is that, unlike the mediator-based model, often there is a tradeoff between the revenue and the capacity, and there is a phase transition from possibly a gain in terms of revenue to a loss as the *fitness* increases, meaning that there is a critical fitness value beyond which the auctioneer always loses in revenue. However, there exist scenarios where the revenue of the auctioneer could indeed increase by increasing capacity. The following results formalize some worst case scenarios. Some further discussion is provided in *APPENDIX*.

Remark: *Since there is no s-auction in the ADC scenario, for notational simplicity, henceforth we will drop the superscripts p . We also assume that the i th bidder is the one allocated slot i when ranked in the decreasing order of $e_i b_i$.*

RESULT 3.3. *Let s_i 's satisfy $(j-1)s_j \geq js_{j+1}$ for all $j \geq 2$. Recall that l is the primary slot being forked in to additional slots.*

- (1) *For $l = 1$, if γ_j 's satisfy $(\gamma_1 - \gamma_2) \geq (\gamma_j - \gamma_{j+1})$ for all $1 \leq j \leq K$ then there exists no fitness factor f such that the revenue of auctioneer increases⁵.*
- (2) *For any $l \geq 2$, the gain in the revenue of the auctioneer is a decreasing function of f and L .*

Proof of Result 3.3(1): Let R and R_0 denote the revenue of the auctioneer with and without the additional capacity respectively. Let us define

$$i_0 = \max_{1 \leq i \leq K} \{i : \gamma_1 f \gamma_1 < \gamma_i\}$$

then $\tilde{\gamma}_j = \gamma_{j+1}$ for all $1 \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_1 f \gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq 1$. Now,

$$\begin{aligned} R_0 &= \sum_{j=1}^K (\gamma_j - \gamma_{j+1}) j s_{j+1} = \gamma_1 s_2 - \sum_{j=2}^K \gamma_j [(j-1)s_j - j s_{j+1}] \\ R &= \tilde{\gamma}_1 s_2 - \sum_{j=2}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - j s_{j+1}] \\ \therefore R - R_0 &= (\tilde{\gamma}_1 - \gamma_1) s_2 - \sum_{j=2}^K (\tilde{\gamma}_j - \gamma_j) [(j-1)s_j - j s_{j+1}] \\ &\quad - \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - j s_{j+1}]. \end{aligned}$$

⁵Note that the conditions on γ_j 's hold when they are geometrically decreasing (i.e. when $\gamma_j = r^{j-1}$, $1 \leq j \leq K$ for some $r < 1$ and 0 otherwise), which is a very good approximation in practice [Abrams and Ghosh 2007; Feng et al. 2006].

Therefore, when $i_0 \geq 2$, we have

$$\begin{aligned}
R - R_0 &= -(\gamma_1 - \gamma_2)s_2 + \sum_{j=2}^{i_0-1} (\gamma_j - \gamma_{j+1}) [(j-1)s_j - js_{j+1}] \\
&\quad + (\gamma_{i_0} - \gamma_1 f \gamma_1) [(i_0 - 1)s_{i_0} - i_0 s_{i_0+1}] \\
&\quad - \sum_{j=i_0+1}^K (\tilde{\gamma}_j - \gamma_j) [(j-1)s_j - js_{j+1}] - \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - js_{j+1}] \\
&\leq -(\gamma_1 - \gamma_2)s_2 + (\gamma_1 - \gamma_2) [s_2 - i_0 s_{i_0+1}] \\
&\quad (\text{recall that } (\gamma_1 - \gamma_2) \geq (\gamma_j - \gamma_{j+1}) \text{ and } (j-1)s_j - js_{j+1} \geq 0 \ \forall j \geq 2) \\
&= -(\gamma_1 - \gamma_2)i_0 s_{i_0+1} < 0.
\end{aligned}$$

When $i_0 = 1$, we have

$$\begin{aligned}
R - R_0 &= -(\gamma_1 - \gamma_1 f \gamma_1)s_2 - \sum_{j=2}^K (\tilde{\gamma}_j - \gamma_j) [(j-1)s_j - js_{j+1}] \\
&\quad - \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - js_{j+1}] \\
&< 0. \quad \square
\end{aligned}$$

Proof of Result 3.3(2): Let

$$i_0 = \max_{1 \leq i \leq K} \{i : \gamma_l f \gamma_1 < \gamma_i\}$$

then $\tilde{\gamma}_j = \gamma_j$ for all $1 \leq j \leq l-1$, $\tilde{\gamma}_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_l f \gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq l \geq 2$.

$$\begin{aligned}
\therefore R &= \tilde{\gamma}_1 s_2 - \sum_{j=2}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - js_{j+1}] \\
&= \gamma_1 s_2 - \sum_{j=2}^{l-1} \gamma_j [(j-1)s_j - js_{j+1}] - \sum_{j=l}^{i_0-1} \gamma_{j+1} [(j-1)s_j - js_{j+1}] \\
&\quad - \gamma_l f \gamma_1 [(i_0 - 1)s_{i_0} - i_0 s_{i_0+1}] - \sum_{j=i_0+1}^{K+L-1} \tilde{\gamma}_j [(j-1)s_j - js_{j+1}]
\end{aligned}$$

Now let us increase f to f' and denote the new position based CTRs as $\tilde{\gamma}_j'$'s and the new revenue of the auctioneer as R' then two cases arise - one where i_0 does not change and other where it changes to $i_0 - 1$.

Case 1: when i_0 does not change by increasing f to f' . Clearly, $\tilde{\gamma}_j' \geq \tilde{\gamma}_j$ for all $j \geq i_0 + 1$ as we will be choosing elements from a set with larger values. Also recall that s_i 's satisfy $(j-1)s_j - js_{j+1} \geq 0$ for all $j \geq 2$. Therefore, the second last term in the expression of R strictly decrease and the last term also decreases and we get $R' < R$.

Case 2: When i_0 changes to $i_0 - 1$ by increasing f to f' . In this case, $\tilde{\gamma}'_j = \gamma_j$ for all $1 \leq j \leq l - 1$, $\tilde{\gamma}'_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 2$, $\tilde{\gamma}'_{i_0-1} = \gamma_l f' \gamma_1$, and $\tilde{\gamma}'_j \geq \tilde{\gamma}_j$ for all $j \geq i_0$. Therefore,

$$\begin{aligned} R' - R &= \gamma_{i_0} [(i_0 - 2)s_{i_0-1} - (i_0 - 1)s_{i_0}] - \gamma_l f' \gamma_1 [(i_0 - 2)s_{i_0-1} - (i_0 - 1)s_{i_0}] \\ &\quad - \sum_{j=i_0}^{K+L-1} (\tilde{\gamma}'_j - \tilde{\gamma}_j) [(j - 1)s_j - j s_{j+1}] \\ &\leq (\gamma_{i_0} - \gamma_l f' \gamma_1) [(i_0 - 2)s_{i_0-1} - (i_0 - 1)s_{i_0}] < 0. \quad \square \end{aligned}$$

Recall that the bidders are characterized by their true valuations v_i^p 's and their relevance scores e_i 's and $s_i = e_i v_i$. Thus, there is a wide pool of bidders satisfying the conditions in the above result, and therefore indicating a significant tradeoff between revenue of the auctioneer and the capacity. Intuitively, the conditions $(j - 1)s_j \geq j s_{j+1}$ state that the s_i 's are well separated, and therefore the payments that the bidders make at **SNE** are also well separated. Increasing capacity (via increasing f or L) means essentially selling a fraction of the clicks at a lower price. When s_i 's are well separated, the extra revenue coming from the newly accommodated bidders still fall short of that lost due to lower payments from the other bidders. Further, the Result 3.3 suggests that there is a phase transition from possibly positive gain in the revenue to negative as f increases, and there is a critical f beyond which the auctioneer always loses.

Now let us look at the change in efficiency due to added capacity in **ADC**. In this case, the result is more in consonance with **MDC**, i.e., the efficiency increases as fitness increases. However, unlike in **MDC**, the efficiency could indeed decrease by increasing capacity.

RESULT 3.4. *The efficiency is an increasing function of fitness f .*

Proof: Let E denote the efficiency when additional capacity is added. Let us define

$$i_0 = \max_{1 \leq i \leq K} \{i : \gamma_l f \gamma_1 < \gamma_i\}$$

then $\tilde{\gamma}_j = \gamma_j$ for all $1 \leq j \leq l - 1$, $\tilde{\gamma}_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 1$, $\tilde{\gamma}_{i_0} = \gamma_l f \gamma_1$, and $\tilde{\gamma}_j \geq \gamma_j$ for all $j \geq i_0 + 1$. Clearly, $i_0 \geq l$. Then,

$$E = \sum_{j=1}^{K+L-1} \tilde{\gamma}_j s_j = \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{i_0-1} \gamma_{j+1} s_j + \gamma_l f \gamma_1 s_{i_0} + \sum_{j=i_0+1}^{K+L-1} \tilde{\gamma}_j s_j.$$

Now let us increase f to f' and denote the new position based CTRs as $\tilde{\gamma}'_j$'s and the new efficiency as E' then two cases arise - one where i_0 does not change and other where it changes to $i_0 - 1$.

Case 1: when i_0 does not change by increasing f to f' . Clearly, $\tilde{\gamma}'_j \geq \tilde{\gamma}_j$ for all $j \geq i_0 + 1$ as we will be choosing elements from a set with larger values. Therefore, the second last term in the expression of E strictly increase and the last term also increases and we get $E' > E$.

Case 2: When i_0 changes to $i_0 - 1$ by increasing f to f' . In this case, $\tilde{\gamma}'_j = \gamma_j$ for all $1 \leq j \leq l - 1$, $\tilde{\gamma}'_j = \gamma_{j+1}$ for all $l \leq j \leq i_0 - 2$, $\tilde{\gamma}'_{i_0-1} = \gamma_l f' \gamma_1$, and $\tilde{\gamma}'_j \geq \tilde{\gamma}_j$ for all

$j \geq i_0$. Therefore,

$$\begin{aligned} E' &= \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{i_0-2} \gamma_{j+1} s_j + \gamma_l f' \gamma_1 s_{i_0-1} + \sum_{j=i_0}^{K+L-1} \tilde{\gamma}_j s_j. \\ &\geq E + (\gamma_l f' \gamma_1 - \gamma_{i_0}) s_{i_0-1} > E. \quad \square \end{aligned}$$

4. DISCUSSIONS

Having established some results on how improving the capacity via mediator driven model and auctioneer driven model effect the revenue and efficiency, we would now like to compare the two models in terms of these two parameters, which are considered two fundamental bench-marking metrics in mechanism design theory [Krishna 2002].

First, recall that the auctioneer's revenue always increases in **MDC** (Result 3.1) and in fact the revenue of the auctioneer increases as the *fitness* of the mediator increases, thus there is no conflict between the revenue and the capacity in that scenario. However, as we saw that in the **ADC** often there is a tradeoff between the revenue and the capacity (Result 3.3). Therefore, in terms of revenue, **MDC** is superior to **ADC**. A typical tradeoff curve is shown in the Figure 1. Further, recall that the efficiency always increases in **MDC** and in fact the efficiency increases as the *fitness* of the mediator increases, thus there is no conflict between the efficiency and the capacity in that scenario. However, in **ADC**, although the efficiency increases when fitness increases, it could go well below the efficiency in the scenario without any additional capacity. A typical tradeoff curve is shown in the Figure 1. Therefore, even in terms of efficiency the **MDC** is superior to **ADC**. Hence, we can conclude that the **ADC** is indeed inferior to the **MDC** and the market becomes more capacity efficient by the participation of mediators.

It is instructive to note that, in the **MDC** scenario, as long as the p -auction and the s -auction are not coupled, and the traffic for the s -auction site is drawn from the p -auction site, it does not matter who adds capacity and runs the s -auction, a mediator or the primary auctioneer. But in order to add the necessary capacity and run the secondary auctions effectively, the mediators have to specialize in the particular sector (e.g., loans, finance, business logistics etc.) that they are adding capacity to. Thus, if the primary auctioneer (e.g., a search engine portal) wants to also play the role of the mediator then it will also have to develop the necessary sales force and business infrastructure. For example, if Google wanted to take on the role of business.com and personalloans.com (see APPENDIX) then it will have to develop a support and sales force that will reach out to small-businesses and to loan companies, which might detract it from its core business. Thus, we expect that separate mediator entities, specializing in different sectors, will continue to coexist with the giant search portals, who specialize on being the primary auctioneers and the source of primary traffic or leads. Our results are further confirmed by a recent empirical study [Gunawardana et al. 2008].

APPENDIX

A. MDC: ADVERTISERS' PAYOFFS

Clearly, for the newly accommodated advertisers, that is the ones who lost in the p -auction but win a slot in s -auction, the payoffs increase from zero to a positive number. Now let us see where do these improvements in the revenue of the auctioneer (Result 1), in payoffs of

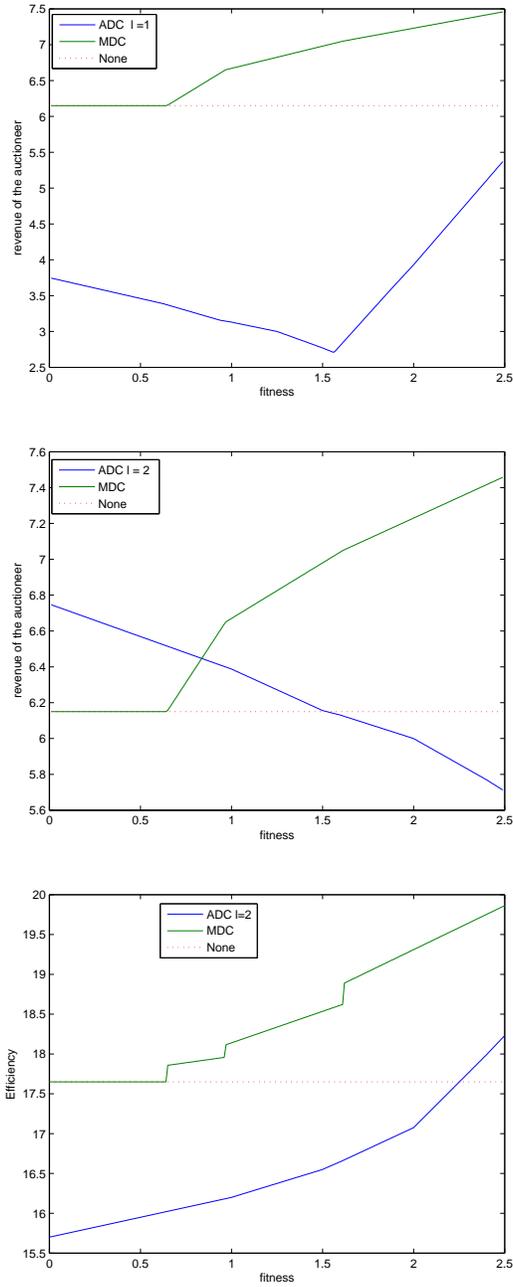


Fig. 1. Tradeoff curves for Auctioneer's Revenue/Efficiency: MDC vs ADC. The data used is the following:
 $N = 8, K = 5, L = 3, \gamma = [0.4 \ 0.25 \ 0.2 \ 0.15 \ 0.10],$
 $s^p = [25 \ 20 \ 8 \ 5 \ 3 \ 2 \ 1.5 \ 1], s^s = [0 \ 0 \ 10 \ 4 \ 3 \ 6 \ 0 \ 0].$

newly accommodated advertisers, and in the efficiency (Result 2) come from? Only thing left to look at is the change in the payoffs for the advertisers who originally won in the p -auction, that is the winners when there was no mediator. The new payoff for j th ranked advertiser in p -auction is

$$u_{\sigma(j)} = \gamma_j s_{\sigma(j)}^p - \sum_{i=j}^K (\gamma_i - \gamma_{i+1}) s_{\sigma(i+1)}^p + u_{\sigma(j)}^s$$

where

$$u_{\sigma(j)}^s = \gamma_l f \gamma_{\tau^{-1}(\sigma(j))} \left(s_{\sigma(j)}^s - r_{\tau^{-1}(\sigma(j))+1}^s \right)$$

is her payoff from the s -auction. Also, for $j \leq l-1$, her payoff when there was no mediator is

$$\begin{aligned} u_{\sigma(j)}^0 &= \gamma_j s_{\sigma(j)}^p - \sum_{i=j}^K (\gamma_i - \gamma_{i+1}) s_{\sigma(i+1)}^p \\ &= \gamma_j s_{\sigma(j)}^p - \sum_{i=j}^{l-2} (\gamma_i - \gamma_{i+1}) s_{\sigma(i+1)}^p - \sum_{i=l-1}^K (\gamma_i - \gamma_{i+1}) s_{\sigma(i+2)}^p. \\ \therefore u_{\sigma(j)} - u_{\sigma(j)}^0 &= u_{\sigma(j)}^s - \sum_{i=l-1}^K (\gamma_i - \gamma_{i+1}) (s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p) \end{aligned}$$

Similarly, for $j \geq l+1$, her payoff when there was no mediator is

$$\begin{aligned} u_{\sigma(j)}^0 &= \gamma_{j-1} s_{\sigma(j)}^p - \sum_{i=j-1}^K (\gamma_i - \gamma_{i+1}) s_{\sigma(i+2)}^p \\ \therefore u_{\sigma(j)} - u_{\sigma(j)}^0 &= u_{\sigma(j)}^s - \sum_{i=j-1}^K (\gamma_i - \gamma_{i+1}) (s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p) \end{aligned}$$

Therefore, in general we have,

$$u_{\sigma(j)} - u_{\sigma(j)}^0 = u_{\sigma(j)}^s - \sum_{i=\max\{l-1, j-1\}}^K (\gamma_i - \gamma_{i+1}) (s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p).$$

Thus, for the j th ranked winning advertiser from the auction without mediation, the revenue from the p -auction decreases by $\sum_{i=\max\{l-1, j-1\}}^K (\gamma_i - \gamma_{i+1}) (s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p)$ and she faces a loss unless compensated for by her payoffs in s -auction. Further, this payoff loss will be visible only to the advertisers who joined the auction game before the mediator and they are likely to participate in the s -auction so as to make up for this loss. Thus, via the mediator, a part of the payoffs of the originally winning advertisers essentially gets distributed among the newly accommodated advertisers. However, when the mediator's fitness factor f is very good, it might be a win-win situation for everyone. Depending on how good the fitness factor f is, sometimes the payoff from the s -auction might be enough to compensate for any loss by accommodating new advertisers. Let us consider an extreme

situation when $L = K$ and $\tau = \bar{\sigma}$. The *gain* in payoff for the advertiser $\sigma(j)$ is

$$\gamma_l f \sum_{i=j}^K (\gamma_i - \gamma_{i+1})(s_{\sigma(j)}^s - s_{\sigma(i+1)}^s) - \sum_{i=\max\{l-1, j-1\}}^K (\gamma_i - \gamma_{i+1})(s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p)$$

Therefore as long as

$$f \geq \frac{\sum_{i=\max\{l-1, j-1\}}^K (\gamma_i - \gamma_{i+1})(s_{\sigma(i+1)}^p - s_{\sigma(i+2)}^p)}{\gamma_l \sum_{i=j}^K (\gamma_i - \gamma_{i+1})(s_{\sigma(j)}^s - s_{\sigma(i+1)}^s)}$$

the advertiser $\sigma(j)$ faces no net loss in payoff and might actually gain.

B. MDC: EXAMPLES OF FOR-PROFIT MEDIATORS

Please refer to the Figures 2, 3, 4, 5.

C. ADC

Remark: Since there is no s -auction in the ADC scenario, for notational simplicity, henceforth we will drop the superscripts p . We also assume that the i th bidder is the one allocated slot i when ranked in the decreasing order of $e_i b_i$.

Value of capacity:

Definition C.1. Let R_0 be the original revenue of the auctioneer without added capacity and R be the new revenue of the auctioneer after adding capacity at their corresponding minimum SNE [Edelman et al. 2007; Varian 2007], then the “value of capacity” is defined as $\frac{R-R_0}{R_0}$ i.e. the relative gain in the revenue of auctioneer per impression.

OBSERVATION C.2. For a given L , if $\exists l \leq K$ such that

$$\eta > 1 - \left(\frac{(\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1})j s_{j+1}}{\sum_{j=l}^K (\gamma_j - \gamma_{j+1})j s_{j+1}} \right) \text{ where}$$

$$\eta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}},$$

then the value of capacity is positive, i.e., revenue of the auctioneer increases by adding capacity.

PROOF. Let $\eta = \min_{l \leq j \leq K} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}}$ then we have $\tilde{\gamma}_j - \tilde{\gamma}_{j+1} \geq \eta(\gamma_j - \gamma_{j+1})$ for $l \leq j \leq K$. At their corresponding minimum SNE [Edelman et al. 2007; Varian 2007], the original revenue of the auctioneer without added capacity and the new revenue of the auctioneer after adding capacity are $R_0 = \sum_{j=1}^K (\gamma_j - \gamma_{j+1})j s_{j+1}$ and $R = \sum_{j=1}^K (\tilde{\gamma}_j -$

easy loans - Google Search http://www.google.com/search?hl=en&q=easy+loans&btnG=Google...

Web Images Video News Maps Gmail more

Google **easy loans** Search

Results 1 - 10 of about 7,210,000 for easy loans. (0.16 seconds)

Web

Easy Loans Sponsored Links
www.Mycashnow.com Overnight Cash Advance Loans Up To \$1000.
 No Credit Check. Act Now!
www.personal-money-loans.com Get a Loan regardless of Credit. Easy Application - Instant Approval
www.Need100.com Bad Credit Personal Loans Fast Personal Loans for Bad Credit. No Credit Check - Fast Approval

Easy Loans
 Easy loans are provided by Myspaydayloan - no faxing, quick application, fast approval. www.mypaydayloan.com/payday-advance/easy-loans.htm - 12k - Cached - Similar pages

Non-Conforming Loans, Personal Loans and Home Loans from Home Loans
 Home Loans - Personal, home, business, investment, non-conforming loans - Easy Loans Australia - Wholesale Loans. www.easy-loans.com.au/ - 23k - Cached - Similar pages

LaSalle Bank: LaSalle Bank's Quick and Easy Loan Application
 Your loan application should take approximately 5 to 10 minutes to complete and will require you to provide the following information about yourself and ... www.lasallebank.com/loans/loan_app.html - 20k - Cached - Similar pages

No Limit Loans - Fast, Easy Confidential Loans from \$100.00 to \$10,000.00
 No Limit Loans - Fast, Easy Confidential Loans from \$100.00 to \$10,000.00. www.no-limit-loans.com/ - 10k - Cached - Similar pages

America First Credit Union :: Personal Car Loan, Special Finance
 America First Credit Union :: Personal Car Loan, Special Finance. www.americafirst.com/loans/personal_car_loan.cfm - 14k - Cached - Similar pages

Easy Loans: Cash Advance
 Cash advances were born out of the need many people have who need an easy, quick short-term loan. Many times getting a cash advance may be the only way that ... www.adfreeshiprovider.com/ - 11k - Cached - Similar pages

Loans | Cheap Secured Loans Online | Quick Easy UK Loan Quotes
 Apply for cheap UK loans online, secured and unsecured personal loans. good or bad credit. www.allaboutloans.co.uk/ - 21k - Cached - Similar pages

BBB Alerts & News
 Consumers to the Better Business Bureau state that they responded to enticing newspaper and web site advertisements promising easy credit and easy loans. ...

Personal Loans
 Borrow up to \$25,000. As seen on Good Morning America. Join today. www.prosper.com California

Quick Cash Loan \$100-5000
 20 sec. pre-approval online. www.fast-cash-personal-loans.com California

California Mortgage
 \$200,000 for \$938/month! When Banks Compete, You Win! www.lendingtree.com California

\$1,500 Cash Loans
 Get \$1,500 Quick Payday Loan. www.CityCashLoans.com/CashLoans

Quick Loans
 Easy Repayment Based on VCMC Sales. www.AmericanCapitalAdvance.com

\$1000 Payday Cash Loans
 Up to \$1000 Payday Loans in Seconds. www.PersonalLoansAll.com California

Loans for anyone
 Home ownership not required. www.loanfinders.com

[More Sponsored Links](#)

1 of 2 9/5/07 9:44 PM

Fig. 2. For-Profit Mediator: Shaded links are the ads (the primary slots), and the doubly shaded link is the ad of the mediator personalloans.com

The screenshot shows the homepage of personalloans.com. The main header features the site name 'personal.com loans' and the tagline 'YOUR FINANCIAL SOLUTION'. Below this is a navigation menu with links for Home, Consolidate Loans, Conventional Loans, Easy Loans, and Home Equity Line Credit Loan. The main content area is titled 'Personal Loans' and contains an article with a photo of a smiling man. The article text discusses secured and unsecured loans, interest rates, and the importance of credit history. To the right of the article is a purple sidebar with several promotional banners for 'Easy Loans' and 'Easy Loan - No Credit Checks'. At the bottom of the page, there is a section titled 'Additional Articles' with a grid of links to various financial topics.

Personal Loans

There are two kinds of personal loans, secured and unsecured. Secured loans are backed by assets or collateral such as an automobile, a home or property. Unsecured loans are for larger amounts than secured loans. Secured loans risk with the presence of collateral. Because of the lowered risk they generally have lower interest rates. Secured loans are best for borrowing large amounts, people with bad or imperfect credit history and those that want longer repayment periods.

A higher credit score will give you a lower interest rate. Obtain a copy of your credit report from any of the major credit reporting agencies and review it for errors. Correct any errors and make sure all your bills are current, this will save you money. Lenders will use your FICO to determine your eligibility and your interest rate.

Unsecured loans do not require collateral; they are normally for less than secured loans. The upper borrowing limit is usually \$10,000. Secured loans are cash advances, payday loans and revolving lines of credit. Unsecured loans can be used for debt consolidation, unexpected expenses, vacations, home repairs, and more. Unsecured loans are normally used by people who do not own a home or property or homeowner who does not wish to pledge their home or property.

Requiring less paperwork than other loans, you can usually apply for an unsecured loan online with as little as your credit score and history, debt information and your earning history. Secured loans are normally used for large purchases. The money can be available to you in as little as 24 hours.

Sponsored Results for Easy Loans

Short Term Loans to \$2000
 Apply Today and Take Advantage of Citibank's Low 5.99% Intro Rate.
www.thinkcash.com

Easy Home Equity Loan
 Apply Today and Take Advantage of Citibank's Low 5.99% Intro Rate.
MyHomeEquity.com

Easy Loan
 Up to 5 Free Quotes with 1 Form. Refi or Home Equity. Intro Terms.
www.NextEq.com/mortgages

Up to \$1,500 Cash Loans
 Instant approval. No credit checks. Get up to \$1,500 cash with no credit checks.
www.cashwest.com

Easy Loan - No Credit Checks
 Up to \$1,500 cash with no credit checks. Earn an extra \$100 with our referral program.
www.paydaymax.com

[Next>>](#)

Additional Articles

- Credit
- Credit Building
- Credit Reports
- Credit Scores
- Credit Lines
- Debt Consolidation
- Debt
- Home Equity Loans
- Auto Loans
- Personal Loans
- Consolidative Student Loans
- Payday Loans
- Refinancing Loans

Fig. 3. Secondary slots at personalloans.com

888 number - Google Search http://www.google.com/search?hl=en&q=888++number&b...

Web Images Maps News Products Gmail more

888 number Search

Results 1 - 10 of about 2,910,000 for 888 number. (0.17 seconds)

<p>888 Number Smart800now.com Boost Your Business: Get Toll-Free Calls Anywhere for Just 4.9¢/min. Buy 1-800 Number 2.9¢ Search now for a custom 800 number. Same day, 7 day no-risk trial 888 Number www.phonepeople.com 888 Number www.frogcentral.com Sign Up For Free Trial! Rated "Best For Toll Free Numbers." Easy Setup.</p>	<p>888 Business Numbers 888 Numbers, 800 Numbers, Toll Free Numbers, VMail Web Access, Email delivery. More www.FreedomVOICE.com 888 number Advanced 800 numbers with follow me forwarding. Low cost. Instant setup Freedom800.com 888 number Search The National Database & Choose Your Own Vanity Number. www.kallio.com 1(888) Phone Services Order Toll-Free 1-888 Phone Service www.888NumbersOnline.com Toll Free Phone Numbers 1-800-888-8888 www.callit.com Your Own Toll Free Number Make Your Small Business Sound Big \$9.95/Month with Instant Activation www.GoVMail.com 888 Number 60 mins Free Trial. Risk Free. Activate Today. No Contract. www.Call800Fax.com/1-888-Toll-Free 888 Number Find Business Toll Free Solutions. www.Distinct.com</p>
---	---

[More Sponsored Links »](#)

TollFreeNumbers.com Ebay! You may have seen toll free numbers listed on ebay. There have been over 1.2 million dollars worth of toll free numbers ...
[www.tollfreenumbers.com/ - 28k - Cached - Similar pages](#)

888 - Number of Jesus If you want to decrease returns by as much as 50%, use an 800/888 number on product literature. This encourages customers to call in and resolve ...
[www.800info2.htm - 19k - Cached - Similar pages](#)
[\[More results from inter800.com \]](#)

800/888 Information
 If you want to decrease returns by as much as 50%, use an 800/888 number on product literature. This encourages customers to call in and resolve ...
[www.800info2.htm - 19k - Cached - Similar pages](#)

Internet 800 Directory - Directory of free toll free, tollfree, ...
 The Number 888 is a most symmetrical Number - it can be reflected, inverted, and has its digits permuted - all with no change in its appearance.
[www.bibleverseel.com/gr/GR_888.asp - 35k - Cached - Similar pages](#)
[www.bibleverseel.com/gr/GR_888.asp - 35k - Cached - Similar pages](#)

Toll Free Phone Numbers Directory 800 888 Reverse Lookup
 The Number 888 is a most symmetrical Number - it can be reflected, inverted, and has its digits permuted - all with no change in its appearance.
[www.tollfreephones.com/ 14k - Cached - Similar pages](#)

Internet 800 Directory - Directory of free toll free, tollfree, ...
 The Number 888 is a most symmetrical Number - it can be reflected, inverted, and has its digits permuted - all with no change in its appearance.
[www.tollfreephones.com/ 14k - Cached - Similar pages](#)

800/888 Information
 If you want to decrease returns by as much as 50%, use an 800/888 number on product literature. This encourages customers to call in and resolve ...
[www.800info2.htm - 19k - Cached - Similar pages](#)
[\[More results from inter800.com \]](#)

TollFreeNumbers.com Ebay! You may have seen toll free numbers listed on ebay. There have been over 1.2 million dollars worth of toll free numbers ...
[www.tollfreenumbers.com/ - 28k - Cached - Similar pages](#)

888 - Number of Jesus If you want to decrease returns by as much as 50%, use an 800/888 number on product literature. This encourages customers to call in and resolve ...
[www.800info2.htm - 19k - Cached - Similar pages](#)

888 Voicemail Numbers 888 phone service with fax, voicemail, email notification.

Fig. 4. For-Profit Mediator: business.com

Toll Free Numbers: 800 Numbers and 800 Number Services... <http://www.business.com/directory/telecommunications/bus...>

/// BUSINESS.COM

Home | Directory | Business Solutions | Business Tools | Business 101 | Business 102 | Business 103 | Business 104 | Business 105 | Business 106 | Business 107 | Business 108 | Business 109 | Business 110 | Business 111 | Business 112 | Business 113 | Business 114 | Business 115 | Business 116 | Business 117 | Business 118 | Business 119 | Business 120 | Business 121 | Business 122 | Business 123 | Business 124 | Business 125 | Business 126 | Business 127 | Business 128 | Business 129 | Business 130 | Business 131 | Business 132 | Business 133 | Business 134 | Business 135 | Business 136 | Business 137 | Business 138 | Business 139 | Business 140 | Business 141 | Business 142 | Business 143 | Business 144 | Business 145 | Business 146 | Business 147 | Business 148 | Business 149 | Business 150 | Business 151 | Business 152 | Business 153 | Business 154 | Business 155 | Business 156 | Business 157 | Business 158 | Business 159 | Business 160 | Business 161 | Business 162 | Business 163 | Business 164 | Business 165 | Business 166 | Business 167 | Business 168 | Business 169 | Business 170 | Business 171 | Business 172 | Business 173 | Business 174 | Business 175 | Business 176 | Business 177 | Business 178 | Business 179 | Business 180 | Business 181 | Business 182 | Business 183 | Business 184 | Business 185 | Business 186 | Business 187 | Business 188 | Business 189 | Business 190 | Business 191 | Business 192 | Business 193 | Business 194 | Business 195 | Business 196 | Business 197 | Business 198 | Business 199 | Business 200 | Business 201 | Business 202 | Business 203 | Business 204 | Business 205 | Business 206 | Business 207 | Business 208 | Business 209 | Business 210 | Business 211 | Business 212 | Business 213 | Business 214 | Business 215 | Business 216 | Business 217 | Business 218 | Business 219 | Business 220 | Business 221 | Business 222 | Business 223 | Business 224 | Business 225 | Business 226 | Business 227 | Business 228 | Business 229 | Business 230 | Business 231 | Business 232 | Business 233 | Business 234 | Business 235 | Business 236 | Business 237 | Business 238 | Business 239 | Business 240 | Business 241 | Business 242 | Business 243 | Business 244 | Business 245 | Business 246 | Business 247 | Business 248 | Business 249 | Business 250 | Business 251 | Business 252 | Business 253 | Business 254 | Business 255 | Business 256 | Business 257 | Business 258 | Business 259 | Business 260 | Business 261 | Business 262 | Business 263 | Business 264 | Business 265 | Business 266 | Business 267 | Business 268 | Business 269 | Business 270 | Business 271 | Business 272 | Business 273 | Business 274 | Business 275 | Business 276 | Business 277 | Business 278 | Business 279 | Business 280 | Business 281 | Business 282 | Business 283 | Business 284 | Business 285 | Business 286 | Business 287 | Business 288 | Business 289 | Business 290 | Business 291 | Business 292 | Business 293 | Business 294 | Business 295 | Business 296 | Business 297 | Business 298 | Business 299 | Business 300 | Business 301 | Business 302 | Business 303 | Business 304 | Business 305 | Business 306 | Business 307 | Business 308 | Business 309 | Business 310 | Business 311 | Business 312 | Business 313 | Business 314 | Business 315 | Business 316 | Business 317 | Business 318 | Business 319 | Business 320 | Business 321 | Business 322 | Business 323 | Business 324 | Business 325 | Business 326 | Business 327 | Business 328 | Business 329 | Business 330 | Business 331 | Business 332 | Business 333 | Business 334 | Business 335 | Business 336 | Business 337 | Business 338 | Business 339 | Business 340 | Business 341 | Business 342 | Business 343 | Business 344 | Business 345 | Business 346 | Business 347 | Business 348 | Business 349 | Business 350 | Business 351 | Business 352 | Business 353 | Business 354 | Business 355 | Business 356 | Business 357 | Business 358 | Business 359 | Business 360 | Business 361 | Business 362 | Business 363 | Business 364 | Business 365 | Business 366 | Business 367 | Business 368 | Business 369 | Business 370 | Business 371 | Business 372 | Business 373 | Business 374 | Business 375 | Business 376 | Business 377 | Business 378 | Business 379 | Business 380 | Business 381 | Business 382 | Business 383 | Business 384 | Business 385 | Business 386 | Business 387 | Business 388 | Business 389 | Business 390 | Business 391 | Business 392 | Business 393 | Business 394 | Business 395 | Business 396 | Business 397 | Business 398 | Business 399 | Business 400 | Business 401 | Business 402 | Business 403 | Business 404 | Business 405 | Business 406 | Business 407 | Business 408 | Business 409 | Business 410 | Business 411 | Business 412 | Business 413 | Business 414 | Business 415 | Business 416 | Business 417 | Business 418 | Business 419 | Business 420 | Business 421 | Business 422 | Business 423 | Business 424 | Business 425 | Business 426 | Business 427 | Business 428 | Business 429 | Business 430 | Business 431 | Business 432 | Business 433 | Business 434 | Business 435 | Business 436 | Business 437 | Business 438 | Business 439 | Business 440 | Business 441 | Business 442 | Business 443 | Business 444 | Business 445 | Business 446 | Business 447 | Business 448 | Business 449 | Business 450 | Business 451 | Business 452 | Business 453 | Business 454 | Business 455 | Business 456 | Business 457 | Business 458 | Business 459 | Business 460 | Business 461 | Business 462 | Business 463 | Business 464 | Business 465 | Business 466 | Business 467 | Business 468 | Business 469 | Business 470 | Business 471 | Business 472 | Business 473 | Business 474 | Business 475 | Business 476 | Business 477 | Business 478 | Business 479 | Business 480 | Business 481 | Business 482 | Business 483 | Business 484 | Business 485 | Business 486 | Business 487 | Business 488 | Business 489 | Business 490 | Business 491 | Business 492 | Business 493 | Business 494 | Business 495 | Business 496 | Business 497 | Business 498 | Business 499 | Business 500 | Business 501 | Business 502 | Business 503 | Business 504 | Business 505 | Business 506 | Business 507 | Business 508 | Business 509 | Business 510 | Business 511 | Business 512 | Business 513 | Business 514 | Business 515 | Business 516 | Business 517 | Business 518 | Business 519 | Business 520 | Business 521 | Business 522 | Business 523 | Business 524 | Business 525 | Business 526 | Business 527 | Business 528 | Business 529 | Business 530 | Business 531 | Business 532 | Business 533 | Business 534 | Business 535 | Business 536 | Business 537 | Business 538 | Business 539 | Business 540 | Business 541 | Business 542 | Business 543 | Business 544 | Business 545 | Business 546 | Business 547 | Business 548 | Business 549 | Business 550 | Business 551 | Business 552 | Business 553 | Business 554 | Business 555 | Business 556 | Business 557 | Business 558 | Business 559 | Business 560 | Business 561 | Business 562 | Business 563 | Business 564 | Business 565 | Business 566 | Business 567 | Business 568 | Business 569 | Business 570 | Business 571 | Business 572 | Business 573 | Business 574 | Business 575 | Business 576 | Business 577 | Business 578 | Business 579 | Business 580 | Business 581 | Business 582 | Business 583 | Business 584 | Business 585 | Business 586 | Business 587 | Business 588 | Business 589 | Business 590 | Business 591 | Business 592 | Business 593 | Business 594 | Business 595 | Business 596 | Business 597 | Business 598 | Business 599 | Business 600 | Business 601 | Business 602 | Business 603 | Business 604 | Business 605 | Business 606 | Business 607 | Business 608 | Business 609 | Business 610 | Business 611 | Business 612 | Business 613 | Business 614 | Business 615 | Business 616 | Business 617 | Business 618 | Business 619 | Business 620 | Business 621 | Business 622 | Business 623 | Business 624 | Business 625 | Business 626 | Business 627 | Business 628 | Business 629 | Business 630 | Business 631 | Business 632 | Business 633 | Business 634 | Business 635 | Business 636 | Business 637 | Business 638 | Business 639 | Business 640 | Business 641 | Business 642 | Business 643 | Business 644 | Business 645 | Business 646 | Business 647 | Business 648 | Business 649 | Business 650 | Business 651 | Business 652 | Business 653 | Business 654 | Business 655 | Business 656 | Business 657 | Business 658 | Business 659 | Business 660 | Business 661 | Business 662 | Business 663 | Business 664 | Business 665 | Business 666 | Business 667 | Business 668 | Business 669 | Business 670 | Business 671 | Business 672 | Business 673 | Business 674 | Business 675 | Business 676 | Business 677 | Business 678 | Business 679 | Business 680 | Business 681 | Business 682 | Business 683 | Business 684 | Business 685 | Business 686 | Business 687 | Business 688 | Business 689 | Business 690 | Business 691 | Business 692 | Business 693 | Business 694 | Business 695 | Business 696 | Business 697 | Business 698 | Business 699 | Business 700 | Business 701 | Business 702 | Business 703 | Business 704 | Business 705 | Business 706 | Business 707 | Business 708 | Business 709 | Business 710 | Business 711 | Business 712 | Business 713 | Business 714 | Business 715 | Business 716 | Business 717 | Business 718 | Business 719 | Business 720 | Business 721 | Business 722 | Business 723 | Business 724 | Business 725 | Business 726 | Business 727 | Business 728 | Business 729 | Business 730 | Business 731 | Business 732 | Business 733 | Business 734 | Business 735 | Business 736 | Business 737 | Business 738 | Business 739 | Business 740 | Business 741 | Business 742 | Business 743 | Business 744 | Business 745 | Business 746 | Business 747 | Business 748 | Business 749 | Business 750 | Business 751 | Business 752 | Business 753 | Business 754 | Business 755 | Business 756 | Business 757 | Business 758 | Business 759 | Business 760 | Business 761 | Business 762 | Business 763 | Business 764 | Business 765 | Business 766 | Business 767 | Business 768 | Business 769 | Business 770 | Business 771 | Business 772 | Business 773 | Business 774 | Business 775 | Business 776 | Business 777 | Business 778 | Business 779 | Business 780 | Business 781 | Business 782 | Business 783 | Business 784 | Business 785 | Business 786 | Business 787 | Business 788 | Business 789 | Business 790 | Business 791 | Business 792 | Business 793 | Business 794 | Business 795 | Business 796 | Business 797 | Business 798 | Business 799 | Business 800 | Business 801 | Business 802 | Business 803 | Business 804 | Business 805 | Business 806 | Business 807 | Business 808 | Business 809 | Business 810 | Business 811 | Business 812 | Business 813 | Business 814 | Business 815 | Business 816 | Business 817 | Business 818 | Business 819 | Business 820 | Business 821 | Business 822 | Business 823 | Business 824 | Business 825 | Business 826 | Business 827 | Business 828 | Business 829 | Business 830 | Business 831 | Business 832 | Business 833 | Business 834 | Business 835 | Business 836 | Business 837 | Business 838 | Business 839 | Business 840 | Business 841 | Business 842 | Business 843 | Business 844 | Business 845 | Business 846 | Business 847 | Business 848 | Business 849 | Business 850 | Business 851 | Business 852 | Business 853 | Business 854 | Business 855 | Business 856 | Business 857 | Business 858 | Business 859 | Business 860 | Business 861 | Business 862 | Business 863 | Business 864 | Business 865 | Business 866 | Business 867 | Business 868 | Business 869 | Business 870 | Business 871 | Business 872 | Business 873 | Business 874 | Business 875 | Business 876 | Business 877 | Business 878 | Business 879 | Business 880 | Business 881 | Business 882 | Business 883 | Business 884 | Business 885 | Business 886 | Business 887 | Business 888 | Business 889 | Business 890 | Business 891 | Business 892 | Business 893 | Business 894 | Business 895 | Business 896 | Business 897 | Business 898 | Business 899 | Business 900 | Business 901 | Business 902 | Business 903 | Business 904 | Business 905 | Business 906 | Business 907 | Business 908 | Business 909 | Business 910 | Business 911 | Business 912 | Business 913 | Business 914 | Business 915 | Business 916 | Business 917 | Business 918 | Business 919 | Business 920 | Business 921 | Business 922 | Business 923 | Business 924 | Business 925 | Business 926 | Business 927 | Business 928 | Business 929 | Business 930 | Business 931 | Business 932 | Business 933 | Business 934 | Business 935 | Business 936 | Business 937 | Business 938 | Business 939 | Business 940 | Business 941 | Business 942 | Business 943 | Business 944 | Business 945 | Business 946 | Business 947 | Business 948 | Business 949 | Business 950 | Business 951 | Business 952 | Business 953 | Business 954 | Business 955 | Business 956 | Business 957 | Business 958 | Business 959 | Business 960 | Business 961 | Business 962 | Business 963 | Business 964 | Business 965 | Business 966 | Business 967 | Business 968 | Business 969 | Business 970 | Business 971 | Business 972 | Business 973 | Business 974 | Business 975 | Business 976 | Business 977 | Business 978 | Business 979 | Business 980 | Business 981 | Business 982 | Business 983 | Business 984 | Business 985 | Business 986 | Business 987 | Business 988 | Business 989 | Business 990 | Business 991 | Business 992 | Business 993 | Business 994 | Business 995 | Business 996 | Business 997 | Business 998 | Business 999 | Business 1000

Toll Free Numbers
Review vendors of toll free phone numbers (800 numbers), review providers for toll free numbers and find an 800 number before you buy. You'll find the best toll free number for your business needs.

Toll Free 800 Numbers - Kallia
Kallia offers toll free numbers from Vanity Number, Free call forwarding, Voice Mail, and Web Access. Instant online setup!
Vanity 800 Number | Instant Setup | Special Number Now
www.kallia.com

Business Toll-Free Number
Business toll-free numbers, call forwarding and many other business features. No extra charge for forwarded calls. Free trial.
www.infglobal.com

New Toll Free and Worldwide Local Numbers Forwarded Anywhere
Get a toll free or local number for your business that delivers calls to any phone, anywhere.
www.tollfreeforwarding.com

Get a Toll Free Number
Month to month, custom numbers. Risk-free 7 day trial. As low as 2.9¢! Very affordable option.
www.infglobal.com

Get a Toll Free Number
Instant online setup. Money saving features. Easy setup. Free trial.
www.infglobal.com

Vonage Small Business
Don't get charged by old-fashioned phone companies! Risk-free trial.
www.vonage.com

More featured listings
View the complete listing for your business. Limited-time free toll local tax numbers also available.
www.afaxcorp.com

Advanced 800 Toll Free Numbers
Offers advanced toll free numbers and voicemail for small businesses. Multiple 800 numbers. 24/7 support. 24/7 support.
800 Numbers | Call Center & Sales | Manage | Evaluate | Sign Up Now
gotvmail.com

Business Toll Free - 60 Minutes Free - Instant Activation
60 minute free trial. Instant Setup. Boost Your Business. Get 1-800-966-8788 Toll Free. No extra charge for forwarded calls. Free trial.
www.60minutefree.com

Business Toll Free
Providers of business toll free vanity numbers designed to enhance marketing programs to increase the response rate in advertising.
www.business-toll-free.com

Advanced Toll-Free Numbers for Small Business
Advanced toll-free services for small business. Includes multi-line call forwarding, routing and screening, email message delivery, fax and more.
800 Numbers | Corporate Email | 24/7 Customer Support | Sign Up Today
www.800tollfree.com

1-800-Telecom. Business Toll Free
Get 24/7 online same day service to business customers. VOIP. Call for instant service.

You may also be interested in:

- 800 Vanity Numbers
- 877 Number
- Personal 800 Number Service
- Buy 800 Number
- 800 VoiceMail
- 800 Number
- 1-800 Phone Number
- 800 Number Service
- Buy Toll-Free Number
- Buy 800 Numbers

Sponsored Links

Experience Business 24/7™
Manage Your Finances w/ Tools that Save Your Business Time and Money. SmallBusiness.com

Flat Rate Toll Free
Unlimited Calling \$49.99/month. Free Toll Free Number. Sign up now.
www.valveco.com

Business Phone Systems
Get the best business phone system. Request Free Evaluations & Compare.
VendorGuru.com/phoneSystem

Business Toll Free #
Buy your business toll free calls. Anytime. Anywhere. Sign up now.
800Numbers.com

Toll Free Max - 3.0¢/min
Advanced 800 Number Services. Instant Fax - 1-800-966-8788
www.tollfreefax.com

Toll Free Number 24/Min
Toll Free number with at \$5.99/min. Add a Virtual PBX for \$3.99/min.
www.prophonedev.com

Free B2B search marketing
Whitepaper, 2008 B2B Search
Marketing Strategy Guide. Advice
from the experts.
Download
To advertise on Business.com, click here

Refine Your Search

Fig. 5. Secondary slots at business.com

$\tilde{\gamma}_{j+1})j s_{j+1}$ respectively.

$$\begin{aligned}
\therefore R - R_0 &= \sum_{j=1}^K [(\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) - (\gamma_j - \gamma_{j+1})] j s_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1} \\
&= \sum_{j=l-1}^K [(\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) - (\gamma_j - \gamma_{j+1})] j s_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1} \\
&\geq (\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=l}^K (\eta - 1)(\gamma_j - \gamma_{j+1}) j s_{j+1} + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1} \\
&= (\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{\tilde{K}} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1} - (1 - \eta) \sum_{j=l}^K (\gamma_j - \gamma_{j+1}) j s_{j+1}
\end{aligned}$$

and hence follows the observation.

We now provide an example to confirm that the above observation does not give a vacuous sufficient condition and the *value of capacity* can indeed be positive.

Example C.3. Let $l = K$ and geometrically decreasing γ_j 's [Abrams and Ghosh 2007; Feng et al. 2006] i.e. $\gamma_j = r^{j-1}, 1 \leq j \leq K$ for some $r < 1$ and 0 otherwise. Then $\tilde{\gamma}_j = r^{j-1}$ for $1 \leq j \leq K-1$ and $\tilde{\gamma}_j = fr^{j-1}$ for $K \leq j \leq K+L-1$ and 0 otherwise. Also, let $j s_{j+1} \geq (j-1)s_j$ for all $K+1 \leq j \leq K+L-1$ and $(K-1)s_K > K s_{K+1}$. Then, the condition for Observation C.2 is satisfied. Detailed calculations are provided below.

We have

$$\eta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j - \tilde{\gamma}_{j+1}}{\gamma_j - \gamma_{j+1}} = \frac{fr^{K-1}(1-r)}{r^{K-1}} = f(1-r).$$

Now,

$$\begin{aligned}
&(\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1}) j s_{j+1} \\
&\geq (\gamma_K - \tilde{\gamma}_K)(K-1)s_K + K s_{K+1}(\tilde{\gamma}_{K+1} - \tilde{\gamma}_{K+L}) \\
&= r^{K-1}(1-f)(K-1)s_K + fr^K K s_{K+1}.
\end{aligned}$$

Also

$$\sum_{j=l}^K (\gamma_j - \gamma_{j+1}) j s_{j+1} = r^{K-1} K s_{K+1} \quad (\text{as } l = K)$$

$$\begin{aligned}
 \therefore 1 - & \left(\frac{(\gamma_l - \tilde{\gamma}_l)(l-1)s_l + \sum_{j=K+1}^{K+L-1} (\tilde{\gamma}_j - \tilde{\gamma}_{j+1})j s_{j+1}}{\sum_{j=l}^K (\gamma_j - \gamma_{j+1})j s_{j+1}} \right) \\
 & \leq 1 - \frac{r^{K-1}(1-f)(K-1)s_K + fr^K K s_{K+1}}{r^{K-1}K s_{K+1}} \\
 & = 1 - \left(\frac{(K-1)s_K}{K s_{K+1}}(1-f) + fr \right) \\
 & = f - fr + (1-f) - \frac{(K-1)s_K}{K s_{K+1}}(1-f) \\
 & = f(1-r) + (1-f) \left(1 - \frac{(K-1)s_K}{K s_{K+1}} \right) \\
 & < f(1-r) = \eta.
 \end{aligned}$$

OBSERVATION C.4. For a given L , if $\exists l \leq K$ such that

$$\beta > 1 - \frac{\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j}{\sum_{j=l}^K \gamma_j s_j} \quad \text{where } \beta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j}{\gamma_j}, \quad (4)$$

then the efficiency improves.

Proof: We have

$$\begin{aligned}
 E_0 & = \sum_{j=1}^K \gamma_j s_j, \quad E = \sum_{j=1}^{K+L-1} \tilde{\gamma}_j s_j = \sum_{j=1}^{l-1} \gamma_j s_j + \sum_{j=l}^{K+L-1} \tilde{\gamma}_j s_j. \\
 \therefore E - E_0 & = \sum_{j=l}^K (\tilde{\gamma}_j - \gamma_j) s_j + \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j.
 \end{aligned}$$

Let

$$\beta = \min_{K \geq j \geq l} \frac{\tilde{\gamma}_j}{\gamma_j}$$

then

$$E - E_0 \geq \sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j - (1-\beta) \sum_{j=l}^K \gamma_j s_j.$$

and hence follows the observation. \square

We now provide an example to confirm that the above observation does not give a vacuous sufficient condition and the efficiency can indeed improve.

Example C.5. Let $l = K$ and geometrically decreasing γ_j 's as in Example C.3 and let s_i 's satisfy $s_{K+j} = \alpha^j s_K$ for some $\alpha < 1$. Then, the condition for Observation C.4 is satisfied when $f > \left(\frac{1-\alpha r}{1-\alpha^L r^L} \right)$. Detailed calculations are provided below.

$$\beta = \frac{\tilde{\gamma}_K}{\gamma_K} = \frac{fr^{K-1}}{r^{K-1}} = f.$$

Also,

$$\begin{aligned}
\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j &= \sum_{j=K+1}^{K+L-1} f r^{j-1} s_j \\
&= f r^K s_K \sum_{j=1}^{L-1} r^{j-1} \alpha^j = \alpha f r^K s_K \left(\frac{1 - r^{L-1} \alpha^{L-1}}{1 - r\alpha} \right) \\
\therefore 1 - \frac{\sum_{j=K+1}^{K+L-1} \tilde{\gamma}_j s_j}{\sum_{j=1}^K \gamma_j s_j} &= 1 - \frac{\alpha f r^K s_K \left(\frac{1 - r^{L-1} \alpha^{L-1}}{1 - r\alpha} \right)}{r^{K-1} s_K} \\
&= 1 - \alpha f r \left(\frac{1 - r^{L-1} \alpha^{L-1}}{1 - r\alpha} \right) = \frac{1 - r\alpha - \alpha r f + f r^L \alpha^L}{1 - r\alpha} \\
&= \frac{f(1 - r\alpha) + 1 - f - \alpha r + f r^L \alpha^L}{1 - r\alpha} = f + \left(1 - f \frac{1 - r^L \alpha^L}{1 - \alpha r} \right) \\
&< f \text{ if } f > \frac{1 - \alpha r}{1 - r^L \alpha^L}. \quad \square
\end{aligned}$$

ACKNOWLEDGMENTS

SKS thanks NetSeer, Inc. for financial support provided during the course of this work.

REFERENCES

- ABRAMS, Z. AND GHOSH, A. 2007. Auctions with revenue guarantees for sponsored search. *Third workshop on Sponsored Search Auctions*.
- AGGARWAL, G., GOEL, A., AND MOTWANI, R. 2006. Truthful auctions for pricing search keywords. In *EC '06: Proceedings of the 7th ACM conference on Electronic commerce*. ACM Press, New York, NY, USA, 1–7.
- ATHEY, S. AND ELLISON, G. 2007. Position auctions with consumer search. *Working Paper*.
- EDELMAN, B., OSTROVSKY, M., AND SCHWARZ, M. 2007. Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American Economic Review* 97, 1 (March), 242–259.
- FENG, J., BHARGAVA, H. K., AND PENNOCK, D. M. 2006. Implementing sponsored search in web search engines: Computational evaluation of alternative mechanisms. *Infirms Journal on Computing*.
- GUNAWARDANA, A., MEEK, C., AND BIGGS, J. 2008. A quality-based auction for search ad markets with aggregators. *Fourth Workshop on Ad Auctions*.
- KRISHNA, V. 2002. *Auction Theory*. Academic Press.
- LAHAIE, S. 2006. An analysis of alternative slot auction designs for sponsored search. In *EC '06: Proceedings of the 7th ACM conference on Electronic commerce*. ACM Press, New York, NY, USA, 218–227.
- LAHAIE, S. AND PENNOCK, D. M. 2007. Revenue analysis of a family of ranking rules for keyword auctions. In *EC '07: Proceedings of the 8th ACM conference on Electronic commerce*. ACM Press, New York, NY, USA, 50–56.
- SINGH, S. K., ROYCHOWDHURY, V. P., GUNADHI, H., AND REZAEI, B. A. 2007. Capacity constraints and the inevitability of mediators in adword auctions. In *WINE '07: Proceedings of The 3rd International Workshop On Internet And Network Economics*. 318–325.
- VARIAN, H. 2007. Position auctions. *International Journal of Industrial Organization* 25, 6, 1163–1178.
- VICKREY, W. 1961. Counterspeculation, auctions and competitive sealed tenders. *Journal of Finance* 16, 8–37.